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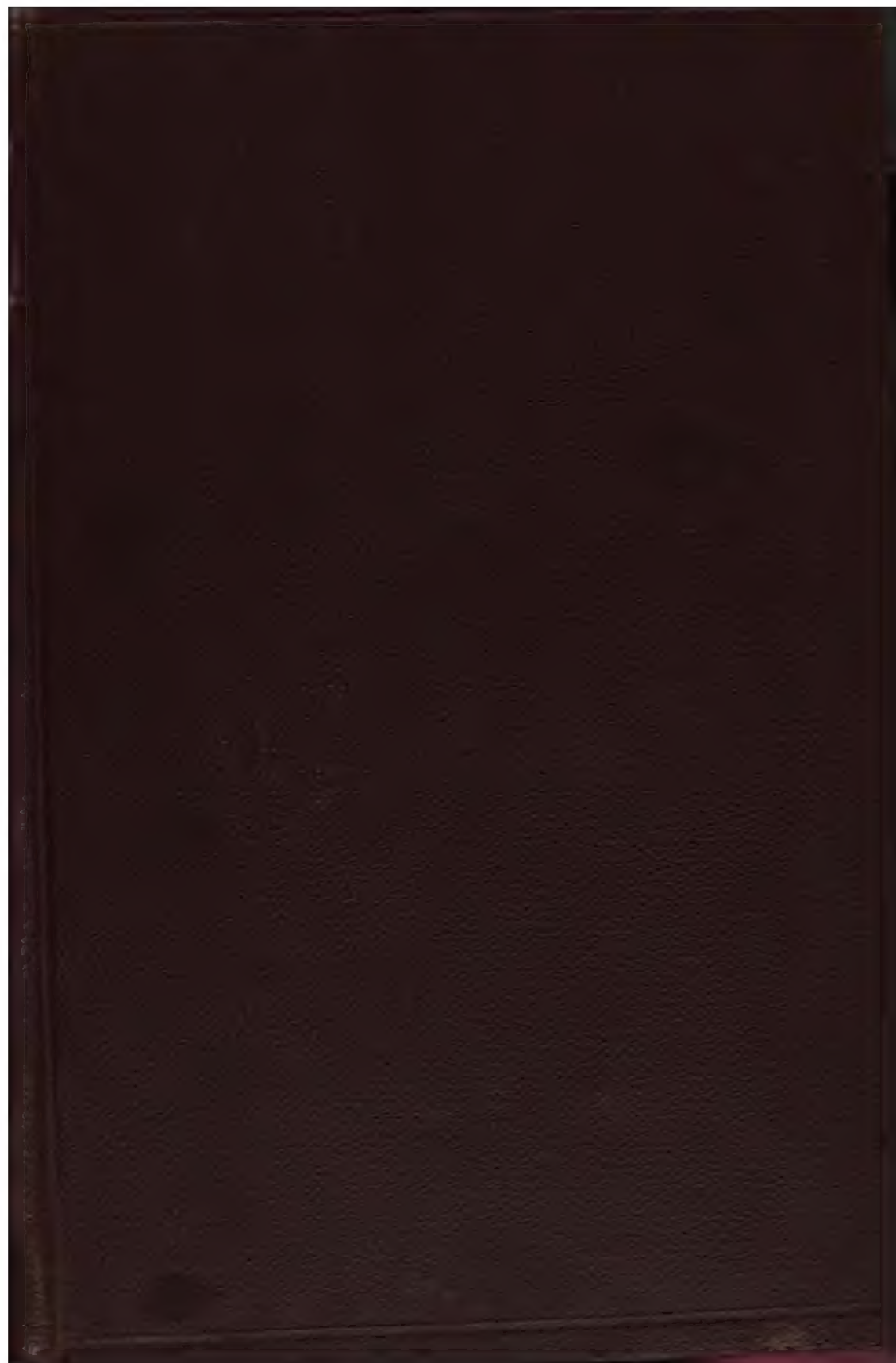
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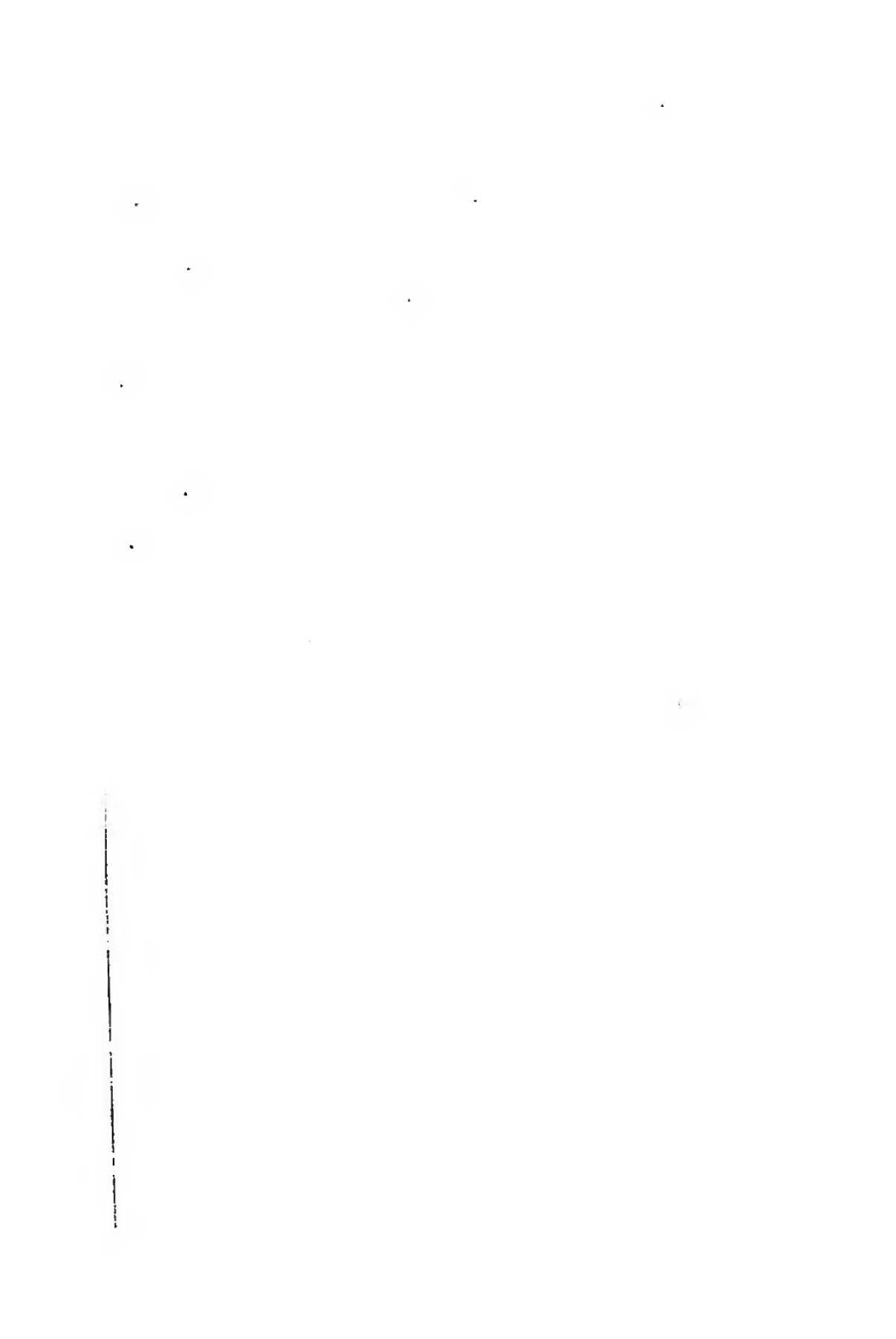
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MARE ISLAND, CALIF.

# EXPERIMENTAL PHYSICS

BY  
EUGENE LOMMEL  
AUTHOR OF "THE NATURE OF LIGHT"

*TRANSLATED FROM THE GERMAN BY*

G. W. MYERS  
OF URBANA, ILLINOIS, U.S.A.

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## PREFACE TO THE FIRST EDITION.

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THE present "Text-book of Experimental Physics," as also the "Lexicon of Physics" (the latter now out of print), have grown out of the writer's lectures. The Text-book undertakes the task of setting forth the fundamental teachings of physics in the light of the present state of knowledge, in such manner as to be generally intelligible and without extended use of mathematical developments. By proceeding always, either from everyday experiences, or from experiments easily performed, facts are everywhere placed in the foreground as the unchanging basis of knowledge. The main text, in coarse print, forms a connected body of principles, for whose understanding only the most elementary mathematical knowledge is required, thus making the book easily accessible to beginners. The subject-matter has been so arranged that an understanding of what is under immediate consideration presupposes nothing save what has been previously discussed. The arrangement of the matter corresponds, therefore, *in a general way*, to the historical order of development of the science. This order has been preferred, since, in the writer's opinion, to avoid the circuitous paths upon which, in special instances, this science has advanced, and to follow the historical route in all other cases, is still the most profitable course to pursue in the development of all its details. Dates have consequently found due consideration throughout the book.

But, to the end that the book may also meet the needs of higher schools and colleges, as well as the initial stages of study, paragraphs in fine print are interspersed, which contain, in closest connection with the main text, the most important mathematical developments in terse and simple form.

Through the instrumentality of a detailed table of contents, of a registry both of names and of things (Index), care has been taken to make the work serve the purposes of a convenient reference-book.

May the author's wish, that the book may be serviceable in teaching, in reviewing the subject, and in self-instruction to the widest circle of readers, be fulfilled.

THE AUTHOR.

MUNICH, Feb., 1893.

## PREFACE TO THE SECOND EDITION.

THIS Second Edition of the "Text-book of Experimental Physics" differs but little from the first; the chief difference being in the purely external circumstance of the introduction into it of the so-called new German orthography now taught in our schools. Some additions for the sake of completeness have been made, in compliance with wishes expressed to me by teachers and investigators, for whose suggestions I am ever grateful.

It would, of course, have been impossible to comply with all such wishes. It has, for example, been a source of surprise to some that Part II., entitled "SOLIDS," follows Part I., on "MOTION (*Mechanics*)," inasmuch as mechanics, so far as it is treated in Part I., refers likewise to solid bodies. Mechanics, however, deals not with *real*, but with *ideal* bodies, which latter are regarded as perfectly rigid, while real bodies suffer change of form when exposed to the action of force. This essential distinction is particularly emphasized in Part I. by the oft-recurring word "rigid," and especially is this done at the beginning of Part II. The captions "Motion" and "Solids" appear to me, therefore, wholly justifiable when used in the order I have followed. They correspond in a general way to the chapters entitled "Mechanics" and "Theory of Elasticity" in theoretical physics.

The demand that a text-book of physics, thoroughly up to date, should necessarily begin with an empirical table of the forms of energy, could for obvious reasons not be complied with in a book for beginners, to whom even the idea of energy

is quite unknown. Its observance would, indeed, have compelled the abandonment of the gradually ascending course of historical development which, in the writer's experience, is the surest way to a vivid understanding and vital appreciation of present attainment. Moreover, this requirement is nevertheless substantially satisfied so far as could be done without surrendering the writer's original aim; for even as early as paragraph 19 (p. 26), the idea of energy, in its all-embracing significance, is introduced, while in the sequel it is reiterated at every opportunity.

Another criticism of the book, that the hypothetical character of the kinetic theory is not set forth with sufficient emphasis, is justified only by the fact that, instead of the word "hypothesis," the German words "annahme" (assumption), "vorstellung" (idea), "anschauung" (view, in common use for this notion have been employed.

As a general thing, pure German words have been employed rather than technical expressions from foreign tongues, *e.g.* "wucht," instead of "energy of motion," or "kinetic energy," a word which immediately conveys the idea to the understanding of every German. In place of "potential" and "difference of potential," the word "spannung" (tension), now generally in vogue in electrotechnica, is frequently used, in the sense of the work of a mass-unit (*e.g.* "electrische spannung" (electric tension) = work per unit of electrical mass). What has heretofore been usually understood as electric tension, I have called electrostatic pressure; for this so-called tension is essentially a pressure, *i.e.* a force per superficial unit.

May this book in its new form be instrumental in promoting the general aims, to the attainment of which the author has sought to be helpful, and find a cordial reception with all to whose service it is dedicated.

THE AUTHOR.

Mexico, June, 1894.

## PREFACE TO THE THIRD EDITION.

---

THIS new edition has undergone but few changes upon the preceding edition, and these few have been made in compliance with wishes expressed to the author. Röntgen's rays have been discussed in the proper place. A plate showing the spectra of the sun, and of several of the elements, has been added, for which the author is indebted to the publishers of the German Edition.

THE AUTHOR.

MUNICH. *March*, 1896.



## PREFACE TO THE ENGLISH EDITION.

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WHILE attending the lectures on Experimental Physics given by Professor von Lommel, at Munich, during the winter semester of '94-'95, the translator became acquainted with the clear and complete exposition of principles contained in the present text-book. The author has pursued the experimental method almost exclusively, throughout the portion of the text given in coarse print. It was in the belief that a fuller acquaintance with the possibilities of this method of imparting instruction, when in the hands of a master of the art, would be as helpful to other teachers of science as it has been to the translator, that the translation of this work was undertaken.

Theoretical considerations have, however, not been ignored. Paragraphs in fine print, containing either further and fuller developments of the subject-matter, or mathematical derivations of principles proved experimentally in the paragraphs they follow, have been inserted in many places. The book is thus adapted to three distinct uses, in American science. The main body of the text is believed to be sufficiently elementary for pupils of the High School grade. Add to this the matter in fine print, and the book is well suited to use as a text in colleges and schools of science. A full index of topics and registry of names at the end render the book valuable also to advanced students for reference purposes. To make the work more readily available for reference to students in astronomy and applied mathematics was not the least reason for translating it into English.

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# EXPERIMENTAL PHYSICS.

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## INTRODUCTION.

1. **Physics—Nature** (Greek, *physis*) is the term used to denote the sum total of everything which can be perceived by the senses. Everything which occupies space and acts upon the senses, is called **matter**. A **body** is a definite portion of space, filled with matter. Changes in the physical world, which, with the course of time, become perceptible to the senses, are **phenomena**.

The word "physics," then, really signifies "Natural Science," or the "Teachings of Nature" in general, and would seem, therefore, from its original signification, to refer to all natural phenomena. In early times, the word physics actually possessed this comprehensive meaning. The great mass and variety of facts, with the progress of time, made it desirable to refer those organic processes which constitute animal and plant life to a special group of sciences—the *biological*—and to treat the rich supply of phenomena, which depend upon a change in the molecular constitution of bodies, in the independent science of *chemistry*. At present, we understand by *physics*, the science of those phenomena in which neither life nor molecular change is essentially involved.

Physics, like the other natural sciences, is an empirical, or experimental science, proceeding from individual truths of experience obtained by observation and experiment. By means of consistent generalizations, it gathers these truths together under a common point of view. In this way it arrives at a knowledge of the laws of nature, each of which expresses, in a purely

empirical manner, the connection of a certain group of phenomena. These laws, then, reveal the *how* but not the *why* of occurrences. The later inquiry into the internal connections of phenomena cannot be answered by experience alone. This answer is to be sought in the promulgation of scientific hypotheses, and in observing whether from these suppositions, the phenomena to be explained are logically deducible. If all inferences from an hypothesis are in accord with the facts, the assumed cause may be considered as possible, and it becomes ever more highly probable the more facts it will explain.

According to the mode of presenting the subject, we distinguish *Experimental Physics*, which takes its teachings directly from experience and illustrates them experimentally (which is the method pursued in the present work), from *Theoretical Physics*, which, from a few propositions and hypotheses of experience, regarded as fundamental, develops its body of doctrines by mere processes of thought, and afterwards proves the agreement of its results with experience. Inasmuch as the latter makes use of mathematics as an indispensable auxiliary to its inferences, it is also called *Mathematical Physics*.

**2. Measure—Units of Measure.**—To follow accurately the course of phenomena, it is necessary to compare the magnitudes, such as space, time, etc., occurring in them, with conventionally chosen homogeneous units of measure. This process is called *measurement*. Space measurements are especially important, and these are all reducible finally to linear measurements.

The meter is to-day almost universally accepted as the unit of length, which, according to original agreement, is the 40,000,000th part of the earth's meridian. This unit of length was adopted in France in 1791, in response to the proposal of an academic commission, in the hope of obtaining a standard of measure directly from nature, and, consequently, not subject to the usual vicissitudes of arbitrary standards. But as the measurements of the meridian then made were shown later to be erroneous, the length of the meter derived from them, and afterwards legally adopted, is not exactly equal to the 40,000,000th part of the meridian. It is shorter than this by

0.0858 of a millimeter. The original purpose of securing a natural standard was, therefore, not attained; and the legal meter is still an arbitrary standard, which, in 1799, was made of platinum by comparison with the final standard which was deposited in the State Archives of Paris as the original. Since 1889, however, it has been represented by a notched scale composed of an alloy of platinum and iridium, consisting of 90 per cent. platinum and 10 per cent. iridium, of the hardness of steel. This scale is deposited and kept in the International Bureau of Weights and Measures in Sèvres, near Paris, as the international prototype of the meter. The true length of the meter is indicated by the distance between two fine lines etched into its surface near its ends, when the temperature of the bar is that of melting ice. The old platinum scale, of rectangular cross section, was too flexible, because too thin. When a scale bends, its convex side becomes somewhat longer and its concave a little shorter; consequently, within the interior of the scale there must be a layer of longitudinal filaments neither lengthened nor shortened by flexure. They constitute the so-called *neutral layer*. For the cross-section of the new scale, the form represented full size in Fig. 1 was chosen. In it, the terminal lines were etched upon the neutral layer, *a b*, which is exposed as the bottom of the upper groove of the scale. This form of section affords the additional advantage, that the scale is better protected against flexure than is the case with most other forms.



FIG. 1 — Cross-section of Prototype of Meter.

The meter (m.) is divided into 10 *decimeters* (dm.), 100 *centimeters* (cm.), 1000 *millimeters* (mm.), and is consequently a decimal standard. The thousandth part of a millimeter is called a *micron* ( $\mu$ ). A length of 1000 m. is called a *kilometer* (km.), and serves as a larger unit for measuring long distances. The most important standards of length, some foreign and some not used at present, are given below:—

Prussian, or Rhine foot	= 318.4 mm.
Bavarian foot	= 291.4 "
Swiss, or Badish foot	= 300.4 "
Foot of Saxony	= 293.2 "

Austrian foot	= 316.1 mm.
Parisian foot	= 324.8 "
English foot	= 304.8 "
German, or geographical mile	= 7.420 km.
English mile	= 1.609 "
Sea mile	= 1.855 "

An important auxiliary to precise linear measurement is the *vernier*. This is a small graduated scale (B, Fig. 2), which moves along a larger one (AA) and makes possible



FIG. 2. - Vernier.

the measurement of smaller parts than can be read directly from the principal scale. If, on the smaller scale, we divide a space equal to nine parts of the larger into ten equal parts,

each of the parts will, of course, equal nine-tenths of one division of the larger scale. To measure a length to the zero of the vernier, we notice which graduation of the vernier (in the figure, the third) most nearly coincides with a graduation of the scale; then the next graduation but one of the vernier is displaced by one-tenth, the next by two-tenths, and the zero graduation by three-tenths, toward the next following graduation of the scale. We have then to add to the length read off in whole parts of the graduated scale up to the zero of the vernier, as many tenths as there are units in the number indicating the graduation of the vernier which coincides with the graduation of the scale. In the example of Fig. 2, the reading is, therefore, 27, 3. This ingenious device is sometimes called a *nonius*, after its alleged inventor, Pedro Nuñez (1542). Its more usual name, *vernier*, comes from that of its true inventor, Pierre Vernier (1631).

For measuring differences of altitude, *e.g.* of columns of fluid, the cathetometer of Dulong and Petit (1818) is used. This instrument consists of a vertical scale, along which a telescope, free to turn in a horizontal plane together with a vernier rigidly fixed to it, may be raised and lowered. The line of sight of the telescope, determined by a reticule, may be directed toward the successive vertical graduations.

The superficial unit is the square meter (sq. m. or  $m^2$ ).

*i.e.* a square whose side is one meter long. A square meter contains 100 sq. dm. ( $\text{dm}^2$ ), 10,000 sq. cm. ( $\text{cm}^2$ ), 1,000,000 sq. mm. ( $\text{mm}^2$ ).

The cubic meter ( $\text{cu.m.}$ ,  $\text{m}^3$ ) is the unit of volume. It is a cube, each of whose edges is 1 m. A smaller volumetric unit is used for measuring fluids; viz. the *cubic decimeter* ( $\text{cu.dm.}$ ,  $\text{dm}^3$ ) or *liter* (l.). It contains 1000 *cubic centimeters* ( $\text{cu.cm.}$ ,  $\text{cm}^3$ ).

Angular measurements are reducible to measurements of the lengths of arcs. The angular unit is the *degree* ( $^\circ$ ), *i.e.* the 90th part of a right angle. The degree is divided into 60 *minutes* ( $'$ ), the minute into 60 *seconds* ( $''$ ). An angle may also be measured by means of the length of the arc which, as a central angle in a circle of unit radius, it intercepts between its sides. The unit arc corresponds to the angle  $57^\circ.296$ , and the semi-circle of  $180^\circ$  corresponds to the arc whose length is  $\pi = 3.1416$  (known as Ludolph's number).

The unit of time in physics is the *second* (sec., second of time), *i.e.* the 86,400th part of the mean solar day—60 seconds make one *minute*; 60 minutes, one *hour*; 24 hours, one *day*.

Other units and instruments for measurement will be mentioned later in their proper places.

## 1. MOTION.

(MECHANICS.)

**3. Motion and Rest.**—A body, or any one of its points, is said to be *in motion*, or *moving*, if, in the course of time, it changes its position in space. When it does not change its position in space, it is at *rest*. We judge of the rest or motion of a body by comparing its position with that of surrounding bodies, which we assume to be in repose.

In case of the motion of a train of cars, for instance, the motion is referred to the surface of the earth, considered at rest, by observing that the train hastens past trees, houses, etc. If the train halts at a station it is at rest, relative to the depôt, or to the earth; but since the earth rotates about its axis, and revolves in its orbit about the sun, the train is not in a state of *real*, or *absolute* rest. It is in what is called a condition of *relative* rest, with respect to the earth, assumed stationary. The locus of the positions which a moving point successively assumes in tracing a line, straight or curved, is called the *path*, or *orbit* of the point. According to the form of the orbit, the motion of the point is said to be either *rectilinear*, or *curvilinear*. In rectilinear orbits the direction of motion is unchanged, and is indicated by the orbit itself. In curvilinear motion, on the contrary, the direction of motion is continually changing. The direction toward which the point is moving at any instant is obviously indicated by the straight line, which touches the curvilinear orbit at the instantaneous position of the moving point. Motion is termed *uniform* when the moving point passes over equal distances in equal times, howsoever small the intervals are taken. It is called *variable*, on the other hand, when unequal distances are traversed in equal times.

**4. Uniform Motion.** The *velocity* of a uniformly moving

point is the distance passed over by it in the unit of time, *i.e.* in the second. Velocity is a "directed magnitude," since it is specified, not alone by its magnitude, but by both its magnitude and its direction conjointly. If we denote the velocity by  $v$ , and the space described in  $t$  seconds by  $s$ , then obviously  $s = vt$ . If two of these three magnitudes,  $s$ ,  $v$  and  $t$  are given, the third may be determined by this equation. If, for illustration,  $s$  and  $t$  are known, we have  $v = \frac{s}{t}$ .

The velocity of uniform motion is, therefore, obtained by dividing the number of linear units traversed, by the number of units of time consumed in traversing them. It is expressed as the ratio of a length to a time. The number which expresses the velocity may for the same motion result differently, according to the choice of the units of length and time, or, what amounts to the same, according to the choice of the unit of velocity. To avoid confusion, therefore, these units must always be indicated. We say, for example, a train has the velocity of  $12 \frac{\text{m.}}{\text{sec.}}$  (meters per second), or  $1200 \frac{\text{cm.}}{\text{sec.}}$ , or  $720 \frac{\text{m.}}{\text{min.}}$

(meters per minute). The notation  $\frac{\text{cm.}}{\text{sec.}}$ , or  $\text{cm.-sec.}^{-1}$ , etc., makes clear at once how the unit of velocity is derived from the fundamental units of length and time.

**5. Gravity.**—Daily experience teaches that every unsupported body falls toward the earth with accelerated motion. Any cause which either produces or changes motion is called *force*. A body is then said to be drawn to earth by a force, which force is immediately perceptible through the muscular exertion necessary to raise a heavy mass from the earth. The body is called *heavy*, and the force which draws it to the earth is called *gravity* (heaviness).

A force is known when we know its *direction* and *magnitude*, or *intensity*. Like velocity, it is a directed magnitude. The direction of gravity is indicated by the direction of a thread on which a heavy body hangs, for the thread can sustain the heavy body only by drawing it upward in the direction exactly opposite to that of gravity. This simple device is called a

*plumb*, and the direction indicated by it, *vertical*, or *perpendicular*. Any plane, perpendicular to the plumb line, as also every line drawn in such a plane, is called *horizontal*.

**6. Weight.**—If the fall of a heavy body is retarded, it exerts, by virtue of its striving to fall, a pressure upon its support, or a pull upon the sustaining cord. This pull or pressure is called the *weight* of the body, and may serve as a measure of the force which draws it to earth. The weight of a body is determined by comparing it, by means of the balance, with the unit of weight. The unit of weight which has been selected as a standard is the *kilogram* (kg.), e.g. the weight of a liter (cu dm.) of pure water at 4° centigrade, or, rather, the weight of a mass of an alloy of platinum and iridium which is deposited as the prototype of the kilogram in the International Bureau of Weights and Measures at Paris, and corresponds with the greatest possible accuracy to the above definition. The thousandth part of a kilogram, i.e. the weight of a cubic centimeter of water, at 4°, is called a *gram* (g.), and is used as a smaller unit of weight. The hundredth part of the gram is called a *centigram* (cg.); the thousandth part, a *milligram* (mg.). Force makes itself manifest as a pressure or a pull. Pressure and pull may always be produced in any desired magnitude and direction through the instrumentality of rollers and pulleys by means of weights. The magnitude of any force, therefore, can be measured in units of weight, or may be expressed as a certain number of g. or kg. In drawings, forces are represented geometrically by straight lines. The direction of the line indicates the direction of force. The number of its linear units indicates the number of units of weight, consequently, the magnitude of the force of gravity.

**7. Uniformly Accelerated Motion—Atwood's Machine.**—The motion of a freely falling body, after a few seconds, becomes so swift as to make it impossible to follow its course with accuracy. Even Galileo, who discovered the laws of falling bodies in 1602, attempted to retard this motion without changing its character by observing the fall of bodies along an inclined plane. A much more appropriate apparatus for accomplishing this, and which may be used in a multitude

of variations of this experiment, is known as Atwood's fall machine (1784).

This (v. Fig. 3) carries upon a column, about 2 m. high, a light wheel, easily movable about its axis, with its periphery hollowed out to receive a slender thread. At each end of the thread equal weights,  $P$ , are attached, each of which, striving to fall, tends to turn the wheel toward its own side; but, this tendency being the same in both directions, no rotation of the wheel, and hence no fall of the weights, can occur. If, however, a small overweight,  $p$ , be placed upon the front weight, an actual fall toward this side then occurs. The moving force is here merely the overweight. If the overweight were permitted to fall freely, this force would have only the mass of the overweight itself to set in motion; but here the same force has not only to move the overweight  $p$ , it must also take with it the two equal weights  $P$ , and the motion is correspondingly reduced. At the side of the upright column of the machine is a pendulum,  $r$ , beating seconds, which, with its first beat, releases a hinged plate situated above at the zero of a scale graduated to centimeters. This plate supports the weight to which the overweight has been added. When the plate is released, this weight begins to sink, and strikes audibly upon a horizontal plate of metal which slides along the column, and may be adjusted to any desired height. The space described in the first second, i.e. where the plate must be clamped in order that the falling weight may strike it precisely at the second beat of the pendulum, or after the lapse of a single second, is determined experimentally. Likewise, the spaces described after the lapse of 2, 3, 4, etc., seconds are determined, and the results are as follows:—

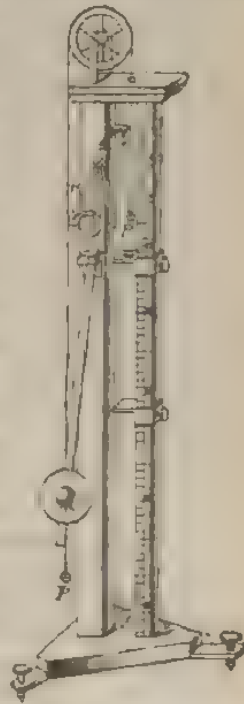


FIG. 3.—Atwood's Machine

The space in two sec. is 4 times as great as in the 1st sec.

" three " 9 " "

" four " 16 " "

etc. It is seen that the spaces described are to each other as the squares of the times employed in their description.

Since the space described by a falling body in the first two seconds is four times as great as in the first, in the second second the body describes a distance three times as great as in the first. Similarly, in the third second, the space traversed is five times; in the fourth, seven times as great as that described in the first second. Therefore, *the spaces described by a falling body in the individual seconds are to each other as the odd numbers 1, 3, 5, 7, 9, etc.*

The motion of falling bodies is accordingly *variable*. It is, moreover, *accelerated*, because in each succeeding second a larger distance is traversed than in the preceding, i.e. its velocity increases uniformly and incessantly.

The question now arises, what is to be understood by velocity in *varied motion*? In case of uniform motion we say the velocity is the distance traversed per second. This definition has no meaning here, for the velocity changes continually. This change, however, can be accomplished only through the instrumentality of the force of the overweight acting incessantly downward, changing the body's condition from that of rest to motion, and then accelerating the body's motion continuously. If, at any instant, the cause of the acceleration, i.e. the overweight, were removed, from that instant the velocity could no longer increase, but would conserve unchanged the value it possessed at *that instant*. We may, therefore, define the velocity which a body in uniform motion has at any instant by the distance which it would traverse uniformly in each succeeding second, if at the instant in question the cause of the change of motion were removed. The correctness of these assertions may be easily proved by means of Atwood's machine. We attach to the column a ring which permits the passage of the sinking weight, but does not allow the overweight land upon its upper edge to pass, and notice carefully that after the removal of this overweight no

further change of velocity occurs, but that, from this instant on, the weight moves uniformly. If now we clamp the ring firmly at the end of the distance traversed during the first second, as found above, so that the overweight is lifted off after the first second of fall, we find that the weight now moving alone strikes the plate with the following beat of the pendulum, provided this plate is clamped below the ring at a distance equal to double the space described the first second. Moreover, in each following second the body traverses double the space fallen the first second, as is at once apparent if the plate is clamped at twice, thrice, etc., this distance below the ring and the striking of the weight after two, three, etc., seconds is noted. Hence *the velocity at the end of the first second is double the space described during the first second.* If we clamp the ring at the end of the space described in two seconds, then at the end of the space described during the first three seconds, etc., and ascertain each time the position which must be given to the plate that it may be struck at the next succeeding beat of the pendulum, we find—

The velocity after 2 sec. is 2 times as great as after 1 sec.

"	3	3	"	"
"	4	4	"	"

etc. We see, then, that the velocities are to each other as the times, or *the velocity is proportional to the time of fall.*

The velocity of a falling body, therefore, increases in each second by double the distance described the first second. This increase of velocity per second is called *acceleration*, and since it is the same for every second, motion of this sort is termed *uniformly accelerated*.

**8. Laws of Uniformly Accelerated Motion.**—These laws may be clearly and concisely expressed in mathematical symbols. Denoting the acceleration by  $a$ , the velocity  $v$  after  $t$  seconds is  $at$  or

$$(1.) \quad v = at.$$

The space described the first second is then  $\frac{1}{2}a$ ; that after  $t$  seconds have elapsed, is  $tt$  or  $t^2$  times as great, consequently, it is  $\frac{1}{2}at^2$ , and we have

$$(2.) \quad s = \frac{1}{2}at^2.$$

By means of these two equations, the first of which gives the velocity, and the second, the distance traversed at any time,  $t$ , all circumstances of uniformly accelerated motion are adequately described, and with their aid every question respecting this motion may be answered. If, for example, the velocity,  $v$ , be desired, which a body possesses after having traversed the distance  $s$  with the uniform acceleration,  $a$ ; the time consumed would be given by equation (2),  $t = \sqrt{\frac{2s}{a}}$ , and according to equation (1), this must merely be multiplied by  $a$ , to obtain the desired final velocity,  $v = \sqrt{2as}$ , or better—

$$(3) \quad v^2 = 2as.$$

That is, we may say the square of the velocity is every instant equal to the double product of the acceleration and distance traversed, and consequently, the velocity itself equals the square root of this product.

**9. Velocity and Acceleration with Varied Motion.**—The change of velocity in varied motion is obviously smaller, the smaller the element of time during which the motion is considered. If this element of time be conceived to become smaller and smaller, the motion approximates ever more nearly to uniform motion, and the velocity is then expressed by the ratio  $\frac{\Delta s}{\Delta t}$ , where  $\Delta s$  denotes the small distance traversed in the short increment of time  $\Delta t$ . Instead of the above definition of velocity of varied motion, we may then give the following general definition. By the velocity which the moving point possesses at any instant, is meant the *limiting value* to which the ratio of the distance traversed in the next succeeding interval of time to the length of this interval approaches, when this interval of time and, therefore, also the corresponding distance, are diminished indefinitely.

With uniformly accelerated motion acceleration is understood to signify the increment of velocity during a unit of time (sec.), or, more generally, the ratio of this increment to the time in which it is produced. Taking a sufficiently small interval of time, however, we may consider variably accelerated

motion as uniformly accelerated, and express its acceleration by the ratio  $\frac{\Delta v}{\Delta t}$ , if  $\Delta v$  denote the small change of velocity occurring during the small interval designated by  $\Delta t$ . By acceleration, therefore, is always meant the *limiting value* toward which the ratio of the change of velocity to the corresponding small interval of time, continually approaches as this interval becomes smaller and smaller. Since a change of velocity is itself a velocity, an acceleration is measured by the ratio of a velocity to a time, and the unit of acceleration is obtained by dividing the unit of velocity, *i.e.* the ratio of the length and time units  $\text{cm.}\cdot\text{sec.}^{-1}$ , by the unit of time. The unit of acceleration is, accordingly, for  $\text{cm.}$  and  $\text{sec.}$  denoted by  $\text{cm.}\cdot\text{sec.}^{-1} : \text{sec.}$ , or by  $\text{cm.}\cdot\text{sec.}^{-2}$ . Since it is reducible to the units of length and time, an acceleration is consequently expressed by the ratio of a length to the square of a time, as was indicated by equation (2), from which we obtain  $a = \frac{2s}{t^2}$ . Such expressions as  $\text{cm.}\cdot\text{sec.}^{-1}$  for velocity and  $\text{cm.}\cdot\text{sec.}^{-2}$  for acceleration which characterize the constitution of these ideas as derived from simple fundamental notions (here of length and of time) are called the *dimensions* of the composite notions.

**10. Acceleration of a Freely Falling Body.**—Experiments with the Atwood machine may be variously modified. If different overweights be allowed to operate upon equal total weights,  $2P + p$ , it is found that the accelerations produced are as the overweights, or as the forces operating. If the total weight,  $2P + p$ , be permitted to fall, first under the influence of the overweight  $p$ , and then, to fall freely, *i.e.* under the action of the force,  $2P + p$ , we find, in the first instance, an acceleration " $a$ " measured on the machine, and in the latter, the acceleration  $g$  of a freely falling body. This  $g$  is yet to be determined. These accelerations must, however, be to each other as the forces operating. We have then—

$$g : a :: 2P + p : p,$$

from which

$$g = a \cdot \frac{2P + p}{p}.$$

The acceleration of a freely falling body, which is usually designated by  $g$ , may be found by means of such experiments as are given above, or better by means of a more accurate method to be explained later. It is found to be—

$$g = 981 \text{ cm.-sec.}^{-2}, \text{ or } g = 9.81 \text{ m.-sec.}^{-2}.$$

and from this the space fallen the first second  $\frac{1}{2}g = 4.9 \text{ m.}$  The laws of falling bodies are given immediately by equations (1), (2) and (3), if  $g$  be substituted for  $a$  in them.

**11. Mass.**—If, with Atwood's machine, the same overweight  $p$  be made to act upon different total weights, it is readily seen that the accelerations produced will be inversely as the total weights, and this will be true, regardless of the kind of matter composing the weights. One may convince himself of this by giving to the weights  $P$  a hollow cylindrical form, and filling them with solids or liquids of various sorts until they become equally heavy. The result of this experiment may be expressed thus: every body opposes to an acceleration produced in it by a force, a resistance which is proportional to its weight. This resistance to acceleration is called the *mass* of the body. Masses of bodies are accordingly proportional to their weights, and may be compared with each other by weighing the bodies. In customary usage, mass is equivalent to the quantity of matter in a body. With bodies composed of the same kind of matter, *e.g.* with water and ice, that the quantity of matter contained in them is proportional to their weight, and that the ideas "mass" and "quantity of matter" are substantially identical, are manifestly true. For bodies composed of different kinds of matter, on the contrary, such as water and mercury, we can assert nothing *a priori* concerning the amount of matter they contain, but we ascribe to such bodies equal masses when the same force imparts the same acceleration to them. This, of course, occurs when their weights are equal.

The acceleration which a force imparts to a body is then directly proportional to the force acting and inversely proportional to the mass moved. Different forces, therefore, produce the same acceleration when the masses upon which

they operate are in the ratio of the forces. This relation holds in case of freely falling bodies. All freely falling bodies are equally accelerated at the same place on the earth's surface. A weight of one kilogram is then drawn to earth with a force a thousand times stronger than the force which acts on a gram. The kilogram, however, contains a thousand times as great a mass as does the gram, and hence, both suffer the same acceleration. All bodies must then fall with equal swiftness. Everyday experience, of course, seems to contradict this principle. Feathers, snowflakes, soapbubbles, and other bodies, whose surfaces are large in proportion to their weights, fall much more slowly than do stones, pieces of metal, and so forth. This is due, however, to the circumstance that the air opposes a resistance to moving bodies, which is directly proportional to their sections at right angles to the direction of motion. This resistance makes itself more apparent, the greater the ratio between the surface of the body and the accelerating force. That all bodies fall equally fast *in vacuo*, may be easily proved by causing feathers, bits of paper and shot to fall within a wide glass tube, from which the air has been removed by an air-pump. After the air has been well-nigh exhausted, it is seen that all these bodies fall with equal velocity.

The resistance of the atmosphere increases also with the velocity of the moving body. In case of the motion reduced by Atwood's machine, atmospheric resistance is of small consequence; but, with freely falling bodies, its retarding influence tends to cause the motion to become ever more nearly uniform. The above laws of uniformly accelerated motion are rigorously true, only on the assumption that no forces, aside from the accelerating force, operate on the moving body. With Atwood's machine, however, besides the atmospheric resistance, the friction of the axis of the pulley, the resistance of the cord to bending, the resistance to the acceleration of the pulley and of the cord, all act to retard the motion. One may easily convince himself that these laws approach the truth more nearly, the more nearly these unavoidable hindrances are eliminated, or the more completely their influence is taken into account.

**12. Units of Mass and Force.**—The units of force and of mass are so chosen that the unit of force acting upon the unit of mass, imparts to it the unit acceleration. The principle, that *the acceleration is directly as the force and inversely as the mass*, may then be briefly written thus—

$$a = \frac{f}{m}, \text{ or } f = ma,$$

where  $f$  is the force which imparts to the mass  $m$  the acceleration  $a$ .

The latter form expresses the principle that *the force is continually equal to the product of the mass moved into the acceleration*, and this holds whether the body is really accelerated, or the force merely exerts a pull, or pressure, against a motionless resistance. This product is called the *resistance of inertia*.

When, for one of the quantities, mass or force, the unit has been selected, the unit for the other is determined by the equation  $f = ma$ . Different systems of measurement result according to the choice of this unit.

If the matter contained in the original Paris kilogram be chosen as unit of mass, or, better, the thousandth part of it, that is, the *mass of one gram's weight*, the corresponding unit of force is that force which imparts to the mass of one gram in one second an increment of velocity equal to one centimeter. This unit of force is called the *dyne*. The force of gravity acting upon the weight of one gram imparts to it an acceleration of  $981 \text{ cm.-sec.}^{-2}$ , and equals therefore 981 dynes. This system of units is called the *absolute*, or *cm.-g.-sec.-system* (*C.G.S.-System*), and is extensively used in physical, and especially, in electrical and magnetic measurements.

For the practical purposes of mechanical engineering, it is much more convenient to choose as the unit of force, the pull which the earth exerts upon the weight of a kilogram; because, as above suggested, the forces operating on the surface of the earth are immediately expressible by weights. If the weight of one kilogram be allowed to fall freely; that is, if the force of 1 kg. be made to act upon the mass contained in it, the weight attains, in one second, a velocity of 9.81 m. In order

that this same force of 1 kg. may produce a velocity of only 1 m., it must be made to operate upon a mass 9.81 times greater; i.e. upon the mass contained in 9.81 kg. This mass, which, under the influence of the unit of force, 1 kg., attains the acceleration 1 m., has been chosen as the unit of mass of the *practical, or terrestrial, system* of measures. Since the mass 1 is the mass contained in 9.81 kgs., 1 kg. contains the mass  $1:9.81$ , or  $\frac{1}{9.81}$ , and  $p$  kgs. the mass  $\frac{p}{9.81}$ . The mass  $m$  of a body

of weight  $p$  kgs. is, accordingly,  $m = \frac{p}{9.81} = \frac{p}{g}$ , or, the number which expresses the mass of a body in terms of the unit just defined, is obtained by dividing the weight in kg. by the acceleration of gravity (9.81).

To the fundamental units of length and time, just used, is to be added that of mass, as a third fundamental unit. To the three units of length, mass, and time, all magnitudes occurring in physics are referable. The unit of force is obtained as the product of the unit of mass into the unit of acceleration, and has, therefore, in the "C.-G.-S.-System," the dimension  $\text{cm.-g.-sec.}^{-2}$  (dyne).

**13. General Laws of Motion.**—The generalization of the experiments made with Atwood's machine leads to many principles of comprehensive significance, from which the laws of all phenomena of motion may be derived. These are called the fundamental principles of mechanics.

We have seen that, after the removal of the moving force (the overweight), a falling body proceeds with the velocity attained, with uniform rectilinear motion, or that it remains in the condition of motion in which it has been put. In order to make the body move more or less swiftly, or to turn it aside from the rectilinear path in which it was previously moving, the operation of an external force is necessary. A body is, of itself, just as incapable of changing its condition of motion as it is of changing its state from that of rest into that of motion. Herein consists the first fundamental principle of the science of motion, the law of inertia, first recognized by Galileo, and expressed by Newton in the following words: "Every body

remains in its condition of rest, or of uniform motion in a straight line, unless compelled to change this condition by the operation of force."

At first glance observed facts seem to contradict this proposition, which is really only the specification of the motion "force." This mistaken idea of a disagreement of theory and fact arises because, with all motions we are able to produce, opposing forces operate which we are never able to eliminate completely with Atwood's machine. The weights come at last to rest, because friction and atmospheric resistance continue to retard their motion. A cannon-ball would move on forever in the direction and with the velocity imparted to it by the gun, were it not that the resistance of the air gradually diminishes its velocity, and the force of gravity continuously pulls it to earth. We may readily convince ourselves that motion once generated, and then left to itself, continues longer and longer, the smaller the opposing obstacles. We observe that a ball rolls much farther upon the smooth surface of ice, where frictional resistances are small, than upon a rough floor. The more rigorous proof of the correctness of this law is furnished by the circumstance that all conclusions drawn from it accord with the facts.

Experiments with Atwood's machine have further taught that the moving force produces, in equal times, equal increments of velocity. These increments are proportional to the force operating. This is true, whether the falling body starts from a state of rest or from any condition of motion whatever. These observations lead to the second fundamental law, also known to Galileo, which, in Newton's words, is, *change of motion is proportional to the force operating, and takes place in the direction of the straight line in which the force acts.* Since nothing is said in this law as to whether the body acted upon by the force is in a state of rest or of motion of any sort whatever, it implies the principle that the effect of a force is independent of the magnitude and direction of any motion already existing, or of the presence of other forces acting on the body. We may also state it thus: Every force acts with its own peculiar intensity and direction, regardless of the

existence of other forces. This principle is sometimes called the *principle of independence*. With Atwood's machine, of course, the moving force had the same direction as the motion already present. But that the principle is universally valid, whatever direction the forces may have is shown by the facts of ordinary experience. In a game of billiards on board ship, the balls are moved by the forces acting on them, when both the ship and balls are in motion, precisely as if the ship were at rest, and a body dropped vertically downward, strikes the same spot of the floor of the cabin whether the ship, and with it the ball, is in motion or at rest.

The third law of Newton is: "With every action there is an equal and contrary reaction, or the forces which any two bodies exert on each other are equal and oppositely directed." Or, briefly: "Action and reaction are equal and opposite." A stone, for example, lying upon a table exerts upon it a pressure downwards, but it suffers also from the table an equally great upward counterpressure. A horse is drawn backward toward the wagon by the traces with the same force it exerts to draw the wagon forward. The wagon retards the progress of the horse in the degree in which the forward motion of the wagon is accelerated. A magnet, attracting a piece of iron, is attracted by the iron equally strongly in the opposite direction. The earth attracts the moon with a force precisely equal to that with which the moon attracts the earth. On firing a gun the ball receives a forward impulse from the explosion of the powder just as strong as the gun is impelled backwards against the shoulder.

**14. Vertical Projection.**—Aided by these fundamental principles, the laws of motion of a body, or of a material point, may be found without experimentation, by mere computation, and their effect may be predicted so soon as the forces operating are given. A force, constant in magnitude and direction, such, for example, as the force of gravity, must generate a constant acceleration ( $g = 9.81 \text{ m.-sec.}^{-2}$ ) and, consequently, also a uniformly accelerated motion, whose velocity,  $v$ , increases uniformly with the time,  $t$ , or  $v = gt$ .

Since the velocity of a falling body increases uniformly, i.e.

in equal times by equal amounts, the body must, in a given time, traverse the same distance it would traverse in the same time with a velocity equal to the mean of the initial and final velocities, or with the velocity which the body had for an instant at the middle of the interval of time, during which the motion is considered. At the beginning of the first second of fall, the velocity was zero; at its end, the velocity was  $g$  and, consequently, the mean velocity for the first second is  $\frac{1}{2}g$ . The body moving uniformly with this velocity, for one second would describe a distance equal to  $\frac{1}{2}g$ . The distance, therefore, which is actually described in the first second with the velocity gradually increasing from zero to  $g$ , is equal to half the acceleration, or  $\frac{1}{2}g$ . With the first two seconds of fall, the initial velocity is again zero and the final velocity is  $2g$ ; the mean velocity is then equal to  $g$ . Moving for two seconds with this velocity, the body would describe a distance equal to  $2g = \frac{1}{2}g \times 4$ , which is four times as great as that described in the first second, and so forth. In general, if the body falls for  $t$  seconds, it will have traversed at the end of this time, the same distance,  $s$ , as though it had moved uniformly for  $t$  seconds with the velocity  $\frac{1}{2}gt$ , or with a velocity equal to a mean between its initial ( $0$ ), and its final ( $gt$ ) values. We have, then,  $s = \frac{1}{2}gt \times t$  or  $s = \frac{1}{2}gt^2$ . From the equations  $v = gt$  and  $s = \frac{1}{2}gt^2$ , we find, as indicated above, for the velocity ( $v$ ) at the end of the space fallen,  $s$ , the expression,  $v^2 = 2gs$ .

If, at the instant of starting, the body is given a downward impulse, that is, if it is hurled downward with the initial velocity  $c$ , it maintains this velocity by virtue of inertia. Superposed upon this, however, is the velocity  $gt$  due to the force of gravity for  $t$  sec., precisely as though the body had started from a state of rest, so that after the lapse of  $t$  sec., the body will have the velocity  $v = c + gt$ . In consequence of inertia alone, the body would have described the space  $ct$ , during  $t$  sec., by reason of its uniform motion with the velocity ( $c$ ) of projection. The force of gravity acting simultaneously, adds to this, the distance  $\frac{1}{2}gt^2$ , so that, after  $t$  secs., the body has actually described the distance  $s = ct + \frac{1}{2}gt^2$ . The body moves with uniform acceleration, precisely as though the falling

motion, which had previously imparted to it the velocity  $c$ , were merely continued.

If a body be thrown vertically upwards, its initial velocity  $c$ , which by virtue of inertia it strives to maintain, will be diminished by the force of gravity acting downwards, by the amount  $g$  every second, so that it will possess, after  $t$  seconds, the velocity  $v = c - gt$ , and instead of describing the distance  $ct$  upwards, which by reason of its inertia it would otherwise do, it will describe only the distance  $s = ct - \frac{1}{2}gt^2$ ; because gravity reduces the space traversed by  $\frac{1}{2}gt^2$  each second. The body rises with uniformly retarded motion until it attains the highest point of its path, where its velocity has become zero. This

happens at the time  $t' = \frac{c}{g}$ . At this point, the body stands

motionless for an instant, and then falls according to the laws of freely falling bodies to the point of starting. Since, in falling, the body's velocity is augmented precisely as, in rising, it was reduced, the time of fall must equal the time of rise, and the body passes every point of its path both in its ascent and in its descent with the same velocity, reaching the starting point with the velocity it had on departing from it. From the equation  $v = c - gt$ , it follows that  $v = -c$ , when  $t = \frac{2c}{g}$ . The

highest point reached is consequently at an altitude, from which, if the body fall freely, it will attain a final velocity equal to the initial. The highest point reached by a body thrown upward is found, from the laws of falling bodies, by dividing the square of the initial velocity by twice the acceleration. From the equation  $s = ct - \frac{1}{2}gt^2$ , the altitude of the highest point is found to be  $s' = \frac{c^2}{2g}$ , by substituting in it

for  $t$  the value  $t' = \frac{c}{g}$ . The same equations are evidently true

for all uniformly retarded motion, it being necessary merely to replace  $g$  by the proper value  $a$ , of the acceleration or retardation, as the case may require.

**15. Horizontal and Oblique Projection.**—If a body be projected upwards in an oblique direction (AG, Fig. 4), by reason

of inertia, it would move on in a straight line with the velocity of projection, describing equal distances  $AB$ ,  $BC$ ,  $CD$ , etc., in equal times, were it not for the action of the force of gravity. Gravity, however, acting on the projectile, draws it downward in such manner that during intervals of time to each other in the ratio of  $1:2:3:4:\dots$  it describes the spaces  $Bb$ ,  $Cc$ ,  $Dd$ ,  $Ee$  . . . which are to each other in the ratio of the squares of the times, that is, in the ratio of  $1:4:9:16:\dots$ , so that the curve  $Abcdefg$ , represents the actual path of the *projectile*. The *projected* body is called a *projectile*, and the path it describes ( $Abcdefg$ ), a *trajectory*. Such a curve as the above, whose separate points are found by laying off from the points of a straight line ( $AG$ ), the parallel distances ( $Bb$ ,  $Cc$ ,  $Dd$  . . .), proportional to the squares of the corre-

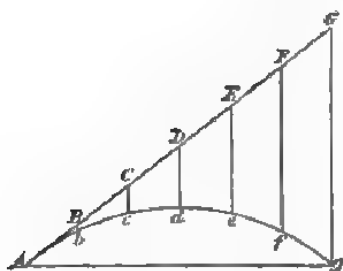


FIG. 4.—Oblique Projection.

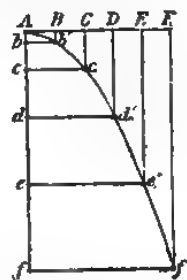


FIG. 5.—Horizontal Projection.

sponding distances ( $AB$ ,  $AC$ ,  $AD$  . . .), measured from the starting point as origin, is called a *parabola*. The path, or trajectory of a body thrown obliquely upward, consists of a rising ( $Ad$ ) and a falling parabolic branch ( $dg$ ) equal to each other, and traversed in equal times. The *range*, i.e. the distance ( $Ag$ ) at which the body again strikes the horizontal plane passed through the point of projection, is, for any given initial velocity, a maximum, when the body is thrown upward at an angle of  $45^\circ$  with the horizon. When a body is projected horizontally, it describes the descending branch only of a parabola ( $Af$ , Fig. 5), and reaches the earth at  $f$  in the same time as would be consumed in falling freely from  $A$  to  $f$ . This

is at once clear from the cut. The parabolic shape of the trajectory may be seen in material form with jets of water flowing from orifices in the sides of vessels. The parabolic form of the curve assumed by the issuing jet may be proved by measuring its co-ordinates at various points. By this agreement of the facts of experience with inferences from fundamental principles, the correctness of the principles is confirmed anew. It has, of course, been assumed in deriving these laws of projection, that no appreciable hindrance to the motion of the projectile occurs. A body projected into the air is, however, exposed to atmospheric resistance, which in case of such high velocities as are produced with heavy guns, becomes of considerable importance; and in such a case, the body is deflected from a purely parabolic path into a somewhat different curve falling more abruptly than it rises, and called the *ballistic curve*.

**16. Quantity of Motion—Shocks.**—Since the acceleration  $a$ , imparted by a force  $f$ , to a mass  $m$ , is  $a = \frac{f}{m}$ , we may introduce the force and the mass into the equations of uniformly accelerated and retarded motion of § 8, by merely replacing  $a$  by  $\frac{f}{m}$  in them. Laws (1) and (3) of the uniformly accelerated motion of a mass  $m$  under the action of a constant force  $f$ , are then expressed by—

$$(1) \, mv = ft; \quad (3) \, fs = \frac{1}{2}mv^2.$$

Equation (1) expresses the principle, that the product of the mass moved into the velocity attained, equals the *product* of the constant force into the time during which it acts. The product of the mass of a body into its velocity is called its *quantity of motion*, or its *momentum*. The product of the force into the time is designated the *impulse* of the force. We may now say "*the impulse of the force equals the momentum produced.*" If the force acts upon a body only during an immeasurably small time, it is called an *impulsive*, or an *instantaneous force*, or a *shock*. It is impossible to determine the magnitude of an impulsive force, but we judge of it by the *quantity of motion* it

will produce. Such a force, measured according to this standard, is in reality, not a force, but an *impulse* of the dimension  $\text{cm.-g.-sec.}^{-1}$ . The body set in motion by the impulse, proceeds with the velocity  $v$  imparted to it, uniformly and in a straight line, so long as no other forces operate upon it. The pressure of the gases, liberated by exploding gun-powder, acting forward against the ball and with equal strength backward, during the same small interval of time against the gun, is an example of impulsive force. Both gun and projectile receive equal impulses, and hence their momenta are also equal. Or, if  $m$  and  $m'$  be their respective masses,  $v$  and  $v'$  their corresponding velocities,  $mv = m'v'$ , or, the velocity of the ball, and that of the gun are inversely as the respective masses.

**17. Work.**—When a force, acting upon a mass, sets it in motion, the force is said to do work, and the result of its action is called **work**. To raise the weight of one kilogram one meter high requires the performance of a definite quantity of work. Twice as much would obviously be required to raise this kilogram two meters high, or to raise two kilograms one meter high, and six times as much to lift three kilograms two meters high. The work performed is, accordingly, on the one side, directly proportional to the resistance overcome or to the equal force exerted in overcoming it, and, on the other, directly proportional also to the distance traversed *in the direction of the force*. If, then, we choose as the unit of work, the work performed by the unit of force over the unit of distance, all work is expressible as the product of the force, or the resistance overcome by it, into the distance along which it acts. In the practical system of units, the kilogrammeter (kgm.) serves as unit of work; but in the absolute system, the *erg*, or the work performed by the force of one dyne, operating through a distance of 1 cm. is used. Its dimension is  $\text{cm}^2\text{-g.-sec.}^{-2}$ . One kilogrammeter is equivalent to 98,100,000 ergs.

In transforming forces into work, the question is, not alone whether work is done, but also, in what time is it accomplished? The work done in one sec. is called the *effect* of the force. In actual practice with machinery, a larger unit, the *horse-power*, is used. It is equivalent to the work of 75 kgm. A strong

man's power to do work is roughly estimated to be from  $\frac{1}{2}$  to  $\frac{1}{3}$  of a horse-power. In the absolute system of units, the unit of *effect*, i.e. the erg per sec. (dimension  $\text{cm}^2\text{-g-sec.}^{-3}$ ), or the larger unit, the *Watt*, equalling 10,000,000 ergs per sec. =  $\frac{1}{746}$  horse power, is sometimes used.

**18. Vis viva, or Living Force.**—In the equation (3)— $fs = \frac{1}{2}mv^2$ , the left member, being the product of a force into a distance, is merely the work performed by the force. In words, this equation asserts that, if the mass,  $m$ , is moved from rest, through the distance,  $s$ , by a constant force, and, at the end of this distance, has attained the velocity,  $v$ , the work performed by this force, equals the half product of the mass into the square of the velocity acquired.

A moving mass has the power to overcome a certain resistance, i.e. to overcome a force,  $f'$ , acting through a certain distance  $s'$ , in a direction opposed to the motion, and, if the opposing force remain constant, the mass assumes a uniformly retarded motion, coming finally to rest. From the laws of uniformly retarded motion of § 14, we learn that a body having an initial velocity  $v$ , comes to rest after having described the distance

$$s' = \frac{v^2}{2a'}$$

where  $a'$  denotes the acceleration opposed to its motion. If in this equation,  $\frac{f'}{m}$  be substituted for  $a'$ , there results,

$$(3') \quad f's' = \frac{1}{2}mv^2,$$

or, in words, the work which the moving mass is capable of performing, if allowed to operate against a resistance,  $f'$ , until its initial velocity,  $v$ , is exhausted, equals the half product of the mass into the square of the initial velocity.

If, now, principle (3') be compared with (3), it is at once seen, that the work a constant force must perform to impart to a mass in repose a definite velocity, is exactly equal to the work this mass can do in overcoming resistance, by virtue of this velocity, before the body comes to rest. This is manifest from the fact that both quantities of work are represented by the same expression, viz. the half product of mass and square

of velocity. The expression  $\frac{1}{2}mv^2$  was formerly called the *living force*, or *kinetic energy* of the moving mass, and this term, as also the term *vis viva*, are still applied to it.

The proposition just stated is valid, not only for constant forces; it is equally true for any variable force, since such a force may be regarded as constant during sufficiently short intervals of time, and changing only at the ends of these intervals. The work of a variable force is expressed by the product of its instantaneous intensity into the short distance traversed during this brief interval. The work performed during any given interval of time is the sum of these small products, and is always equal to the half product of the mass moved and the square of the final velocity. The assumption that the body proceeds from, and returns to, the condition of repose, is not essential to the validity of this proposition. For, granted that the body proceed from a state of rest under the influence of force  $f$ , through the distance  $s$ , and attain in the meantime the velocity  $v$ , and then that it proceed farther to  $s'$ , having attained the velocity  $v'$ , we have  $fs = \frac{1}{2}mv^2$  and  $f's' = \frac{1}{2}mv'^2$ , and consequently also  $f(s' - s) = \frac{1}{2}m(v'^2 - v^2)$ , or, the change in *vis viva* equals the change in work.

We may, thus, consider a moving mass as a magazine, so to speak, in which the work necessary to set it in motion, or to augment its velocity, is stored up and may be drawn upon at will, without either loss or gain, to do work against resistance.

**19. Energy.**—The power of a body to do work is called its *energy*. It is expressed by the quantity of work, measured in work-units, which a body is capable of giving out. The *vis viva*, which resides in a moving body, is energy. But energy is possessed, not alone by moving bodies. Bodies at rest may also possess it. If, for example, a stone, projected upward, be caught by the roof of a building at the highest point of its path, the stone has lost its motion, but not its capability of performing work, and, consequently, not its energy. For, when the stone is released from its support and allowed to fall to earth, it will reach the earth's surface with the same velocity, and consequently with the same *vis viva*, which it possessed at the instant of its vertical projection. It is now capable of

performing just as much work as was expended upon it. The energy residing in the stone lying quietly upon the roof, and rendered apparent during the fall, is due to the body's elevated position or, in other words, to the circumstance that the stone, when on the roof, was further from the earth which attracts it than when lying upon the earth's surface. This capability of performing work, stored up, as it were, in a body at rest, is called its *energy of position*, or *potential energy*. In contradistinction to this, the *vis viva* of a moving body is called its *energy of motion*, or its *kinetic energy*. The work expended in bending a crossbow, is stored up as potential energy in the stretched cord, and is ready at a finger's touch to be transformed into the kinetic energy, or the *vis viva* of the flying arrow. The work performed by the hand in winding a clock passes into the spring, or weight, as potential energy, and remains in this state so long as the mechanism is prevented from running. So soon, however, as the impediment to motion is removed, this potential energy begins to be gradually transformed into the kinetic energy of the moving wheels. By the latter illustration, we see also why energy of position is also sometimes called *tension*. If a stone is thrown upward, the opposing force of gravity diminishes its velocity, but what is lost in kinetic energy during ascent, is gained in potential energy, until the highest point of flight is reached, at which the velocity is consumed, and the entire supply of kinetic is transformed into potential energy. When the stone begins to descend, it starts with a quantity of potential energy equal to the initial *vis viva*. The further the stone falls the smaller grows its potential, and the larger its kinetic energy: but everywhere *the sum of the two is the same*.<sup>\*</sup> At the instant of striking the ground, its potential is wholly reconverted into kinetic energy, the quantity of which is again precisely equal to the initial value. The sum total of the energy of the stone, i.e. the sum of its energy of motion and of position, remains unchanged during the entire period of motion. What actually happened was merely the conversion, without loss or gain, of one sort of energy into the other.

<sup>\*</sup> For from equations (14)  $mgs + \frac{1}{2}mv^2 = \frac{1}{2}mv^2$ , when  $mgs - ps$  is the potential energy.

Let us now inquire what becomes of the energy of the stone on striking the earth, and instantly coming to a state of rest? The energy of its visible fall is, of course, destroyed at the instant of striking the ground. It has been ascertained that whenever energy of motion is apparently destroyed, either by a sudden impulse or by the prolonged resistance of friction, a rise in the temperature of the body occurs. A cannon ball, for example, fired against a plate of armour, is heated to glowing, and the stopping of a moving train by brakes heats both brakes and wheel. Joule and Hirn have proved by experiment that for every 424 units of work, or kilogrammeters, which disappear by shock, or by friction, a quantity of heat sufficient to raise the temperature of water  $1^{\circ}$  Centigrade is produced. If this quantity of heat, called the heat unit, be properly utilized, it will perform again 424 kgm. of work. This 424 kgm. is designated *the mechanical equivalent of heat*. The transformation of work into heat, as also the converse process, is readily intelligible, on the assumption that heat is a kind of motion; that it is a motion of the molecules of bodies, which, by reason of the excessive smallness of the particles, is not perceptible to the eye. This motion, however, does produce an impression upon our consciousness, and *this impression we call heat*. When the energy of apparent, or mass motion of a body is seemingly destroyed, either by shock, or by friction, it does not in reality vanish, but is merely transformed without either loss or gain into the energy of invisible or molecular motion, called heat. Energy can be neither destroyed nor created. A so-called perpetual-motion machine, that is, an apparatus which may perform more work than it consumes, is therefore an impossibility. All the processes of nature, in which energy seems to vanish, rest merely upon the transformation of the energy of one mode of motion into that of another, or more briefly, upon the transformation of the energy of motion into that of position, and conversely. Consequently, the entire supply of energy in the universe is always the same. This fundamental law of nature, which is confirmed by universal experience, is called the principle of the *conservation of energy* (Robert Mayer, 1842; Helmholtz, 1847). It has also been less

appropriately called the principle of the *conservation of force*. Inasmuch as the transformation of all the various forms of natural energy, such as sound, heat, light, electricity, chemical affinity, elasticity, and mechanical energy, conform to this law, we are led to regard them simply as different manifestations of one and the same essence. Recognizing thus their intrinsic relationship, we are in a certain sense justified in speaking of the *unity of the forces of nature*.

#### 20. Composition of Motions—Parallelogram of Forces.—

Suppose a ship to be at A (Fig. 6), near the bank of a river, and that during a certain interval of time, the wind acting against its sails, drives it from A to a point, C, on the opposite shore. Suppose, also, that during this same time it would be carried by the force of the current, acting alone, from A to B. If, now, both these forces act

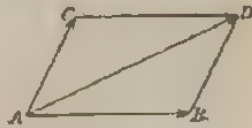


FIG. 6.—Parallelogram of Forces.

contemporaneously, that of the current, wholly undisturbed by that of the wind, will carry the ship through the distance AB, which it would traverse by virtue of the current alone, from the line AC, over which the wind alone would carry it, to the line through B, parallel to AC. At the same time the ship will be carried by the wind from the line AB, which it would describe by virtue of the current, in the direction of the wind, toward the opposite shore, to the line CD, drawn through C parallel to AB. This principle, by virtue of which, when a body is acted upon by any number of forces, each force produces its effect independently of all the rest, is known as the *principle of independence*. At the end of the time the ship must therefore be at the point D, at which the parallels BD and CD intersect. If we wish to consider the place of the ship at any intervening instant, we need only remember that if the two superposed motions are of the same kind, *e.g.* if both are uniform, or both uniformly accelerated, the body remains continually upon the line, AD, or, that it traverses the diagonal, AD, of the parallelogram, ABCD, determined by the two paths AB and AC and the included angle BAC. The ship moves, therefore, precisely as if it were driven by a wind from A to D

upon the smooth surface of the sea. Such a wind might then replace the two simultaneous forces completely. Since, with the same kind of motions, the velocities, accelerations, and the forces themselves are in the same ratio as the distances traversed in equal times, the same graphical method may be used for the composition of these magnitudes as the one just outlined, provided the forces be represented in magnitude and direction by straight lines. We are thus led to the proposition of the *parallelogram of forces*: two forces, called *components*, applied at any point, and acting at any angle with respect to each other, may be replaced by a single force, called a *resultant*, represented, in magnitude and direction, by the diagonal of the parallelogram, constructed upon the components as sides. We need only repeat this same mode of construction to compound any number of forces, acting upon the same point, into a single resultant. We merely compound the third force with the resultant of the first two, the fourth, with the resultant of the first three, and so on indefinitely. The principle embodied in this construction is known as the *polygon of forces*.

The above proposition teaches us, furthermore, how to decompose a given force, AD, into two components, AC and AB, which together produce the same effect as the single force. It is only necessary to draw a parallelogram whose diagonal represents the given force. Since, however, an indefinite number of parallelograms may be constructed upon a given diagonal, the problem is indeterminate unless, either the directions of both components, or both the magnitude and direction of one of them, be also given.

The tendency to motion along the diagonal, AD, produced by the component forces, AB and AC, is overcome, or, speaking technically, *equilibrium* is established, if a force be made to act upon A equal and opposite to the resultant, AD. We may, then, regard any one of the three forces as the reversed resultant of the other two. Such equilibrium of the three forces is automatically established by means of the apparatus represented schematically in Fig. 7.

Near the top of each of two vertical posts, fixed rigidly to a wooden base, is an adjustable sheave, which may be fixed at

any desired height. A string, on which a cord,  $a$ , slides easily toward the right or left, works over these sheaves. If, now, weights be hung at the ends of the cord, as also on the ring, the weights at the sides draw obliquely upward at  $a$ , and give a resultant directed vertically upward, which the middle weight, pulling vertically downward, will hold in equilibrium. If, for example, three hectograms (hg.) be hung at the left end of the cord and four at the right, when five hgs. are attached at the middle, we find, on drawing a parallelogram whose sides are three and four, and whose diagonal is five, that the sides enclose a right angle, and that, in point of fact, the two parts of the string meet at right angles at  $a$ . For other weights, there would result other angles, all of which would

be found to agree with those obtained by geometrical construction. Of this agreement one may easily convince himself, by placing the parallelogram, drawn on paper, with its diagonal vertical and in the prolongation of the suspending cord at  $a$ .

Moreover, the place of a point can always be found by constructing a parallelogram, even when the motions are unlike in kind, *e.g.* when a uniform and a uniformly accelerated motion are compounded. The only difference between this case and the former, is that, in the latter, the point will describe a curvilinear path instead of the diagonal of a parallelogram. This fact is apparent in the phenomena of the motion of projectiles. A projectile is a body whose motion is compounded of a uniform motion, due to an initial impulse, and a uniformly accelerated motion, due to the continuous action of the force of the earth's attraction on the body. It was in the study of the motion of projectiles that the principle of independence was first recognized.

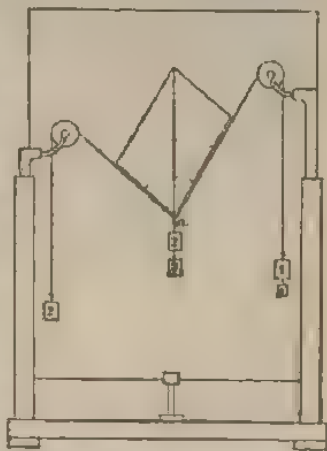


FIG. 7 — Parallelogram of Forces.

**21. Motion and Equilibrium on an Inclined Plane.**—As an example of the application of the principle of the parallelogram of forces, let us first consider the behaviour of a heavy body upon a plane inclined to the horizon. Take the plane of the drawing, perpendicular to the intersection of the inclined plane with the plane of the horizon. This plane of the drawing will intersect the inclined plane in the straight line, AB, which makes, with the horizontal, AC, the angle  $\alpha$ . A perpendicular, BC, dropped from any point, B, of the inclined plane upon the horizontal, i.e. the side opposite the angle  $\alpha$  in the triangle, ABC, is called the height of the inclined plane ( $h$ ). The

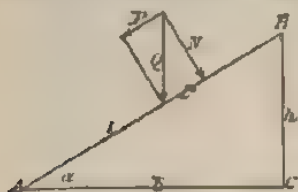


FIG. 8.—Inclined Plane.

hypotenuse, AB, is called its length ( $l$ ), and the side, AC, adjacent the angle,  $\alpha$ , is the base ( $b$ ). Since motion can occur only in a direction parallel to the inclined plane, we will decompose the force,  $Q$ , acting vertically downwards, through the centre of gravity of the body, that is, the

body's weight, into two components perpendicular to each other; the one, the force,  $P$ , parallel to the plane; the other, the normal force,  $N$ , perpendicular to it. Each of the right-angled triangles into which the parallelogram is divided by the diagonal,  $Q$ , is similar to the triangle, ABC, or  $h/b$ , and we have—

$$P : Q = h : l, \text{ and } N : Q = b : l,$$

that is to say, the parallel force is to the weight of the body as the height of the plane is to its length; and the normal force is to the weight as the base is to the length. The components result from these proportions as follows—

$$P = Q \cdot \frac{h}{l} = Q \sin \alpha; \quad N = Q \cdot \frac{b}{l} = Q \cos \alpha.$$

Each of the components is smaller than the weight of the body, because in a right-angled triangle either leg is less than the hypotenuse. The ratio of the height to the length is called the *rise*, and is usually expressed in per cent., the length being put equal to 100.

The normal force  $N$  presses the body against the plane, and is neutralized by the reaction of the plane. Neglecting friction, or, technically, *regarding the plane as smooth*, this force exercises no influence whatever upon the motion. The parallel force, on the contrary, causes the body to slide down the plane with uniformly accelerated motion, and with an acceleration  $g'$ , which is to that of a freely falling body,  $g$ , as  $P$  is to  $Q$ , or as  $h$  is to  $l$ , and consequently,

$$g' = g \cdot \frac{h}{l} = g \sin a.$$

The distances traversed during the time  $t$ , along the plane and in case of free fall, are to each other respectively, as  $g'$  to  $g$ , or as  $h$  to  $l$ . If, now, we drop from  $C$  (Fig. 9) the perpendicular  $CD$  upon  $AB$ ,  $BD$  will be traversed in the same time as  $BC$ , because

$$BD : BC = h : l.$$

If a circumference be described upon  $BC$  as a diameter, it will cut  $AB$  in  $D$ , and hence it follows that the chord  $BD$ , as

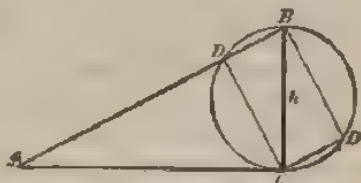


FIG. 9.—Fall along Chords.

also the equal and parallel chord  $CD$ , will be traversed in the same time as the diameter  $BC$ . We may, therefore, state generally, that the vertical diameter of a circle is traversed in the same time as any chord of the circle drawn from either of its ends. The velocity which the body attains after having traversed the entire length of the plane is determined from the equation

$$v^2 = 2g'l = 2g \cdot \frac{h}{l} \cdot l = 2gh.$$

It is thus seen to be the same as if the body had fallen vertically from the height  $h$ , with the acceleration  $g$ ; for the latter velocity is determined from precisely the same equation,  $v^2 = 2gh$ . A body sliding, without friction, down an inclined plane, acquires the same velocity, and consequently also the same momentum, as if it had fallen freely through the same vertical height. If, then, its velocity were reversed, the body would rise

to the same height, or to the same level, whether projected vertically, or along the surface of an inclined plane. Since the increase, or decrease, of momentum which a body experiences during its fall or rise depends only upon the *vertical* height traversed by it, this proposition must also hold for either ascent or descent in curvilinear paths.

To overcome the sliding of the weight it is only necessary to cause a force, equal to the parallel force  $P$ , to act upwards along the plane and parallel to it. By means of an adjustable model of an inclined plane, from which the length, height, base, and angle of inclination can be read off, the law  $P : Q = h : l$  may be experimentally verified by attaching a string to the load  $Q$ , and passing it upward along the plane, over a sheave at the top. The load will then be in equilibrium, if the weight attached to the free end of the string bears the same ratio to the load as the height of the plane to its length. If the parallel force which acts upward along the plane be slightly increased, the weight will move upwards. It is therefore raised by a force which is only a fractional part of that required to raise it perpendicularly. For this reason the inclined plane is often used to raise and lower heavy loads, when the force at command is not sufficient to elevate them vertically, or directly to retard their downward motion. Skids for loading and unloading wagons, hilly roads, railroads, etc., are examples of the inclined plane. No work is saved, however, by its use, for the work along the plane ( $Pl$ ) is, by reason of the above proportion, always equal to the work ( $Qh$ ) required to elevate the load perpendicularly.

**22. The Screw.**—It may, then, be inquired how great a force ( $H$ , Fig. 10), acting parallel to the base of the inclined plane, is necessary to prevent a body of weight ( $Q$ ) from sliding down it? To answer this, let the vertical force  $Q$  be decomposed into two components, one of which,  $H$ , shall be directed horizontally, and the other perpendicularly to the inclined plane, the latter being wholly destroyed by the reaction of the plane. In the parallelogram (Fig. 10), the right-angled triangle,  $HQ$ , is similar to that which represents the inclined plane, and we have —

$$H : Q = h : l,$$

or, the horizontal force is to the load as the height of the plane is to its base. From this results—

$$H = Q \cdot \frac{h}{b} = Q \cdot \tan \alpha.$$

A force, equal and opposite to this, must be made to act upon the body to hold its tendency to slide downward in equilibrium. This mode of applying a force is, however, advantageous only so long as the angle of inclination  $\alpha$  is less than  $45^\circ$ . With higher inclinations,  $h$  is greater than  $b$ , and consequently  $H$  is greater than  $Q$ .

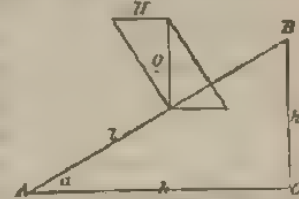


FIG. 10. — Inclined Plane (Screw).

This particular mode of action of a force finds practical application in *screws*, which are merely inclined planes, wrapped around a cylinder, called the *axis* of the screw, and given the form of spiral projections, bounded above and below by winding surfaces, called *threads*. This system of threads, which are usually rectangular or triangular in section, fits accurately into a corresponding system of depressions on the inner wall of a hollow cylinder, called the *nut*. The distance between consecutive threads of the screw, technically the *pitch* of the screw, corresponds to the height of the inclined plane and the circumference of the axis, to its base. If the nut be fixed, and a force be applied to the axis, when in a vertical position, at some point on its circumference, the axis will be elevated or depressed according to the direction of rotation. In the former case a load may be raised, and in the latter, a pressure may be exerted. This load, or pressure, will bear the same relation to the force as the circumference of the axis to the pitch of the screw, or as the base to the height of the plane.

In the foregoing conclusions, all matters relating to the resistance due to friction have been disregarded. By means of this resistance the screw is held with great force by its nut, upon which fact depends the commercial value of the screw as a *clamp*, or device for rigidly fixing bodies together. If the axis of the cylinder be turned one complete revolution, and

the nut be fixed, the screw advances through a distance equal to its pitch. If, on the contrary, the screw is fixed, the nut advances this same distance. If, now, the screw be turned only the fractional part of a circumference, the advance will amount to the same fractional part of the pitch. Upon this principle rests the use of fine motion screws for the accurate setting of apparatus, as also the application of the carefully cut *micrometer screw*, to measurements where extreme accuracy is required, such as testing the graduation of scales and divided circles, as with *graduating engines*, etc. The *spherometer* is an apparatus for measuring the thickness of thin plates, and consists of a micrometer screw working in a nut, the entire mechanism being supported by legs terminating in steel points, borne upon plates of glass. The endless screw, consisting of an axis with but a few threads, and without a nut, is frequently used to communicate motion to a gear wheel, into whose

teeth its threads engage. The force acting on the circumference of the axis, exerts upon the perimeter of the toothed wheel a pressure which is to the force acting, as the circumference of the axis to the pitch of the screw.

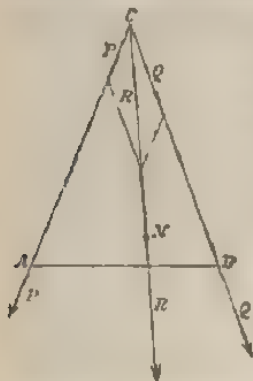


FIG. 11.—Forces at Two Points.

**23. Composition of Two Forces acting in the Same Plane upon Different Points of a Rigid Body.**—If the directions of any two forces,  $P$  and  $Q$ , applied at different points,  $A$  and  $B$  (Fig. 11), of a rigid body intersect, the forces may be compounded, according to the principle of the parallelogram of forces,

into a single resultant,  $R$ . The point of application of each force may be arbitrarily shifted to any point in the direction of the force without in any way modifying its effects, provided the new point of application be rigidly fixed to the old. If, then, we displace the point of application of both forces to the point of intersection,  $C$ , of their directions, they may there be replaced by their resultant,  $R$ , provided  $C$  is rigidly connected with  $A$  and  $B$ . The resultant  $R$  may now, without

alteration of its effect, be displaced to any point, *M*, situated on its line of action. This same construction holds, also, even when the point of intersection, *C*, lies outside of the body; because any point of the body whatever, lying on the direction of the resultant, may be selected as the point of application.

**24. The Wedge.**—As an illustration of the principle of the foregoing paragraph, the wedge (Fig. 12) may be selected. This is a prism, whose cross section is an isosceles triangle, and whose sharp edge may be forced into a body to be split, if a sufficient force be applied to the surface lying opposite this edge and called the *back* of the wedge. The resistance of the body to be split to the entrance of the wedge may be represented by two equal forces, *P*, applied perpendicularly and symmetrically upon the faces of the wedge.



FIG. 12.—Wedge

To compound these into a single resultant, *R*, we replace the point of application of each in its own direction to that point of the medial line of the triangular cross section of the wedge, at which their prolonged directions intersect. If, now, we construct on the forces *P*, the parallelogram *abcb'*, its diagonal, *ac*, represents the force *R*, with which, neglecting friction, the wedge would, so to speak, be squeezed out of the body, if an equal and opposite force were not applied to the back of the wedge to hold *R* in equilibrium. The triangle *abc* is evidently similar to the triangle *ABC*, which represents the outline of the wedge, and there results—

$$R : P = AC : AB,$$

i.e. the force which, acting on the back of the wedge, holds it in equilibrium, is to the opposing resistance as the back of the wedge to its side. Consequently, the smaller the back of the wedge is in comparison with the side, that is, the sharper the wedge, the smaller the force necessary to drive it into the body. Many of our cutting tools, for example, knives, hatchets, planes, chisels, etc., are illustrations of the wedge.

**25. Parallel Forces.**—The method of compounding two

forces, acting at different points of a body, as it is explained in paragraph 23, does not suffice when the directions of the forces are parallel. We will now consider this case separately.

The horizontal bar in Fig. 13 represents a metal scale of uniform thickness, and easily movable about an axis through its middle point. A cord which works over a pulley, fixed at the top of an upright post, carries, at one end, the bar, together with the mechanism for supporting it, and, at the other, a



FIG 13—Parallel Forces

scale-pan. If a weight, equal to that of the bar and its support, be placed on the pan, the bar will remain in equilibrium in any position, either horizontal or inclined. On either side of the middle point  $O$ , and at equal distances on the scale, are the numbers 1, 1', 2, 2', 3, 3', and so forth, near which weights may be attached by little hooks provided for the purpose. If a weight  $p$ , say of 20 g., be hung at equal distances from the middle point, for example, at 5 and 5', the scale will vibrate for a moment, then stop and stand at equilibrium, provided two weights equal to  $p$  be placed on the scale-pan. Since, now, the force  $2p$ , acting upward, holds in equilibrium the

combined effect of the two forces at 5 and 5', both directed downward and each equal to  $p$ , this latter effect must be equal to a single force,  $2p$ , applied at the middle point O, and acting downward. It is clear, therefore, that two equal and parallel forces, acting at different points of a rigid straight line, may be replaced by a resultant, parallel to them, equal to their sum, and applied at the centre of the line connecting their individual points of application. If, then, six forces, each equal to  $p$ , be applied to the scale at the points 1, 1', 3, 3', 5, 5', it will again stand at equilibrium when a weight equal to  $6p$  is placed in the scale-pan. This will become apparent at once, if we reflect that the forces,  $p$ , applied at the points 1, 1', 3, 3', 5, 5', may be compounded in pairs, each pair giving rise to a single resultant, acting downward, and applied at the middle point O, without in any way modifying their combined effect. But we may also, if we choose, divide the six weights,  $p$ , into two unequal groups, one consisting of the two forces, applied at 3 and 5, and the other comprising the four remaining forces, at 1, 1', 3', and 5'. Let us now remove the weights at 3 and 5, and hang them, one underneath the other, at 4, i.e. midway between 3 and 5. The equilibrium will not be disturbed by the rearrangement. Similarly, we might combine the weights at 1' and 3' and at 1 and 5' into a single resultant, applied at a point midway between them, that is, at the point 2', without in any way affecting their combined resultant,  $6p$ , acting always at the middle point O. The force  $4p$ , at the distance 2, and the force  $2p$  at the distance 4, from the point O, give, therefore, a resultant  $6p$ , parallel to each of the components and applied at the point O. The equilibrium is not disturbed when the bar is drawn from the horizontal into any other position. It is, consequently, immaterial whether the parallel forces act perpendicularly, or obliquely, to the line connecting their points of application. The following principle may, therefore, be stated :—*The resultant of two parallel forces, acting in the same direction, on two different points of a body, is parallel to them, similarly directed and equal to their sum. Its point of application will divide the line connecting the points of application of the individual forces into segments inversely proportional to*

the adjacent forces. The position of the point is independent of the direction of the parallel forces.

If a cord be attached to the scale at the point 4', and passed upward over a fixed pulley, and the weight  $2p$  be removed from 1 and attached to the other end of the cord, the bar will be acted upon by two opposite parallel forces, viz.: the force  $2p$  at 4 directed upward, and the force  $4p$  at 2' directed downward. Equilibrium will then be established if the weight  $4p - 2p = 2p$  be put upon the scale-pan, whence it follows that the combined action of the two forces is the same as the action of the single force  $2p$  directed downward at O. Hence the principle: *Two oppositely parallel (sometimes called anti-parallel) and unequal forces may be replaced by a resultant, equal to their difference and directed with the larger. The point of application of the resultant lies upon the line connecting the points of application of the two components, prolonged in the direction of the larger and so situated that its distances from the points are inversely as the components.*

Conversely, any force may be resolved into two parallel, or oppositely parallel, components, whose sum, or difference, is equal to it, and whose point of application is determined according to the two foregoing propositions.

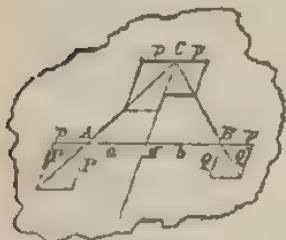


FIG. 14.—Parallel Forces.

forces  $P$  and  $Q$ . The resultant of  $P$  and  $p$  at  $A$ , is the force  $P'$  determined by constructing a parallelogram on  $P$  and  $p$ . At  $B$ , the force  $Q'$  represents the combined effect of  $Q$  and  $q$ .  $P'$  and  $Q'$  may be considered as having their points of application at  $C$ , the intersection of their directions. At their new point of application, imagine them again resolved into their former components. The components  $p$  and  $q$  will destroy each other and leave only  $P$  and  $Q$  to be considered. These latter, acting in the same direction and on the same point, will add their effects, and give the single resultant force  $P + Q$ .

The resultant may now be shifted arbitrarily in its own direction, and

The two preceding propositions may be derived also from the following considerations, based upon the principle of the parallelogram of forces. At the points  $A$  and  $B$  Fig. 14 of the body where the like directed parallel forces  $P$  and  $Q$  act, the two equal forces,  $p$  and  $q$ , are so applied as to work oppositely to each other, in the direction of the rigid connecting line  $AB$ . Since  $p$  and  $q$  destroy each other, they in no way modify the effect of the

may be considered as having its point of application anywhere in this line, as at M, its intersection with AB. From the similarity of the triangles AMC and BMC with the triangles into which the parallelograms are divided, there will result, if  $ma$  be designated by  $a$  and MB by  $b$ —

$$a : MC = p : P \text{ or } P \cdot a = p \cdot MC$$

$$b : MC = p : Q \text{ or } Q \cdot b = p \cdot MC,$$

and, consequently—

$$P \cdot a = Q \cdot b \text{ or } a : b = Q : P,$$

hence the first of the two propositions stated above.

If two oppositely parallel and unequal forces be applied at the points A and B (Fig. 15), the larger, P, may be decomposed into two components, one of which, Q, is oppositely equal to the force Q, and is applied at the same point, B, the other, R, will be equal to the difference of P and Q, and will act at the point M, which lies upon the prolongation through A, of the connecting line of the points of application, A and B, and divides this line into segments, such that MA is to AB as Q is to the difference between P and Q, or, what is the same thing, so that  $MA : MB = Q : P$ . Since the forces, Q and Q, destroy each other, there will remain as a resultant, which may fully replace the two forces, the single force R. This will equal the difference of the two given forces, and its point of application lies on the prolongation beyond A, of AB, at distances from A and B, which are inversely proportional to the corresponding forces.

This is the second of the propositions enunciated above. If the given distance, AB, be designated by  $a$ , and the distance, AM, by  $x$ , we shall have—

$$P \cdot x = Q \cdot (a + x) \text{ or } x(P - Q) = a \cdot Q,$$

from which there results the equation—

$$x = a \cdot Q \div (P - Q).$$

**26. Couples.**—If the two oppositely parallel forces P and Q (Fig. 15) approach equality, the difference,  $P - Q$ , approaches the value zero, and the point of application, M, recedes indefinitely. If P becomes, ultimately, equal to Q, we shall have a resultant force equal to zero, and acting upon an infinitely remote point. This is only the mathematical form of statement that, for this case, no resultant is obtainable. *Two equal and anti-parallel forces have therefore no resultant; they constitute what is called a couple—a mechanical combination of forces not capable of reduction to a simpler form.* It is evident that the forces of a couple cannot produce a motion of translation in the body. Their entire tendency is to produce a motion of rotation about an axis perpendicular to the plane, determined by the directions of the forces, i.e. perpendicular



FIG. 15.—Anti-Parallel Forces.

to the plane of the drawing. The rotational tendency, due to the couple, obviously increases both with an increase of the intensity of the forces  $P$  (Fig. 16), as also with an increase of the perpendicular distance between their parallel directions. The product,  $P \cdot a$ , of the force, into this distance, which is called the *arm* of the couple, serves as a measure of the rotational tendency, and is termed the *moment* of the couple. The effect of a couple is not altered if it be transferred and rotated by any arbitrary amount, either in its own plane or in any other parallel plane rigidly connected with the former. It may also be replaced by any other couple whose moment



FIG. 16.—Couple.

is the same as that of the given couple. A couple is completely determined by the direction of its rotation, its moment, and the position of its plane. The position of this plane, as also the direction of rotation, may be indicated by a perpendicular erected upon this plane, and extended toward the side from which the rotation appears right-handed, or in the sense of the hands of a watch. If the length of the perpendicular be made

proportional to the moment of the pair, the couple will be represented geometrically, by this so-called *axis*, in both magnitude and direction. Couples whose planes are parallel and whose axes coincide, may be replaced by a single couple whose moment, or axis, equals the sum of the moments or axes of the component couples. In the composition of couples, it must, of course, be borne in mind that moments in one direction are to be regarded as positive, while those in the opposite direction must be considered negative. They compound, therefore, in the case considered, precisely as forces acting on the same point and in the same straight line are compounded, i.e. by simple addition. Two couples whose planes and, consequently, also whose axes are inclined to each other, are compounded in accordance with the same rule as forces. The axis, or moment, of the resulting couple is in magnitude and direction, the diagonal of the parallelogram, whose sides are

the axes or moments of the given couples. Conversely, any couple may be resolved into two component couples.

**27. Composition of Forces acting upon Different Points of a Rigid Body.**—At any point  $O$  (Fig. 17) of the body let two forces be applied, one equal and parallel to one of the given forces, and the other equal and opposite to it. These two forces will wholly destroy each other, and, consequently, leave the body's condition as to rest or motion entirely unaltered. The oppositely directed

force and the given force form a couple. Hence, any force  $P$ , acting on the point  $A$ , may be replaced by an equal parallel force applied at the point  $O$ , rigidly connected with the point  $A$ , and a couple, whose forces are the given force and one equal and opposite to it. If, then, the same mode of



FIG. 17.—Composition of any Number of Forces.

treatment be applied to each of the forces acting on the various points of the body, and the point  $O$  be used for each force, all the forces will ultimately have been transported, each parallel to its own direction, to the point  $O$ , and in addition to these forces there will be also a couple for each of the given forces. The forces at  $O$ , on the one hand, and the axes of the couples on the other, may now be compounded according to the principle of the parallelogram of forces. It is thus apparent that any number of forces applied to a body at various points and in various directions may be reduced to a single force and a single couple, which would, in general, produce a simultaneous translatory and rotatory motion of the body.

The kite may serve to illustrate what has been said. Let  $k$  (Fig. 18) represent the central line of the surface of the kite, which is inclined at an angle  $\phi$  to the horizon.  $k$  designates the head, and  $s$  the tail. The pressure of the wind  $V$ , directed horizontally, and acting on the point  $q$  (the centre of gravity [24] of the surface), may be decomposed into the component  $W$ , acting along the surface and producing no effect, and the effective component  $Q$  acting upward against the surface at the point  $q$ , and directed along the perpendicular to the surface. At the centre of gravity of the mass of the kite  $p$ , which, by reason of the tail attached at  $s$ , lies below  $q$ , the weight  $P$  of the kite acts downward. Let the string be attached to the kite at  $o$ , which lies

between  $g$  and the head  $k$ . If now the forces  $P'$  and  $P''$ , as also  $Q'$  and  $Q''$ , equal and opposite to  $P$  and  $Q$  respectively, be applied at  $o$ ,  $P'$  and  $Q'$  give the resultant  $R$ , and the string, fixed at  $o$  and held at its lower end, assumes the direction of this resultant, and, by its tension, holds the kite in equilibrium. In addition to these forces, the kite will also be acted upon by the oppositely directed couples  $PP''$  and  $QQ''$ . To prevent rotation about  $o$ , the moments of these couples must, of course, be equal to each other, i.e. if  $ro$  denote the length of the perpendicular dropped from  $o$  upon the direction of  $P$ , we must have—

$$P \cdot ro = Q \cdot oq;$$

or, if  $qp$  be designated by  $a$ , and  $qo$  by  $b$ , we must have—

$$P(a+b) \cos \phi = Qb.$$

The kite will then float in equilibrium, rising meanwhile steadily in the direction of the force  $R$  as the string is let out gradually.

If  $U$  denote the normal component of the wind-pressure acting upon a superficial unit of the surface, the pressure  $V$ , upon the surface  $s$ , inclined to the direction of the wind at the angle  $\phi$ , is given by

$$V = U s \sin \phi,$$

and the effective component  $Q$  by

$$Q = U s \sin^2 \phi.$$

From the foregoing equation, viz.

$$P(a+b) \cos \phi = U \cdot b s \sin^2 \phi,$$

it is at once evident that, for any given intensity of wind-pressure, the angle between the string and the horizontal is greatest, and consequently also the kite will rise highest if the position of the points  $o$ ,  $q$ ,  $p$ , be so chosen that—

$$a = 2b.$$

**28. Centre of Parallel Forces—Centre of Gravity.**—By repeated application of the foregoing proposition for the composition of parallel forces, any number of parallel forces whatever may be compounded into a single resultant. To accomplish this we need merely to compound the resultant of the first pair of parallel forces with the third, this new resultant with the fourth, and so on. In this way we should finally obtain a total resultant, equal to the sum of all the given forces, and acting at a definite point called the *centre of the parallel forces*. The point of application of the resultant of all the elementary forces of gravity, acting upon the various particles of a heavy body, is called the *centre of gravity*

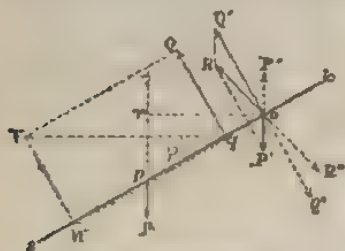


FIG. 18.—The Kite.

of the body's mass. Since these elementary forces are all directed vertically, and consequently are parallel to each other, their resultant equals their sum, i.e. equals the entire weight of the body, and the centre of gravity is the centre of the system of parallel forces of gravity. In this latter point the entire weight of the body may be regarded as concentrated, and it must be supported in order that the body maintain its equilibrium against gravity. If, for example, a body be suspended in any manner, it will be in equilibrium when its centre of gravity is vertically under the point of suspension. Upon this principle depends an experimental method of finding the centre of gravity of a body. When a body is suspended by means of a thread attached to the point



FIG. 19.—Lines of Gravity.

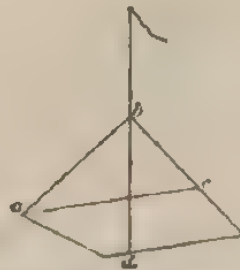


FIG. 20.—Centre of Gravity.

A (Fig. 19) of its perimeter, the prolongation of the thread must pass through the centre of gravity. Any straight line, as  $ac$ , drawn through a body's centre of gravity is called a *line of gravity*. If now the body be suspended from a second point  $b$  (Fig. 20), the centre of gravity must also lie on the prolongation of the line of suspension, and another line of gravity,  $bd$ , will be located. The centre of gravity must, therefore, lie at the intersection of  $ac$  and  $bd$ . With homogeneous bodies of determinate geometrical form, the centre of gravity may frequently be found from simple mathematical considerations. The geometrical centre of a sphere, or of an ellipsoid, is obviously also the centre of gravity of the body. The centre

of gravity of a cylinder with parallel ends is evidently at the middle of its axis, and that of a parallelopiped at the intersection of its diagonals. The centre of gravity of a triangular surface lies in the medial line and two-thirds of the distance from any vertex to the opposite side; that of a pyramid, or of a cone, upon the line from its vertex to the centre of gravity of its base, and at a distance equal to one-fourth of its length from the base. The centre of gravity of a body may even lie wholly without its mass, as is the case with a hollow sphere, a shell, a flask, etc., where it lies within the hollow of the body.

Conceive, for example, of a triangle, ABC (Fig. 21), cut from a piece of tin and divided into slender strips by lines parallel to the side AC. The centre of gravity of each strip obviously lies in its middle point; consequently, the



FIG. 21.—Centre of Gravity of a Triangle.



FIG. 22.—Centre of Gravity of a Pyramid.

centre of gravity of the entire triangle lies on the line BD, drawn from the middle point D, of the line AC, to the vertex B, because this line bisects each strip. Or we may say the "transversal" BD is a line of gravity of the triangle. For similar reasons the line CE is also a line of gravity, and hence the centre of gravity must lie at their intersection S. Through this point the third transversal must also pass. Connect the points D and E and the triangle DSE formed by it, SD and SE is similar to the triangle BSC. We have, then—

$$DS : BS = ES : CS = DE : BC = 1 : 2,$$

since, by geometry,  $DE = \frac{1}{2}BC$ ,  $DS = \frac{1}{2}BS$ , and  $ES = \frac{1}{2}CS$ , or  $DS = \frac{1}{3}BD$  and  $ES = \frac{1}{3}CE$ . To find the centre of gravity of a polygon, it is decomposed by diagonals into triangles, and the centre of gravity of each triangle is determined. At the centre of gravity of each triangle a weight proportional to the area of the corresponding triangle is supposed to be applied, and the centre of gravity of this system of parallel forces is then determined.

A triangular pyramid ABCD (Fig. 22) may be conceived as decomposed by planes parallel to one of its faces, ABC, into thin laminae. The straight line DE, drawn from the vertex D to the centre of gravity E of this face, contains the centre of gravity of all the laminae, and is accordingly a line of gravity of the pyramid. The same is true of every line CF drawn from any other vertex C to the centre of gravity of the opposite face, and consequently

the point  $S$ , at which the lines  $De$  and  $Cf$ , lying in the plane  $CGD$ , intersect, is the centre of gravity of the pyramid. Since  $GE = \frac{1}{3}GC$ , and  $GF = \frac{1}{3}GD$ ,  $EF = \frac{1}{3}ED$ , and hence  $ES = \frac{1}{3}SD$ , or  $ES = \frac{1}{4}DE$ . The centre of gravity of a pyramid lies, therefore, upon the straight line drawn from the apex to the centre of gravity of the opposite face, and at a distance equal to three fourths of its length from the vertex. This proposition holds for any polygonal pyramid, or cone. Similar considerations suffice to locate the centre of gravity of many other surfaces and bodies.

The position of the centre of gravity may also be found by computation. For example, in Fig. 23, suppose any number of material points,  $m, m', m'', \dots$  to act upon a rigid straight line at distances  $r, r', r'', \dots$  respectively, from a given point  $A$  of the line. The moment with respect to the point  $A$  of the entire mass ( $M = m + m' + m'' + \dots = \Sigma m$ ), supposed concentrated at the centre of gravity  $S$ , at the distance  $s$  from  $A$ , is equal to the sum of the moments of the individual forces, or  $mr + m'r' + m''r'' + \dots = \Sigma mr$ ; and we have, therefore,  $Ms = \Sigma mr$ , and consequently,

$$s = \frac{\Sigma mr}{M} = \frac{\Sigma mr}{\Sigma m}.$$

If, as a special case, only two masses be given, at distances  $r$  and  $r'$ , there results—

$$s = \frac{mr + m'r'}{m + m'}.$$

**29. Lever.**—Let a body which is movable about a fixed axis at  $U$ , perpendicular to the plane of the drawing, be acted upon at  $A$  and  $B$  (Fig. 24) by two forces,  $P$  and  $Q$ , lying in the same plane. The body would, under these circumstances, have a motion of rotation about this axis, but no motion of translation. This rotation will also be neutralized, or the body will be in equilibrium as regards rotation, if the resultant  $R$ , determined according to § 23, passes through a point,  $M$ , of the fixed axis. If the point  $M$  be connected with the vertices  $D$  and  $E$  of the parallelogram  $CDFE$ , used to construct the resultant  $R$ , and perpendiculars  $a$  and  $b$  be dropped from  $M$  upon the direction of the components  $P$  and  $Q$ , the triangles  $MDC$  and  $MEC$ , having a common base  $MC$ , and equal altitudes  $h$ , will have equal areas. The double areas of these

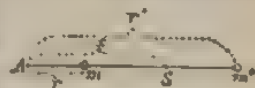


FIG. 23.—Computation of the Centre of Gravity.

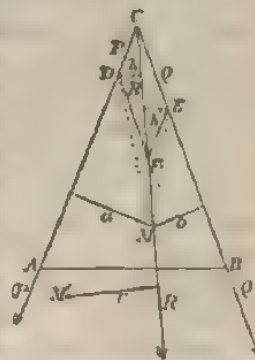


FIG. 24.—Moment.

triangles are also expressed by the products  $Pa$  and  $Qb$  respectively. In order that the resultant pass through the fixed axis and no rotation occur,  $Pa$  must equal  $Qb$ . The product of a force into the perpendicular from any point upon its direction is called the *static moment* of the force with respect to the point. It expresses the tendency of the force to produce rotation, and has the same value as the moment of the couple, which remains after the force, together with another equal and opposite to it, has been transformed, according to § 27, to a fixed point of the axis, which resists the force. The body will be in equilibrium when the moments of the two forces are equal, i.e. when  $Pa = Qb$ , or what is the same thing, when the moment of the resultant couple  $Pa - Qb = 0$ . If the axis should pass through any other point,  $M'$ , at the distance,  $r$ , from the resultant,  $R$ , a rotation about  $U'$  with the moment,  $Rr$ , would ensue.

A body which may be rotated about a fixed axis, and on which forces act in planes perpendicular to this axis, is called a *lever*. The perpendicular distance of any force from the axis is called the *lever-arm* of the force. The lever is in equilibrium if the sum of the products of each force into its lever-arm, or the sum of all the moments, is zero, those forces acting in one direction being considered positive, and those in the opposite direction, negative; because, according to § 27, the given forces may be reduced to a single resultant acting upon some point of the axis and a single couple. The former is destroyed by the reaction of the axis, and the latter can produce no rotation when its moment, which equals the sum of the moments of all the opposite couples, is equal to zero. The simplest form of the lever is an inflexible, weightless, straight line, movable about one of its points, at whose ends like-directed, parallel forces, such as suspended weights, act perpendicularly to its length. The fixed point, about which the line moves, is called the *fulcrum*. With such a lever (Fig. 25) the two segments of the line,  $MA$  and  $MB$ , from the fulcrum to the points of application of the forces, are the lever-arms. When a lever is of this description it is called a *two-armed lever*, or a lever of the *first class*. It will be in

equilibrium if the product of the one force into its lever-arm equals that of the other force into its lever-arm; or, what amounts to the same, when the forces are in the inverse ratio of their lever-arms. An *equal-armed* lever is in equilibrium when the forces applied at the ends are equal, as is exemplified by beam balances.

If the movable bar be acted upon by forces having opposite directions, in order that each may destroy the rotational tendency of the other, both must act upon the same side of the point about which the motion takes place (Fig. 26), as in the foregoing case. They will hold each other in equilibrium if they are inversely proportional to their distances from the fulcrum. Although in this case also there is a lever-arm corresponding to each force,  $MA$  and  $MB$ , since only the longer

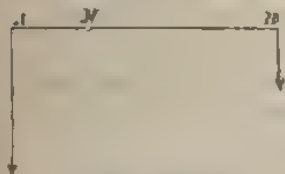


FIG. 25.—Lever.



FIG. 26.—Lever.

obtrudes itself upon consideration, and the shorter is only a part of the longer, this sort of lever is sometimes designated as a *one-armed* lever, or, more generally, as a lever of the *second class*.

The experimental proof of the laws of the lever has been given above, by means of the movable metal scale of Fig. 13; for we need only fix the support of the axis of the scale to convert it into a lever, whose fixed axis destroys the resultant, and at the same time supports the weight of the bar, and so operates precisely as though it had no weight. If the centre of gravity of a lever lies without the axis, its weight must be regarded as an additional force, acting downward at the centre of gravity of the bar.

By the aid of the lever, a heavy load may be held in equilibrium by a small force, and by slightly increasing the force, the load may be raised, provided the lever arm of the force be made just as many times longer than that of the load as the latter

exceeds the force. The crowbar affords a simple illustration of this. To move a heavy block of stone from its position, the workman pushes one end of the heavy iron rod under the block, places near this end a small stone, to serve as a fulcrum, and by his muscular force depresses the long arm of the lever, thereby raising slightly the mass of stone. A well-known application of the one-armed lever is had in the familiar *truck*. The point of rotation is on the axis of the wheels, and the muscular force of the arm directed upward, holds in equilibrium the load placed in the car and near the axis of the wheels. By the aid of wheels, which exercise no influence upon the leverage of the apparatus, the workman is then able to move the load about.

Levers of the most varied forms find frequent application in daily life. The iron handle to which the wires of a door-bell are fastened, and which serves to convert the vertical pull of the hand into a horizontal pull on the bell, is a bent lever, with arms forming a right angle to each other. A key is a lever, turning about the axis of its length; the ward representing the one arm, and the handle the other. Scissors, tongs, nut-crackers, etc., are combinations of levers.

The windlass, which is an apparatus consisting of a cylinder with an attached circular disc of greater diameter than that of the cylinder, is a lever. The fulcrum of the lever is in the axis of the cylinder. The load, fastened to the end of a rope wound about the cylinder, will be held in equilibrium by the force acting at the circumference of the wheel, provided the force bears the same ratio to the load as does the radius of the cylinder to that of the wheel. The wheel is sometimes replaced by a single spoke, called the *crank*.

A series of levers, which act upon each other at their ends, is called a compound lever, or a *train of levers*. Such a lever will be in equilibrium when the force at the end of the last lever is to that at the beginning of the first as the continued product of all the arms next the load is to the continued product of all the arms next the power. Systems of toothed wheels engaging in each other are compound levers whose arms are the radii of the wheels.

Any force acting on a body, free to turn about an axis, may,

without alteration of effect upon the body, be replaced by any other force whose rotational tendency is the same as that of the replaced force. The intensity of the new force must be to that of the old, inversely as their respective arms, else the effect upon the body will not remain unaltered.

**30. Pulleys.**—The *pulley* is a special kind of lever. It consists of a circular disc free to turn about an axis passing through its centre and supported in a case, called a *block*. Around the circumference of the disc a groove is cut for the reception of a rope, or cord. The pulley is said to be *fixed*, when its block is suspended to a fixed point, in such manner as to prevent any translatory motion of its centre. It will evidently be in equilibrium when equal forces act at the ends of the cord, since the rotational tendencies, or the moments of the two forces, are then equal and opposite, and their resultant acts merely as a pressure against the fixed axis. Such a pulley, of course, has no tendency either to increase or diminish the weight of a load; it merely serves to change the direction of the force. It may be advantageously used to convert the downward pull of a mass of stone, attached to the end of a rope into an equally strong upward pull, or to make use of the horizontal pull of a horse to raise heavy loads in a vertical direction. A pulley whose block is free to move is called *movable*. An illustration of such a pulley may be had by attaching one end of a rope to a fixed point, hanging a load to the block and applying a force at the other end of the rope. To hold this system in equilibrium both ends of the rope must be stretched by equal forces, whose resultant, passing through the axis of rotation of the pulley, keeps the weight balanced. If, for example, the ends of the rope are parallel and vertical, obviously, to maintain equilibrium, each end of the rope must carry half the load. By means of the movable pulley, therefore, the load is, so to speak, diminished by one half. If the free end of the rope be attached to the block of a second movable pulley, this free end attached to a third movable block, and so on, and the last end be passed over a fixed pulley, the load will be reduced by half with each movable pulley. By means of such an apparatus, called a *train of pulleys*, a small force acting at the last end of the rope may

be made to hold in equilibrium a large weight attached to the block of the first movable pulley, and by a slight increase of the power applied, above what is needed to establish equilibrium, the load may be raised. This same purpose is subserved by what is called a system of *blocks and falls*. This consists of a fixed and a movable block, each containing a number of pulleys set on a common axis, and so connected, that the cord, one end of which is attached to the fixed block, passes alternately over a pulley of the movable and fixed blocks, and is finally connected with the force applied to the system. Since the load attached to the movable block is distributed over twice as many cords as there are pulleys in each block, to hold the system in equilibrium, it is necessary to apply to the end of the cord, which hangs from the fixed block (called the *tackle fall*), a force equal to such a fractional part of the load as is represented by the number of pulleys in both blocks.

31. *Machines* are contrivances by which forces may be made to subserve useful purposes. The *simple machines*, sometimes called the *mechanical powers*, to which the parts of all compound machines may be reduced, are the various forms of the *lever*, including the pulley, the windlass, etc., and the different species of *inclined planes*, such as the screw, the wedge, etc. Machines simply transmit and modify the action of forces. They do not either save, or waste, the working forces applied to them. The

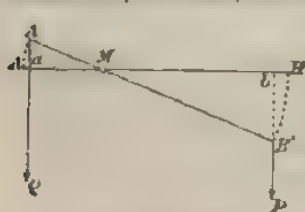


FIG. 27. — Lever.

work expended by the moving force is always equal to that performed upon the load, or upon whatever resistances are to be overcome, as was above proved for the inclined plane.

A lever (Fig. 27) is in equilibrium when the product of force into lever arm is the same on both sides of the fulcrum.

If, now, the lever be drawn out of its horizontal position of equilibrium,  $AMB$ , into the inclined position,  $A'M'B'$ , and the load be raised by the smaller force, the work of the force equals the product of the force  $P$ , into the distance  $BB' = \delta$ , by which the point of application is depressed, and the work to be done

to raise the load equals the load  $Q$  into the distance  $aA' = \delta$ . Since the distances  $aA'$  and  $bB'$  are to each other as the corresponding lever arms,  $MA'$  and  $MB'$ , we must have  $P\delta = Q\delta'$ , i.e. the work of the force equals the work of the load.

But if the load  $Q$  be raised by means of a tackle and falls, by a force,  $P$ , equal to  $\frac{1}{2}$  of  $Q$ , the load will rise with  $\frac{1}{2}$  of the velocity, with which the point of application of the force is depressed, and the work performed by force and load will again be equal, or  $P\delta = Q\delta'$ , if  $\delta$  and  $\delta'$  represent the respective simultaneous displacements of the points of application of  $P$  and  $Q$ , in the direction of the forces  $P$  and  $Q$ . This general proposition has been expressed in the following form as the "Golden Rule of Mechanics." *What is gained in force is lost in velocity.*

Instead of  $P\delta = Q\delta'$ , we may also write  $P\delta - Q\delta' = 0$ , or,  $P\delta + Q\delta' = 0$ , if displacement in the direction of the motion of the point of application of the force acting be considered positive, and that in the opposite direction, negative. If, then, a machine is in equilibrium, and any small displacement of the point of application possible to the machine, or, more briefly, any *virtual* displacement of the point of application, occurs, the sum of all the *effective*, or *positive*, quantities of work and the *resisting*, or *negative*, quantities of work will equal 0. This holds not only for simple machines, and for a single pair of forces, but it is also true of any number of forces operating in any manner whatever upon a system of rigidly connected points. We are thus led to the following general principle of *virtual work*, variously referred to as the principle of virtual moments, or the principle of virtual velocities. *If any rigidly connected system of material points whatever, acted upon by forces, be in equilibrium, the sum of the virtual work of the forces equals zero for all possible, or virtual, displacements of the points of application, or if  $P, P', P'', \dots$  represent the forces,  $\delta, \delta', \delta'', \dots$  the components of the displacements in the directions of the corresponding forces,*

$$P\delta + P'\delta' + P''\delta'' + \dots = 0.$$

As an illustration, the differential pulley may be cited (Fig. 28). About the peripheries of two pulleys, fixed to a common axis and rigidly connected with each other, and a movable pulley to which the load  $P$  is connected, let

an endless cord, or chain, be passed, as shown in Fig. 28, but in such manner as to be secure against slipping with respect to the pulley. Let the radii of the rigidly connected pulleys be denoted by  $R$  and  $r$ , and suppose them nearly equal to each other. If, now, the cord be drawn in the direction of the arrow with the force  $P$ , through the distance  $s$ , the portion of the cord designated by  $a$  will be rolled up by an amount  $s$  on the pulley  $R$ , while the portion  $b$  will be unrolled by an amount  $s \cdot r$  and, consequently, the load  $P$  will rise by an

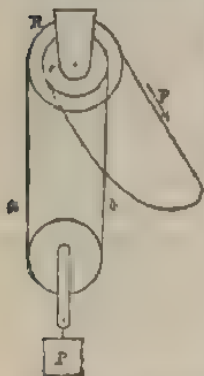


FIG. 28.—Differential Pulley.

amount  $s' = \frac{1}{2}s \left(1 - \frac{r}{R}\right)$ . To maintain equilibrium  $P_0 - P_0'$  must equal 0, and hence, since the arbitrary displacement  $s$  divides out, we have—

$$P = \frac{1}{2}P' \left(1 - \frac{r}{R}\right).$$

**32. Stability.**—The capability of a body to maintain its position against gravity is called *stability*. A body remains in its position upon a horizontal plane, if the vertical line drawn through its centre of gravity, where the weight is conceived as concentrated, pierces the supporting surface, or *base*, of the body. When a body is supported upon separate points of its figure, the base is to be regarded as the surface obtained by connecting the outermost supporting points with straight lines. The base of a man, when standing, is not merely the soles of his feet, but it includes all that surface lying between his feet, bounded on either side by the sole of a foot, in front by a straight line connecting the points of his toes, and in the rear, by a straight line connecting the heels. If he attempt to carry a burden, to



FIG. 29.—Stability.

prevent falling, he must so incline his body as to bring the vertical line through the common centre of gravity of both load and body within this base. To overturn a body it must be rotated about an edge, or a point ( $a$ , Fig. 29) of the perimeter of its base, until its centre of gravity stands vertically above that point. If the body be released before this position is attained it will return to its original position, but if it be

turned slightly beyond this position, it will fall over and assume a new position of equilibrium. The turning may be supposed produced by a horizontal force,  $K$ , applied at the centre of gravity,  $S$ , in which case the moment of this force must be at least equal to the opposite moment of gravity,  $G$ , or the force,  $K$ , multiplied by its distance,  $ab$ , from the centre of rotation, i.e. by the height of the centre of gravity above the base, must be equal to the force  $G$ , or to the weight of the body, multiplied by the distance  $ac$  (half the width of the base) from this same centre of rotation. The *stability* of a body, of which the force,  $K$ , affords a measure, is, therefore, directly proportional to the weight and to the width of the base of the body, and inversely proportional to the height of the centre of gravity above the base. We may, then, say a body stands in more stable equilibrium the greater its weight, the broader its base, and the lower its centre of gravity. To give candelabra, cloak-racks, and various other articles of furniture the greatest stability, they are made with long and widely-extended legs, that the base may be as extensive as possible, consistently with convenience. To prevent lamps from too easy overturning, they are provided with heavy bases, to the end that the centre of gravity may have as low a position as possible. For this same reason, in loading wagons, the heaviest portions of the freight are put underneath and the lighter portions toward the top of the load.

A body, free to move about a fixed horizontal axis, will be in equilibrium against gravity in any position whatever, provided the centre of gravity lies exactly upon the axis. The body is, under such circumstances, said to be in *indifferent* or *neutral* equilibrium. If the centre of gravity be vertically above this axis, and the body be slightly turned, it will be drawn by gravity still farther toward the side toward which the displacement occurred, and in this case the body is said to be in *unstable* equilibrium. The rotation will continue until the centre of gravity lies vertically below the axis, and in this condition, the body is said to be in *stable* equilibrium; for, if it be displaced from this position, it will seek to return to it. The centre of gravity of any body always seeks the lowest possible

position, and this is the position of stable equilibrium. A chain, for example, which hangs slack, and is supported at its ends, assumes of its own accord that position in which its centre of gravity is as low as possible. Upon this principle various forms of toys depend, such as cylinders, and double cones that roll uphill, standing figures, and so forth, which at first glance seem to contradict, but, in point of fact, confirm this principle.

Another illustration of the principle that the centre of gravity always seeks its lowest position is afforded by a bar suspended by two fibres,  $aA$ ,  $bB$ , of equal length, as shown in Fig. 30. Such form of suspension is known as *hyilar*.



FIG. 30.—Bifilar Suspension.

If the bar be twisted out of its position of equilibrium, its centre of gravity will be raised from  $O$  to  $O'$ , the latter lying in a higher horizontal plane,  $A'O'C$ , than the former. At  $A'$ , and also at  $B$ , the force  $AD = \frac{1}{2}P$ , i.e. one half the weight of the bar acts downward. This force may be resolved into two components perpendicular to each other, the one,  $A'E$ , in the direction of the thread, and tending to stretch it, and the other,  $A'F$ , tending to draw the end  $A$  of the bar in the direction of the tangent to the circular arc  $A'A$ , back to the original position  $A$ . From the similarity of the triangles  $A'DE$  and  $A'aC$ , we have  $A'E : AD = A'C : A'a$ , or, designating the angle  $A'O'C$ , which the new position of the bar makes with the position of equilibrium, by  $\alpha$ , half the length of the bar by  $r$ , and the length of fibre by  $l$ , this proportion may be written thus—

$$A'E : \frac{1}{2}P = r \cdot \sin \alpha : l, \text{ or } A'E = \frac{Pr \sin \alpha}{2l}.$$

Consequently, the moment of the couple for an equal and opposite force acts at  $B$ , tending to draw the bar back into the position of equilibrium, is given by—

$$A'E \cdot 2r; \text{ or, } l^2 \cdot \frac{\sin \alpha}{l}.$$

**33. The Balance.**—The balance serves the purpose of comparing *unknown* with *known* masses, the latter being called *weights*, or *counterpoises*. The customary form of the apparatus is essentially a lever with equal arms. It consists of a light bar, or beam, turning about a steel knife-edge, attached near the middle of its length and supported upon a steel, or agate, plate. The ends of the bar are also supplied with knife-edges, on which hooks, or bearings, carrying the scale-pans, are borne. When not in use, or when putting on or taking off weights, by means

of an attachment known as the *arrest*, the weight of the beam and scale-pans may be taken off the knife-edges, to prevent unnecessary wear and consequent impairment of the delicacy of the balance. In order that the beam may be in stable equilibrium and in a horizontal position, when not in use, or when equal weights are in both scale-pans, the centre of gravity of the combined mass of all the parts of the apparatus must lie beneath the middle knife-edge. A needle, or pointer, fixed perpendicularly to the beam, and oscillating with it, in front of a graduated arc, shows by its position with respect to the index, or the zero-point of the graduated arc, when the mechanism is in equilibrium. If a smaller weight be placed in one of the scale-pans, the beam will incline toward the side of this pan, and the centre of gravity will be raised on the opposite side until the weight of the beam, applied to the centre of gravity of the apparatus, has attained a rotational tendency equal and opposite to that of the overweight, applied at the end of the beam. The angle which the pointer in this new position makes with the vertical is called the *angle of displacement*. That the same overweight may always give the same displacement for any magnitude of total load upon the scales, the middle knife-edge must lie in the same plane with those at the ends. The sensitiveness of the balance is greater, the greater the angle of displacement for a given overweight. That the lever-arm of the beam may be sufficiently long, its centre of gravity must be raised higher, and the angle of displacement made greater; the smaller the weight of the beam, the nearer its centre of gravity lies to the knife-edge, and the greater the lever-arm of the overweight, or the longer the beam of the balance. To secure as high a degree of sensitiveness as possible, the centre of gravity is located quite near the knife-edge, and the beam is made as long and light as is consistent with the desired rigidity. This is ordinarily effected, where a high degree of precision is sought, by giving the beam a broken form. A fine screw, on which a heavy nut travels, is usually placed above the beam to regulate the position of the centre of gravity. For weighing bodies lighter than 1 cg., a bent wire of platinum weighing one cg. is placed

upon an arm of the beam, graduated into ten equal parts, and the wire is then shifted along the arm until equilibrium is restored. The number of the graduation then indicates how many milligrams would have to be placed in the scale-pan to hold the unknown weight in equilibrium.

If the lever-arms of a balance should not be exactly equal, which for mechanical and practical reasons is always the case, it is still possible to weigh accurately with it. This is done either by *double weighing*, i.e. by weighing the body in each pan, and taking the mean, or by *substitution*, i.e. by bringing the body into equilibrium by means of any objects, such as shot, for example, placed upon the other scale-pan and then removing the body, and again restoring the equilibrium by placing known weights upon the scale-pan formerly supporting the body. The *sensitiveness* of a balance is expressed by the ratio of the least weight which gives a distinct displacement to the greatest weight in one of the pans. A balance of ordinary sensitiveness will give, for a weight of one kg., and an overweight of one mg., an appreciable displacement, and the sensitiveness is therefore one-millionth. The more delicate balances used for scientific purposes have a much higher sensitiveness than this. Many of them give an appreciable displacement with a weight of 1 kg. and an overweight of 0.003 mg., and have accordingly a sensitiveness of one three-hundred-millionth. Accuracy of weighing is also materially enhanced by placing a small mirror above the middle of the beam and perpendicularly to its length, and then viewing by means of a telescope the image of a finely graduated scale placed beside the mirror.

For scientific purposes balances, with equal arms, are used almost exclusively, whereas, in commerce, those with unequal arms are in more general use.

The Roman balance, or steelyard, is a lever having two unequal arms, to the shorter of which the body to be weighed is suspended. The body is held in equilibrium by a single weight, called the *poa*, which may be shifted along the longer arm to any desired position. This arm is graduated, and the graduations are numbered in such manner that the weight of

the body can be read off directly from the notch in which the pea must be placed to establish equilibrium.

To weigh heavier loads the platform scales are used. Here a wooden platform, or bridge, is carried by a combination of levers so disposed with reference to each other as to keep the platform always in a horizontal position, and to exert upon the load a pull equal to the weight, regardless of the particular point of the platform at which the body to be weighed is situated. In case of the decimal platform scale, introduced by Quintenz in 1823, the arrangement of the parts is as follows:

The weight,  $Q$  (Fig. 31), lying upon the platform,  $AB$ , which is movable about a knife-edge near  $A$ , may be resolved into the two forces,  $q$  and  $q'$ , whose sum shall equal  $Q$ , the one,  $q$ , acting at  $B$ , directly upon the rod,  $BF$ , attached to the beam,  $GH$ , at  $F$ , and the other,  $q'$ , acting at  $A$ , upon the lower lever,  $CD$ , turning about the knife-edge,  $C$ , and exerting a pull at  $D$  upon the rod,  $GD$ . If the lever-arm,  $CD$ , is  $n$  times as long as  $CA$ ,  $q'$  may be replaced by the force,  $\frac{q'}{n}$ , at  $D$ . If, now,  $EG$  is made  $n$  times as great as  $EF$  ( $E$  being the point about which the beam rotates), we may again replace the force,  $\frac{q'}{n}$ , at  $G$  by the force,  $q'$  ( $n$  times as great as  $\frac{q'}{n}$ ), acting at  $F$ . The load,  $Q = q + q'$ , will act, therefore, as if it were suspended directly to the rod,  $BF$ , and this is true, no matter at what point of the platform the load,  $Q$ , may lie. If the lever-arm,  $EH$ , carrying the scale-pan, be made 10 times (or 100 times) as long as the arm,  $EF$ , the load may be held in equilibrium by one-tenth (or one one-hundredth) of its own weight. This scale is consequently called the *decimal* (or *centesimal*) scale.

In various other weighing apparatus, such as the *pointer balance*, the longer arm of a bent lever moves over an experimentally graduated, circular scale when the load is placed on the shorter end.

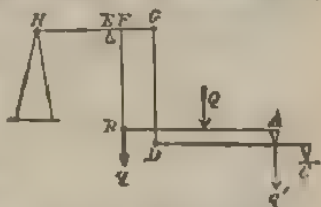


FIG. 31.—Platform Scales.

A balance used extensively in commerce, and known as the *counter scales*, was devised in 1670 by Roberval, and is illustrated in its essential parts in Fig. 32. It consists of the beams  $AB$  and  $A'B'$  equal in length, the one movable about the point  $M$ , and the other about  $M'$ . These beams are connected by two vertical rods,  $AA'$  and  $BB'$ , so as to form a parallelogram. Each vertical rod carries at its upper end a horizontal scale-pan rigidly connected with it. If the parallelogram is in equilibrium in a horizontal position, and is then displaced into the inclined position indicated by the dotted lines, the pan  $A$  rises by just as much as the pan  $B$  sinks, and since, according to the principle of virtual work, when the apparatus is in equilibrium, the



FIG. 32.—Counter Scales.

work on both sides must be the same, the force acting on one side must equal that acting on the other, i.e. the weights of the bodies in the scale-pans are equal.

**34. Central Motion.**—We will now consider the motion of a body, which, after an initial velocity has been imparted to it,

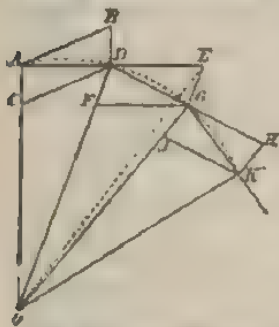


FIG. 33.—Central Motion.

is subject to the continuous action of a force directed toward a fixed point called the *centre*. By virtue of its inertia, the body strives to move with this initial velocity always in the direction of the line,  $AB$  (Fig. 33). The force, which is always directed to the point  $O$ , and is called the *centripetal force*, strives continually to draw the body out of its path. If  $AC$  denotes the distance this force

causes the body to traverse toward the centre, during the time required for its inertia to carry it from  $A$  to  $B$ , the actual place,  $D$ , of the body after the lapse of this interval of time is the point of intersection of the lines,  $BD$  and  $CD$ , drawn parallel to  $AB$  and  $AC$  respectively. The path traversed by the body from  $A$  to  $D$  is really a curve, but it approaches the line  $AD$  more and more

closely, the shorter the interval of time considered. If we take this interval sufficiently small (and we may take it as small as we choose), the path from A to D may be regarded as *rectilinear*. During the next equally small interval, the body, by virtue of its inertia, would traverse in the direction, and with the velocity it has at D, the distance  $DE = AD$ , if it were not acted upon by the force directed from D to O. This latter force, however, draws the body aside from the line DE by the distance DF, and brings it at the end of the interval to the vertex, G, of the parallelogram, DEGF. Similarly, during a third equal interval, the body, instead of traversing the distance GH, equal to and similarly directed with DG, moves to K, the vertex of the parallelogram, GHKI, and so on for succeeding intervals of time. The body, therefore, under the influence of the force continually drawing it towards the centre O, will traverse the curved line, ADGK, which comes more and more nearly into coincidence with the broken line, ADGK, the shorter we make the intervals of time considered. The direction of motion at any point of the curved path is obviously determined by the tangent to the curve at this point. The rectilinear motion the body would have by virtue of its inertia, if at any given point the central force should cease, is called its *tangential motion*. The straight line, conceived as drawn from any instantaneous position of the moving body to the centre of the circle, is called the *radius vector* of the body. While the body moves from A to D, the radius vector sweeps out the area AOD, and while passing from D to G, it sweeps out the area DOG, etc. These areas, which are in reality bounded by the arcs AD, DG, etc., differ but little from the triangles, AOD, DOG, etc., and these differences become less and less the smaller the corresponding intervals of time. It is readily seen that the triangles DOG and AOD have equal areas, and so with any pair of consecutive triangles.<sup>1</sup> The following proposition, therefore, results:—*When a body is acted upon by a*

<sup>1</sup> This readily appears by drawing the line OE, and noticing that triangle DOG is equivalent to triangle DOE, since EG is parallel to their common base DO. But AOD is equal to DOE, because these triangles have equal bases ( $AD = DE$ ) and their vertices lie at the same point O. Consequently  $DOG = AOD$ .

central force, its radii vectores sweep out equal areas in equal times. The converse of this proposition is also true. If a body moves so that the radii vectores to any fixed point sweep out equal areas in equal times, it must be acted upon by a force always directed toward the point. For the special case of a rectilinear motion the force equals zero.

**35. Centripetal Force.**—Suppose a material point, A (Fig. 34), to be moving in a circular path with radius  $r$  and velocity  $v$ ;



FIG. 34.—Centripetal Force.

the distance, AB, traversed along the circumference during the short interval of time  $\tau$ , equals  $v\tau$ . Let the perpendicular, BC, be dropped from B upon the diameter AE through A. AC is then the small distance by which the point during the time  $\tau$  is drawn aside from the tangent, AD, along which it would move by virtue of its inertia. The right triangle, whose hypotenuse is the chord, AB, is similar to the triangle, AEB, whose hypotenuse is the diameter,  $AE = 2r$ , of the circle. We have, therefore—

$$AC : AB = AB : AE.$$

The shorter the interval,  $\tau$ , and hence, also the distance, AB, be made, the more nearly does the arc ( $AB = v\tau$ ) approach equality with the chord, AB. For a small enough interval, therefore, we have—

$$AC : v\tau = v\tau : 2r.$$

and, consequently,

$$AC = \frac{v^2 \tau^2}{2r}.$$

During a very small interval of time, the force directed toward O may be regarded as invariable, and the motion produced by it as uniformly accelerated. Denoting the corresponding acceleration by C, the space traversed during the time  $\tau$  becomes  $AC = \frac{1}{2}C\tau^2$ .

Equating the two values of AC, we obtain—

$$\frac{1}{2}cr^3 = \frac{v^2 r^3}{2r},$$

and, consequently, for the centripetal acceleration,

$$c = \frac{v^2}{r}.$$

The centripetal force C is obtained by multiplying the acceleration, c, by the mass, m, of the moving point. There results,

$$C = \frac{mv^2}{r}.$$

*The centripetal force is directly proportional to the mass and to the square of the velocity and inversely proportional to the radius of the path.*

This proposition, which results from the consideration of uniform motion in a circular path, holds generally for all curvilinear motion. For, at any point of a curve, a circle (the osculating circle) may be drawn, whose circumference coalesces with the given curve at the chosen point, so closely that the material point, while traversing this portion of its path, moves for a short time on the circumference of this circle. For curvilinear paths, therefore, we have only to substitute in the foregoing expression, the value of the radius of curvature,  $\rho$ , corresponding to the point in question for the radius,  $r$ . The quotient,  $\frac{1}{\rho}$ , is taken as the measure of the curvature of the curve.

It is, moreover, *a priori* apparent that the force required to deflect a body from the straight line must be greater, the greater the *vis viva*, or energy ( $\frac{1}{2}mv^2$ ), of the body and the sharper the curvature,  $\frac{1}{\rho}$ , of the path.

With uniform circular motion, the period of revolution, T, is ordinarily used in place of the velocity,  $v$ . T being the time of revolution about the entire circumference  $2\pi r$ , we may write

$$v = \frac{2\pi r}{T}.$$

Substituting this value of  $v$  in the foregoing value of  $C$ , we find

$$C = \frac{4\pi^2 mr}{T^2},$$

and we may say:—*The centripetal force of a body moving uniformly in a circle is directly proportional to the mass and the radius, and inversely proportional to the square of the period.*

**36. Centrifugal Force.**—A locomotive running on a curved track strives continually, by reason of its inertia, to leave the track in the direction of the tangent, AB (Fig. 35), and to

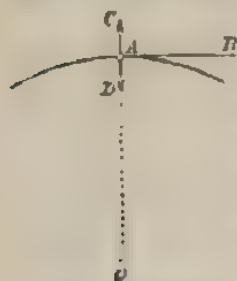


FIG. 35.—Centrifugal Force.

move in a direction which would carry it away from the centre of curvature, O, of the curved track. This tendency to tangential motion shows itself in a pressure, AC, which is exerted by the flange of the outer wheels against the inner side of the rail farthest from the centre, O. This pressure, or force, is called the *centrifugal force*. From the unyielding inner surface of the rail, a force, AD, equal to the centrifugal force, is exerted to keep the locomotive upon the track.

Centripetal force and centrifugal force are to be regarded, then, merely as an action and reaction, and are, therefore, always equal and opposed. The centrifugal force is only the resistance which the moving mass, by reason of its inertia, opposes to acceleration in the direction toward the centre of curvature and may be called *resistance of inertia*. It always exists with curvilinear motion. If, for example, a stone, fastened to the end of a string, or placed in a sling, be whirled rapidly about on the circumference of a circle, the string undergoes a stretch, which, acting as a centripetal force, constrains the stone to turn aside from a rectilinear path and describe a circumference, while, at the same time, acting as a centrifugal force, it exerts a pull upon the hand holding the other end of the string. If the string suddenly breaks, or if one end be suddenly released, the centripetal, and with it the centrifugal force, suddenly cease, the stone flying off, by reason of its inertia, in the direction of a

tangent and with the velocity attained at the instant of its release.

When mill-stones, grindstones, and fly-wheels, turn with too high velocities, the centrifugal force may become so great as to break them in pieces with such violence that the fragments, hurled off in tangential directions, oftentimes produce serious damage. A useful application of this principle of centrifugal force is had in the *centrifugal drying-machine*, used for drying clothing, for obtaining the juice from the pulverized beet-root, for separating crystals from their disintegrated matrices, etc. To dry clothing, for example, the damp fabric is placed in a cylindrical drum, whose circumference consists of a network of wire gauze, and the drum is set in rapid rotation. The articles to be dried are pressed (by the centrifugal force) with great power against the inner surface of the drum, and the water, thus



FIG. 36.—Centrifugal Railway.

squeezed out of the cloth, passes through the meshes of the wire gauze and flies off as spray in the direction of a tangent. The action of the centrifugal force is made very clearly apparent by filling a glass tumbler with water, fastening a string to the tumbler and swinging it over the head in the circumference of a circle, as is done with a sling. Even when the tumbler is at the highest point of its path and its mouth is turned downward, the water does not flow out, because the centrifugal force here acts directly against gravity and with an intensity greater than that of gravity. Phenomena in some sense similar to these are exhibited by the *centrifugal railway* (Fig. 36). Here a little car, descending from the platform, A, with a velocity corresponding to that due to a vertical fall through the height, AB, traverses the circular loop, BC, of the

track at C, having its trucks turned upward, and finally it arrives safely at the platform, D.

Since centrifugal force is always equal to centripetal, its intensity is given by the same expressions, viz. :—

$$C = \frac{mv^2}{r}, \text{ and } C = \frac{4\pi^2mr}{T^2}.$$

To prove both the fact and the law of centrifugal force, the so-called *centrifugal machine* is used.

Two wheels with parallel axes (Fig. 37), a larger, called a *drive wheel*, and a smaller, with its axis so arranged that various apparatus for experimentation may be attached, are connected by an endless cord, or band, which works in grooves on their circumferences. The larger wheel is turned with the hand by



FIG. 37.—Centrifugal Machine.

means of a pin, and the axis of the smaller wheel is then set rotating with a velocity as many times greater than the velocity of the axis of the larger, as the radius of the larger

wheel exceeds that of the smaller. Suppose, for example, a wooden frame, carrying a wire stretched horizontally, be attached to the axis of the smaller wheel, and a pair of metal balls, provided with holes for the reception of the stretched wire, be fastened together with a cord, and slipped on the wire so as to shift easily back and forth along the wire. If the balls lie on opposite sides of the axis, and rotation has begun, the centrifugal force will spread them apart, and that ball whose centrifugal force is greatest will draw the other toward it along the wire. A position may be readily found in which the balls remain at rest, since the centrifugal forces here hold each other in equilibrium. This position will be found to be that in which the distances of the bodies from the axis of rotation are inversely as their masses, or that in which the product of the mass of a ball by the radius of the circle it traverses is the same for both balls. With the same periods of revolution, therefore, the centrifugal force is as the mass and as

the radius of the path, or *orbit*. If a vertical shaft, to which two rods, each carrying a ball at its lower end, are hinged so as to act like pendulums, be set upon the axis of the centrifugal machine and the axis be rotated, the balls will separate farther and farther from each other with increased rapidity of rotation and the spreading will occur with sufficient force to raise a weight along the axis. A practical application of this apparatus, called the *centrifugal regulator*, is made in the governors for steam engines. If a hollow glass globe (Fig. 38), containing a little mercury and a quantity of coloured water, be rapidly turned about its vertical axis by means of the centrifugal machine, the mercury being more massive than the water, will attain a greater centrifugal force, and will, consequently, be driven farther from the axis than is the water. The mercury will form a brilliant zone about the equator of the globe, above and below which a band of coloured water will be formed. An elastic hoop of metal, attached to the vertical shaft of the centrifugal machine in such way that the prolongation of the shaft forms the vertical diameter of the circle, is deformed by the centrifugal force, which acts most strongly on those points of the hoop most remote from the shaft, into an ellipse, and represents, in miniature, the flattening of the earth. To prove that the centrifugal force is proportional to the square of the velocity, we may use a centrifugal machine, which, in addition to the drive wheel, has two smaller wheels, one having a diameter only half as great as the other, and, when driven by the same cord, rotating in half the time of the larger wheel. Upon each of the two vertical axes let a frame carrying a horizontal wire be placed, along which a ball may slide with gentle friction. To each ball let a thread be attached and passed over two small rollers, one above the other, supported by a vertical post at the middle of the frame. The end of the cord, after passing over the roller, is attached to a scale-pan for the reception of weights. Both frames are identical in all respects. If the machine be set rotating, both pans will be raised simultaneously



FIG. 38. -For the Centrifugal Machine

by the centrifugal forces acting on the cords, when the pan belonging to the frame rotating with double velocity, carries four times as great a weight as that upon the other.

If a small metal cylinder, or bar, be attached by a cord to the lower end of the axis of the centrifugal machine, it will rotate at the outset about its own vertical axis (Fig. 39, A). If, however, the cylinder be accidentally drawn aside from the vertical position by the least conceivable amount, there will arise, through the action of the centrifugal forces, striving to

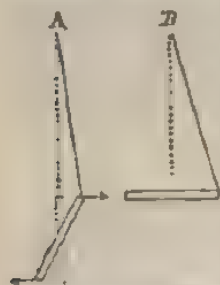


FIG. 39.—Rotating Bar.

draw the body as far as possible from the axis, a couple, by which the cylinder will finally be drawn into the horizontal position, B. If the cylinder be drawn out of this position of stable equilibrium, to establish which the centrifugal force has performed its mechanical work, it will return to it again of its own accord. In a similar way a ring, suspended by a thread and set rotating about a vertical axis, assumes a horizontal position by virtue

of the centrifugal force acting upon its circumference.

The centrifugal force caused by the diurnal rotation of the earth is everywhere perpendicular to the earth's axis and directed away from it. Since for all points of the earth's surface the period of rotation is twenty-four hours (sidereal time), the centrifugal force at any point is proportional to the radius of the parallel of latitude described by the point in its diurnal motion. At the equator, where it acts directly against gravity, the centrifugal force is a maximum and equals  $\frac{1}{289}$  of gravity.

**37. Gyroscopic Motion.**—If the mass of a rigid body rotating about an axis be distributed uniformly about the axis, the resultant force acting on the body, arising out of the rotation, will be equal to zero, i.e. there will be no such force. For, the centrifugal force of each particle will be destroyed by the equal and opposite centrifugal force of that particle of the body diametrically opposite to the former. In this case the axis is called a *free axis*. Since every particle of matter rotating about a free axis strives by its inertia to remain in its plane of

rotation perpendicular to this axis, the free axis shows a tendency to maintain its direction in space, and opposes, therefore, to any external force, seeking to draw it out of this direction, a resistance proportional to the *vis viva*, or *energy* of the motion of rotation. Thus it happens that a heavy, rapidly rotating disc does not fall, when its axis is inclined (Fig. 40), and that wheels, *e.g.* of bicycles, hoops, pieces of coin, etc., do not fall over while rolling upon an edge, or spinning upon a vertical diameter. The effect of the disturbing force of gravity upon the gyroscope is shown in the deviation of the axis of rotation in a direction perpendicular to the direction of the action of the disturbing force, and its consequent slow motion on the surface of a cone, without the occurrence of any change whatever in the inclination of the axis to the horizon (Fig. 41). If a circular disc, ACBD (Fig. 42), turns about an axis of symmetry, MS, supported at M, in the direction of the curved arrows (seen



FIG. 40.—Gyroscope.



FIG. 41.—Gyroscope.

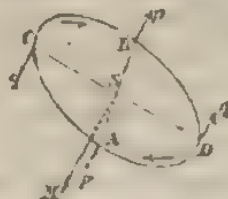


FIG. 42 Gyroscope.

from above, in the direction of the hands of a watch), and the weight of the disc acting at the centre of gravity *S* causes the axis to sink by a small amount; that is to say, causes a slight turning of the disc about its horizontal diameter, *AB*, or some line parallel to *AB*; the direction of motion of the highest and lowest points, *C* and *D*, will remain unchanged, since they

have suffered only a parallel displacement. But at the ends of the diameter,  $AB$ , a change of direction occurs, and in lesser degree at all other points of the circumference. At  $A$  and  $B$  there will arise out of the tendency of the disc to persist in its plane, the forces  $p$  and  $p$  perpendicular to the plane of the disc. These equal and opposite forces constitute a couple, which produces a rotation of the disc about the diameter,  $CD$ , perpendicular to  $AB$ , and with it, a progressive motion of the end of the axis in the direction of the hands of a watch. The points  $C$  and  $D$  are, therefore, compelled to leave their directions of motion, and the resistance to this change of direction gives rise to the couple,  $gg$ , which turns the disc about its horizontal diameter,  $AB$ , in such way as to elevate the axis.

The tendency of a free axis to maintain its direction in space may be proved by means of Bohnenberger's rotating apparatus (Fig. 43). This apparatus consists

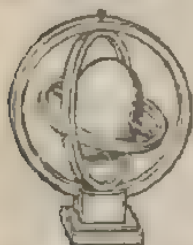


FIG. 43 — Bohnenberger's Rotating Apparatus.

of a globe whose axis of rotation is so supported on three rings turning within one another, that it may assume any position whatever. If the globe be set in rapid rotation by first winding a cord about its axis and then suddenly pulling it off, this axis will remain continually parallel to itself, howsoever the apparatus may be turned, or inclined. Examples of rotation about a free axis are afforded us in the planets, and, as a special case, by the earth. If the earth were a perfect sphere, its axis would remain for ever parallel to itself and always point to the pole star. The attraction of the sun, however, upon the earth's equatorial belt of matter, gives rise to a disturbing force, which is ever striving to bring the axis of the earth, now inclined at an angle of  $66^{\circ}5'$  to the earth's orbit-plane (called the plane of the *ecliptic*), to exact perpendicularity to this plane. But as with the gyroscope, the axis of the earth does not alter its inclination to the plane of the earth's orbit. It merely describes, during an interval of something more than 25,800 years, a conical surface of approximately  $47^{\circ}$  opening, about the perpendicular to the plane of the *ecliptic*. The effect of this

is that in the course of thousands of years, widely different stars will take their turn as pole star to the earth. For example, 12000 years hence, the brilliant star Vega ( $\alpha$  Lyrae) will be the pole star. This slow conical motion of the earth's axis causes the equinoxes to run toward the west upon the ecliptic by about 50 seconds of arc yearly, and this westward motion of these points is known as the *precession of the equinoxes*.

**38. Angular Velocity.**—When a body rotates about an axis, each of its points describes a circumference, whose centre lies upon the axis and whose plane is perpendicular to it. The velocities of the various points are directly as the radii of the circles traversed, or what is the same thing, as the distances of the points from the axis. If, then, the velocity be known for any particular distance, e.g. for the distance 1 from the axis, it is known for all points of the rotating body. This velocity at distance unity, i.e. the length of a circular arc traversed by a point at unit distance from the axis during one second, is called the *angular velocity* of the rotating body. If it is given, the velocity,  $v$ , of any point is obtained by multiplying the angular velocity by the distance,  $r$ , from the axis (i.e.  $v = r\omega$ ). This magnitude is called “angular velocity;” because the arc of the circle of radius unity may be regarded as a measure of the magnitude of the angle, through which the body revolves in one second; or if the rotation is not uniform, the angle through which the body would rotate, if from the instant considered, no further change of velocity should occur. By *angular acceleration* is meant the change of angular velocity during a unit of time.

**39. Moment of Inertia.**—If a body turn about an axis, each of its particles possesses by virtue of this rotation a certain quantity of the energy of motion, or *vis viva*, which is expressed by the half product of its mass ( $m$ ) into the square of its velocity ( $v$ ), i.e. this energy equals  $\frac{1}{2}mv^2$ , or, if  $\omega$  be the angular velocity, or velocity at distance unity and  $r$  denote the distance of the particle from the axis, it equals  $\frac{1}{2}mr^2\omega^2$ . A mass,  $\mu$ , at the distance unity from the axis has, with the same angular velocity, the energy  $\frac{1}{2}\mu\omega^2$ , which equals the foregoing if the mass,  $\mu$ , be so chosen that  $\mu = mr^2$ . The particle,  $m$ , at the distance,  $r$ , from the axis may, therefore, be replaced by the mass,

$mr^2$ , at the distance unity from the axis, without in any way affecting the energy of the body. If the mass of every particle be thus multiplied by the square of its distance from the axis of rotation, and all these values be summed, the result will be the mass, which, applied at the distance unity from the axis, might replace the actual mass of the body without the energy of rotation of the body in any way suffering modification. This sum, written  $\Sigma mr^2$ , is called the *moment of inertia* of the body. When its value is known, experimentally, or otherwise, the problem of rotatory motion is extraordinarily simplified, because by its means, instead of the countless number of material particles at different distances from the axis, we need consider but a single mass concentrated in a single point, or uniformly distributed along the circumference of a circle of radius unity. The energy, or *vis viva*, of a rotating body is, then, equal to the half product of its moment of inertia into the square of the angular velocity.

40. The Pendulum.—A small, but heavy, material body,

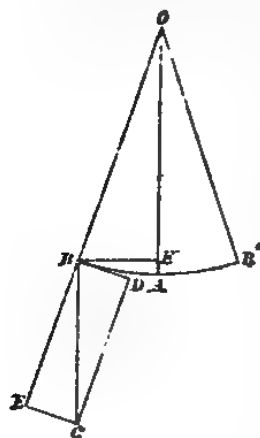


FIG. 44.—Pendulum.

suspended by a slender line of the least possible weight, is called a *simple pendulum*. If the pendulum cord is considered as having no weight, and the body as being a single material particle, it is called a *mathematical pendulum*. If the pendulum be drawn aside from its vertical position of equilibrium (OA, Fig. 44) and then left to move of its own accord, it will return, under the action of gravity, along the circular path (BA), with increasing velocity to this position again. The inertia of the mass will not allow the body to remain here, but will carry it across this position with the velocity due to the fall through the

vertical distance FA, thereby causing the pendulum to describe an arc, AB', equal to AB, but with continually diminishing velocity. At the highest point, B', the velocity will have been exhausted by the opposing force of gravity, and the pendulum

will here come to rest. The motion of the pendulum from the extreme point (B) on the one side, to the extreme point (B') on the other, is called a *vibration*; the angle BOA', which the suspending fibre in its extreme position makes with the position of equilibrium, is called the *amplitude of vibration*. During the second vibration the pendulum returns to its initial position (B) in precisely the same way as during the first vibration. It would continue to vibrate incessantly with equal amplitude, were it not that external hindrances, such as friction of the point of suspension, resistance of the air, and so forth, continually diminish the amplitude, and finally bring the pendulum to rest in its position of equilibrium.

The force which compels the pendulum to return to its position of equilibrium is a component of gravity. In the figure, let BC denote the vertical downward pull of the weight of the pendulum. According to the parallelogram of forces, this force may be resolved into the two rectangular components, BE and BD. The former of these acts in the direction of the suspending fibre, and the latter in the direction of the tangent to the circular arc, or in the direction of the motion of the body at the point, B. Only the latter can be instrumental in producing motion; the former being wholly expended in stretching the cord. If, now, BF be drawn perpendicular to OA, we have from the similarity of the triangles, BCD and BOF, that the moving force, BD, is to the entire force of gravity, BC, as the distance, BF =  $r$ , is to the length of the pendulum, OB =  $l$ , or that—

$$BD : p = r : l.$$

From this it follows that the moving force,  $BD = \frac{pr}{l}$ , for one and the same pendulum, is proportional to the distance of the pendulum bob (the heavy body) from the position of equilibrium of the suspending fibre.

When the amplitude of vibration is small, i.e. not exceeding  $2^\circ$  or  $3^\circ$ , the curved path, BA, described by the pendulum bob from any point of its path to the position of equilibrium, is not appreciably different from the right line, BF. Since, then, the moving forces are proportional to the distances traversed, it

is obvious that the time required to attain the position of equilibrium will be the same whether the amplitude be  $2^\circ$  or  $3^\circ$ , or only a few minutes, or seconds of arc. In case of a small amplitude, therefore, all vibrations are performed in equal times, or they are *isochronous*. Galileo, at the age of twenty years, discovered this law in 1583, by accidentally observing the vibrations of a bronze lamp suspended in the cathedral at Pisa. By counting his pulse-beats he convinced himself that the vibrations, though growing ever smaller, still consumed the same time in their performance. Upon this law of the isochronous vibration of pendulums, Huyghens' application of the pendulum to clocks in 1657 depends. In this application, the problem was to convert the varying motion of a

system of wheels produced by a weight, or a spring, checked at equal intervals of time and for only an instant, into a uniform motion.

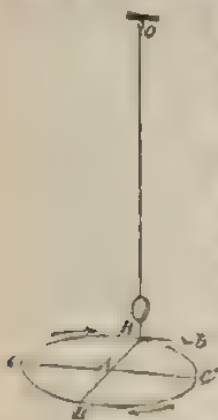


FIG. 45.—Circular Pendulum.

Let us now consider the pendulum, (Fig. 45), which, at rest, would hang in the position,  $OA$ . If the pendulum bob be drawn aside to  $B$  and then let go, or if an impulse at  $A$  be imparted to it in the direction  $AB$ , it will vibrate back and forth in the plane,  $OBB'$ , along the straight line,  $BB'$ . Likewise it could be made to vibrate along any desired line, *e.g.* along  $CC'$  perpendicular to  $BB'$ , by giving it an impulse in this direction, or by pulling the pendulum aside to  $C$ , or  $C'$ , and letting it go. Suppose, now,

the pendulum to be set vibrating along  $BB'$  and at the instant it reaches  $B$ , an impulse to be given to it in the direction  $Bb$  perpendicular to  $BA$ , which would carry it, if free to move in this direction, just as far from  $B$  toward the side, as the distance of the body from the position of equilibrium,  $A$ , at the instant of the impulse. The pendulum would then describe the circle  $Bc'B'CB$ , in the direction of the arrows and with uniform velocity. From this we readily recognize that circular motion may be regarded as a compound motion, consisting of two

simultaneous rectilinear, mutually perpendicular vibratory motions along  $BB'$  and  $CC'$ ; or conversely, that circular motion may be decomposed into two such motions. It is also easy to see that if the pendulum vibrates rectilinearly (along  $BB'$ ) precisely the same time is needed for an oscillation to and fro, as the circular pendulum requires for a complete revolution. For, if the circular pendulum be illuminated by a horizontal beam of light, parallel to  $CC'$ , the shadow upon a vertical wall parallel to  $BB'$  moves in precisely the same way as the pendulum vibrating rectilinearly along  $BB'$ . Both shadow and rectilinear pendulum complete a double vibration in the time required for the circular pendulum to make a revolution. We know from (35), that for circular motion, the centrifugal force—

$$C = \frac{4\pi^2 mr}{T^2},$$

where  $m$  is the mass of the moving body,  $r$ , the radius of the circular path, and  $T$ , the time of revolution. The centripetal force, which alone draws the pendulum to the position of equilibrium,  $A$ , at the centre of the circular path is here, where only small amplitudes,  $r$ , are considered, merely the component of gravity obtained above (Bd, Fig. 44) and equals  $\frac{pr}{l}$ , where  $p$  is the weight of the pendulum bob,  $l$ , the length,  $OB$ , of the thread, and  $r$ , the distance,  $AB$ , which is the radius of the circular path. If  $m$  denote the mass of the pendulum bob and  $g$  the acceleration of a freely falling body, we have  $p = mg$ . Therefore, the centripetal force acting on the circular pendulum is expressed by—

$$C = \frac{mgr}{l}.$$

This expression must become equal to that above, and we have —

$$\frac{mgr}{l} = \frac{4\pi^2 mr}{T^2}.$$

Since  $m$  and  $r$  divide out of both sides of the equation, it reduces to—

$$\frac{g}{l} = \frac{4\pi^2}{T^2},$$

or to— 
$$T^2 = \frac{4\pi^2 l}{g},$$

and extracting the square root of both sides we obtain—

$$T = 2\pi\sqrt{\frac{l}{g}},$$

for the time of a double vibration of a pendulum swinging rectilinearly in a vertical plane, which is the time required. The time of a single vibration is then half of this. The time of vibration,  $t$ , of a pendulum is then expressed by—

$$t = \pi\sqrt{\frac{l}{g}},$$

provided the amplitude is sufficiently small. In this equation (where  $\pi$  is the ordinary Ludolphian number 3.14159...), all laws of vibration of the pendulum are contained. The law of isochronism is expressed by the fact that  $t$  is independent of the amplitude, since  $r$  divided out of the above equation. It is also independent of the mass, or weight, of the body, since  $m$  also divided out. Newton verified that the time of vibration of a pendulum is independent of the mass, by suspending a hollow box by a slender cord, then filling the box with various substances and noting the time of vibration. The form of the box being always the same, the same atmospheric resistance was always encountered by the pendulum, and he found the time of vibration to be always the same. From this it follows that *to all bodies, regardless of weight, or molecular constitution, the force of gravity imparts the same acceleration, or that all bodies fall with equal rapidity.* This experiment with the simple pendulum is much more accurate than the one formerly adduced to verify the same principle. For, despite atmospheric resistance, a pendulum can perform several thousand vibrations before coming to rest, and all these vibrations, from the first ones of large amplitude, to the later ones of less amplitude, are of the same duration; so that, by counting the number of vibrations occurring in a definite time, and dividing the number of seconds by the number of vibrations, an extremely accurate value for the time of vibration may be obtained.

The formula shows, further, that, (1) *the time of vibration is directly proportional to the square root of the length of the pendulum*, and (2) *the time of vibration is inversely proportional to the square root of the acceleration of gravity*. The first of these was discovered by Galileo by observing the vibrations of simple pendulums of unequal lengths. The latter may be verified by means of a pendulum of the following form. Two rods are rigidly fixed perpendicularly to each other, one of them resting at its lower end, on a plane inclined at an angle with a vertical plane and kept perpendicular to this plane, and the other rigidly fixed to the former, the combination of the two being free to oscillate about the axis of the former. This mechanism was introduced by Mach. The vibrations take place more slowly than when the plane of vibration is vertical, because instead of the acceleration,  $g$ , only the component  $g \cos \alpha$  is operative, and the time is found to be increased in the ratio of  $1 : \sqrt{\cos \alpha}$ .

By the number of vibrations ( $n$ ) of a pendulum is meant the number of vibrations occurring in a second. It is given by  $n = \frac{1}{t}$ , or  $n = \frac{\sqrt{g}}{\pi \sqrt{l}}$ . If a pendulum be allowed to vibrate under the action of two different accelerations,  $g$  and  $g'$ , the number of vibrations,  $n$  and  $n'$ , will be to each other as the square roots of the accelerations, or *the accelerations are as the squares of the number of vibrations*, or,

$$g : g' = n^2 : n'^2.$$

**41. Seconds-Pendulum. Determination of  $g$ .**—A pendulum, whose time of vibration is one second, is called a seconds-pendulum. If the length of the mathematical seconds-pendulum be designated  $l_1$ , from the formula given above, there results, since  $t = 1$ ,—

$$1 = \pi \sqrt{\frac{l_1}{g}}, \text{ or } 1 = \pi \sqrt{\frac{l_1}{g}}.$$

whence,

$$g = \pi^2 l_1.$$

We may, therefore, find the acceleration of gravity by multiplying the length of the seconds-pendulum by the square of  $\pi$ . Since the length of the seconds-pendulum may be very accurately measured, this is the most precise known method

of determining  $g$ . Borda found for the length of the seconds-pendulum at Paris 99.392 cm., whence the acceleration of free fall,  $g = 980.95$  cm.-sec.

When, in 1672, the French astronomer Richer, of Cayenne (lat.  $5^\circ$  N.), made and discussed a series of observations, he remarked that the pendulum clock he had brought with him from Paris lost  $2\frac{1}{2}$  minutes daily. To make his clock run correctly, he was compelled to shorten the seconds-pendulum by a few millimeters. Returning to Paris, he found that his clock gained  $2\frac{1}{2}$  minutes daily. If, however, the same pendulum vibrates more slowly in Cayenne than in Paris, it can only be due to the fact that the force of gravity is weaker there than in Paris, and, consequently, a freely falling body would experience a smaller acceleration there than in Paris.

By determining the length of the seconds-pendulum at different points of the earth's surface, it has been found that the length increases from the equator toward the poles. At the equator its length is 99.092 cm.; in latitude  $45^\circ$  it is 99.355 cm.; at Berlin, 99.424, and these values indicate that at the pole its length would be 99.613 cm. From this, it is evident, that the acceleration of gravity will increase correspondingly from the equator toward the poles. From the equation,  $g = \pi^2/l$ , we find for the acceleration of gravity at the equator 978.40; in lat.  $45^\circ$ , 980.60; at Paris, 980.95; at Berlin, 981.28; and at the poles, 983.1 cm.-sec.

**42. Foucault's Pendulum.**—By virtue of its inertia, a pendulum strives to remain in its plane of vibration precisely as the gyroscope persists in its plane of rotation. If a pendulum be suspended within a frame attached to the centrifugal machine, and set vibrating, the plane of vibration will be found to preserve its direction in space, when the frame is turned slowly about its axis. Similarly, as was remarked by Foucault in 1851, a pendulum will maintain its plane of vibration despite the rotation of the earth.

Imagine a pendulum to be suspended directly above the north pole of the earth. Its plane of vibration will keep its direction in space, while the earth and all objects upon it below the pole, turn under the pendulum from west to east. The

observer, who thinks his position fixed, sees the direction of vibration turn slowly with respect to the earth's surface, from the east, through the south, toward the west—that is, toward the right, completing a whole revolution in 24 hours. At all other points, the motion of the earth's surface may be considered as compounded of a slow rotation about a vertical axis, and of a motion of translation from west to east. Only the former can become apparent, manifesting itself in the northern hemisphere as rotation of the direction of vibration toward the right, or *dextrorsum*, and in the southern, toward the left, or *sinistrorsum*. This rotation becomes slower as the place of observation approaches the equator, and just at the equator it will be zero.

Let the circle, NQS (Fig. 46), represent the meridian through the place, M, whose geographical latitude,  $MOQ = \phi$ . The velocity of rotation of the earth ( $15^\circ$  per hour) about the axis, NS, may be represented by the distance, OA, laid off from O upon the axis. This may be resolved by (26) into a rotation, OB, about the vertical, OMZ, of the place of observation and a rotation, OC, about the horizon, MH, parallel to OC'. The former, which is alone effective, equals  $OB \sin \phi$ , or  $15 \sin \phi$ , i.e. the velocity of apparent rotation of the plane of vibration is proportional to the sine of the geographical latitude. The duration of a complete vibration is inversely proportional to this sine, and equals  $\frac{24}{\sin \phi}$  hours. In Munich, for example, the plane of vibration makes a complete revolution, in 32 hours, 13 minutes; at Berlin, in 30 hours, 15 minutes.

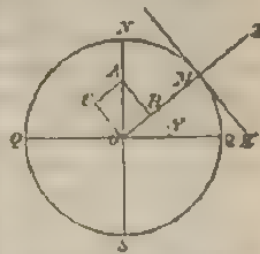


FIG. 46 — Principle of Foucault's Pendulum.

For the celebrated experiment of Foucault, which makes the rotation of the earth apparent, a pendulum with great inertia must be used, so that, when once set in motion, it will vibrate for a considerable time. Foucault accordingly used a heavy mass of lead suspended by a long fibre from the dome of the Pantheon in Paris.

**43. The Physical Pendulum.**—The simple, or mathematical pendulum hitherto considered, consisting of a single material point suspended to a cord without weight, is in reality a physical impossibility. All pendulums capable of construction

are so-called *physical*, or *compound* pendulums, and they consist of an indefinitely great number of material particles, situated at different distances from the centre of suspension. They may be regarded, therefore, as compounded of an infinitely great number of simple pendulums. Since each material particle of the pendulum strives to oscillate more rapidly, the nearer it lies to the point of suspension, and since, by reason of their rigid connection, all particles must oscillate simultaneously, those particles lying near the point of suspension are retarded by those more remote from it, and the remote particles are in turn accelerated by those lying nearer the point of suspension. There is somewhere an intermediate point, which is neither accelerated nor retarded, but which vibrates precisely as its distance from the point of suspension requires. This point is designated the *centre of oscillation*, and its distance from the point of suspension, called the *reduced length of the pendulum*, is the *length* of the mathematical pendulum, whose time of vibration is the same as that of the given physical pendulum. If, by the length of the pendulum, this reduced length is understood, the same laws of vibration hold for the physical as for the mathematical pendulum.

If the centres of oscillation and of suspension of a physical pendulum be interchanged, it will be found that the time of vibration in both positions is the same. By means of this fact the required length is easily found. The form devised by Huyghens in 1673, and known as the *reversion pendulum*, can be best used for this purpose. It consists of a bar, or rod, provided with not only the customary steel knife-edges for suspension, but also with a second adjustable knife-edge, faced opposite to that for suspension. The adjustable knife-edge is set experimentally in such position that the pendulum, when suspended by it, vibrates in precisely the same time as when suspended in the usual way. The reduced length is then the distance between the two knife-edges. By this process, the very accurate values of the seconds-pendulum, mentioned above, were obtained by Kater in 1818, and by Bessel in 1828.

A physical pendulum (*i.e.* any heavy body of arbitrary form, and free to oscillate about a horizontal axis) which reaches its position of equilibrium

with angular velocity,  $\omega$ , has in that position, the energy  $\frac{1}{2}\omega^2 \Sigma mr^2$ , where  $\Sigma mr^2$  denotes the *moment of inertia* of the pendulum with respect to the axis of rotation. Denoting by  $h$ , the vertical fall of a material point at distance unity from the axis; by  $r$ , the distance of the centre of gravity from the axis;  $hr$  is then the distance fallen vertically by the centre of gravity, and  $hgr \Sigma m$  is the work of free fall, which the total weight,  $g \Sigma m$ , of the pendulum, applied at the centre of gravity, must perform to produce this energy. We have, therefore—

$$hgr \Sigma m = \frac{1}{2}\omega^2 \Sigma mr^2.$$

For a simple pendulum having the same time of vibration and amplitude, of length  $l$  and mass  $\mu$ , which, after its mass point  $\mu$  has sunk by  $h'$ , reaches its position of equilibrium with velocity  $l\omega$ , the same relation (work of free fall equals energy) holds,

$$h'g\mu = \frac{1}{2}\omega^2 \mu l^2, \text{ or } h'g = \frac{1}{2}\omega^2 l.$$

If this value of  $h'g$  be substituted in the foregoing equation, there will result for the reduced length of the pendulum—

$$l = \frac{\Sigma mr^2}{\Sigma m},$$

or, since for a straight bar,  $\Sigma m = \Sigma mr$ , by (28) —

$$l = \frac{\Sigma mr^2}{\Sigma mr}.$$

If now the pendulum be reversed, so that centre of oscillation  $S$  (Fig. 47) becomes centre of suspension, the new length  $l'$  is given by—

$$l' = \frac{\Sigma m(l-r)^2}{\Sigma m(l-r)} = \frac{l^2 \Sigma m - 2l \Sigma mr + \Sigma mr^2}{l \Sigma m - \Sigma mr},$$

since here  $r' = l - r$ , is to be used instead of  $r$  above. But  $\Sigma mr^2 = l \Sigma mr$ , and we have—

$$l' = \frac{l^2 \Sigma m - l \Sigma mr}{l \Sigma m - \Sigma mr} = \frac{l(l \Sigma m - \Sigma mr)}{l \Sigma m - \Sigma mr} = l,$$

and, consequently, the reduced length, and hence also the time of vibration, is the same with the new suspension as with the old.



FIG. 47.—Reversion Pendulum.

**44. Kepler's Laws of Planetary Motion.**—The planets offer examples of central motion on a gigantic scale. The so-called major planets, in order of their distance from the sun, are: Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, and Neptune. From the observations of Tycho Brahe on the planet Mars, Kepler, between 1609 and 1618, discovered the following laws for the motion of the planets about the sun:—

1. The orbits of the planets are ellipses with the sun at one of the focii.

2. The radii vectores of the planets sweep out equal areas in equal times.

3. The squares of the periods of any two planets are to each other as the cubes of their mean distances from the sun.

The deviations of the elliptical paths from circles are so slight, however, that they may be neglected and the paths may, therefore, be regarded as approximately equal to circles with the sun at the centre.

**45. Universal Gravitation.**—From the second of Kepler's laws, together with the law of areas, or of *areal velocity* (34), it follows that the planetary motions are due to a centripetal force always directed toward the sun.

Since the orbits may be regarded as very closely approximating circles, the centripetal forces ( $C$  and  $C'$ ) of two planets, from the laws of central motion (35), are as their masses  $m$  and  $m'$ , and as the radii  $r$  and  $r'$  of their orbits (i.e. as their distances from the sun), and inversely as the squares of their periods,  $T$  and  $T'$ . We have, then,

$$C : C' = \frac{mr^2}{T^2} : \frac{m'r'^2}{T'^2}.$$

Since, by Kepler's third law, the squares of the periods are as the cubes of the distances, or since  $T^2 : T'^2 = r^3 : r'^3$ , there results,

$$C : C' = \frac{m}{r^2} : \frac{m'}{r'^2},$$

that is to say, the centripetal forces are directly as the masses, and inversely as the squares of the distances from the sun. Or, differently stated, every planet is attracted by the sun with a force varying directly as the mass and inversely as the square of its distance. This was discovered by Newton in 1686.

Not only is the planet of mass  $m$ , drawn toward the sun, but inasmuch as there can be no action without an equal and contrary reaction, the sun, of mass  $M$ , is also drawn with an equal force toward the planet. The force exerted by the two bodies in the direction of their line of centres must therefore be proportional also to the mass  $M$ . After Newton had recognized this law for the bodies of the planetary system, the suspicion arose in his mind that this mutual action is but a particular expression for a universal property of matter, which may be called *gravitation*, or *general mass-attraction*, and which reveals itself by making

two masses,  $m$  and  $M$ , at a distance,  $r$ , between centres to attract each other with a force,  $f \cdot \frac{mM}{r^2}$ , i.e. with a force, directly as the product of their masses, and inversely as the square of their distances. The factor,  $f$ , which is constant for all masses, and which must be derived from observation, is called the *constant of gravitation*, or the *Newtonian constant*.

Tradition tells us that a falling apple suggested to Newton the idea that gravity, or heaviness, is only the attraction of the earth's mass for bodies on its surface, and that it must manifest itself by the fall of bodies, not alone at the earth's surface, but with decreasing intensity for all points of space to the distance of the moon, and even beyond. It occurred to him in this connection that this force may be what holds the moon in its orbit, and compels it to circle about the earth as the planets are held in their orbits about the sun by the attractive force of the latter. From astronomical observation, it is known that the moon, whose inertia continually strives to carry it in the direction of a tangent to its path, receives during every second, from the earth's attraction, an acceleration of 0.271 cm. in the direction of the earth's centre. If, now, this acceleration is a result of the force of gravitation, which imparts to a body falling freely at the equator of the earth, an acceleration of 978 cm., it must be possible to compute the acceleration of the moon from the latter acceleration according to the law derived above. Since the distance to the moon is 60 times the earth's radius, the moon is 60 times as far from the earth's centre of gravity as is a point on the terrestrial equator, and, consequently, the lunar acceleration can be but  $\frac{1}{60 \times 60}$ , or  $\frac{1}{3600}$  part of that of a body falling freely at the earth's surface:  $978 \div 3600 = 0.271$  cm. From the complete accordancy of this result with that derived from astronomical observations, the proof that gravity and general mass-attraction are identical may be regarded as established.

The attraction of a body on a material point arises from the combined action of all the separate forces proceeding from the individual particles of the body. If the body is a homogeneous

sphere, or a sphere composed of concentric spherical shells, its total attraction upon a particle situated outside of it, is obviously directed toward the centre of the sphere, and is exactly the same as if the entire mass of the sphere were concentrated at its centre of gravity, as will be proved later. Consequently the centre of gravity of the earth may be regarded as the seat of an attractive force, whence all distances are to be estimated, as was done above in considering the earth's attraction on the moon. A hollow spherical shell exercises no action whatever on a point situated either on its inner surface, or within its cavity, since the forces exerted upon it by points of the shell on one side of the centre, are exactly balanced by equal and opposite forces due to the symmetrically situated points on the opposite side. A point on the interior of the earth, *e.g.* at the bottom of a mine, will then in no way be affected by those parts of the earth's mass that lie farther from the centre of the earth than does the point in question, and will therefore be drawn toward the centre of gravity of the earth with a force due solely to the attraction of the nucleus of the earth lying below it.

If, then, the earth were composed of homogeneous matter throughout, at the bottom of a deep shaft the acceleration of free fall would be less, and the time of vibration of a pendulum would be greater, than at the earth's surface. Airy found, on the contrary, in 1856, from pendulum observations made at the surface of the earth, and at the bottom of the Harton coal mine, 383 m. in depth, that the acceleration was *greater* at the bottom of the mine than at the surface of the earth. From this it must be inferred that the interior of the earth is composed of matter of greater specific gravity than that of its crust. From his experiments it was found also that the earth is about 6.5 times as heavy as an equal volume of water.

It has also been found, from pendulum observations, that the force of gravity on the surface of the earth diminishes from the poles toward the equator. The cause of this diminution is to be sought, first, in the centrifugal force due to the earth's rotation. Since the angular velocity and the radius of the earth are known, the magnitude of the centrifugal force may be easily computed. At the equator, where it acts directly

against gravity, its magnitude is found to be  $\frac{1}{280}$  of the force of gravity, and, consequently, the acceleration of gravity will here be 34 mm. less than at the poles. Pendulum observations show, however, that the decrease in the acceleration from the poles to the equator is really greater than this, and amounts to more than 51 mm. For this decrease there must, therefore, be a different cause. It has its explanation in the fact that the pole is nearer to the centre of the earth than are points on the equator, or that the earth, instead of being an exact sphere, is flattened like an orange at the poles. From the values of the acceleration resulting from pendulum experiments and the magnitude of the centrifugal force, the polar *flattening* may be computed. It is found to be equal to  $\frac{1}{257}$ . This value, which expresses the fact that the diameter of the earth from pole to pole is  $\frac{1}{257}$  shorter than the equatorial diameter, agrees quite closely with the value,  $\frac{1}{259}$ , found from geodetic surveys.

As mentioned above, the attraction between two masses is mutual. A stone drawn toward the earth with a certain force attracts the earth toward itself with precisely the same force. With equal forces, however, the accelerations are inversely as the masses moved. Consequently, the acceleration of the earth's mass, which is enormous in comparison with that of the stone, may be considered evanescent.

If now every body attracts every other, when passing a large building, why are we not drawn toward it? The answer is: We are, in point of fact, drawn toward it, but the effect is so small in comparison with the attraction of the enormous mass of the earth, as to be wholly imperceptible. Nevertheless, with sufficiently sensitive apparatus, the attraction of a large ball of lead upon a smaller one can be, not only proved, but even measured, as Cavendish in 1798, Reich in 1852, Baily in 1842, Cornu and Baille from 1870 to 1878, have done by means of the torsion balance. If we know the force of attraction exerted by a known mass of lead upon a metallic sphere at a known distance, from a comparison of this with the force of attraction exerted by the earth upon the same sphere, i.e. with its weight, we may readily determine the mass of the earth. From the measurements of the above-mentioned physicists,

the mass of the earth is found to be 5.5 times as great as that of an equal volume of water. Maskelyne, in 1775, showed that a plumb-line suspended by the side of a somewhat regular chain of mountains is deflected from the vertical toward the mountain. From the magnitude of this deviation and the estimated weight of the mountain, he was also able to determine the mass of the earth. His determination agrees quite satisfactorily with the above value. Jolly has, as recently as 1880, measured by a balance the attraction of a ball of lead 1m. in diameter upon a known mass of mercury, and found the weight of the earth in comparison with water to be 5.692. The diminution of gravity, according to the inverse square of the distance from the earth's centre, as required by the law of gravitation, was here to be ascertained by means of the balance. If two masses, weighing one kilogram each, be suspended at different altitudes on the arms of a balance, the lower mass, being nearer the earth's centre of gravity, will appear the heavier. It was found that when the difference in the heights of the two masses was 5.2m., the lower mass was 1.5 mg. heavier. This value is smaller by 0.152 mg. than that computed from the law of gravitation. The discrepancy is, however, sufficiently well explained by the disturbing attraction of neighbouring buildings.

**48. The Tides.**—That the periodic ebb and flow of the waters of the sea are connected with the moon's motion was early recognized. Newton was the first to explain the phenomenon on the law of gravitation. The moon, *M* (Fig. 48),



FIG. 48.—The Tides.

attracts those portions of the water, *A*, of the ocean on the side of the earth turned toward the moon, more strongly than the more remote centre, *C*, of the earth, and this, in turn, more strongly than those portions of water at *B* on the opposite side. Since the earth and moon move relatively to each other by reason of their orbital motions, those portions of the water near *A* must move toward the moon more rapidly than the main body of the earth, and those near *B* less rapidly. Consequently, the

pressure of the water against the earth is diminished both at A and B, and the water rises on the side opposite the moon as well as on the same side, while at points  $90^\circ$  from A and B the water falls. When the water is rising it is *flood tide*; when falling, it is *ebb*. During the diurnal rotation of the earth on its axis, the rise and fall of the water circles about the entire earth and causes an elevation and depression of the surface of the water at every point on the sea twice daily. The sun also produces a tidal wave, which, notwithstanding the sun's enormous mass, is much smaller than that of the moon by reason of its immensely greater distance. At *full* and *new moon*, when both earth and sun are in the line MC, their effects are additive, and the highest tides, called *spring tides*, are then produced. At the time of the first and third quarters, when the sun and moon are  $90^\circ$  apart, the crest of the lunar and the hollow of the solar tides coincide, giving rise to the so-called *neap tides*. These periodic fluctuations of the tides occur twice a month. The regular course of the phenomena, as they would occur if the entire surface of the earth were covered by the ocean, is, moreover, very greatly complicated by the configuration of the shore lines of the continents.

## II. SOLIDS.

**47. General Properties of Bodies.** — In the foregoing considerations on motion and equilibrium, it was assumed that solids are perfectly *rigid*, *i.e.* that all points of a body are invariably connected with each other, so that opposite equal forces, tending to draw two points of a body toward, or from, each other, neither lengthen nor shorten the line connecting them. It was assumed that, no matter how great the forces, they could only produce reactions equal in intensity to their own. This, however, does not correspond to facts. If any two points of a solid are acted upon by forces, a change in their distance actually occurs. The volume of any body is increased by a pull and diminished by a pressure, and the change of volume is proportional to the intensity of the force applied. The body, however, opposes to this deformation, a force which holds the applied force in equilibrium, and if the force does not exceed a certain limit, it will return to its original state as soon as the force is removed. If the force exceed this limit, however, it will separate the particles of the body, producing a *rent*, or *tear*. Since all bodies when subject to a pull, or pressure, or to change of temperature, undergo a change of volume, this *variability of volume* is termed a *general property* of bodies.

*Impenetrability* is also regarded as a general property of bodies. By this we mean merely that a body opposes to a pressure exerted upon it by another body, a resistance which increases with the diminution of its volume until a point is reached at which, if the pressure be increased, the body will be crushed, or torn. A saw cuts a block of wood, into which it appears to penetrate by removing from its path a portion

of the mass in the form of sawdust. Water and air may be pushed aside, but not penetrated by the hand.

*Porous bodies* are only apparently penetrated by liquids and gases, for such bodies merely absorb these fluids into their *pores*. Considerable quantities of gases or liquids may often be taken up by porous bodies without appreciable change of volume. It was shown by the Florentine Academy, in 1661, that even metals under pressure are permeable to water, and consequently porous. Glass, on the other hand, is impermeable to either fluids or gases, and consequently is not porous, as was shown by Quincke. If by pores we mean perceptible interstices, *porosity* is, then, not a general property of bodies.

*Divisibility* is that property by virtue of which a body is capable of subdivision into parts. It is common to all bodies. This decomposition may be performed either mechanically by cutting, crushing, tearing, etc., or by natural agencies, such as evaporation, dissolution, etc. In both cases experience teaches that there is no limit beyond which this subdivision may not be carried. Wollaston has succeeded in drawing platinum wires 0.0008 mm., or  $0.8\mu$  ( $\mu$ , or *micron* = 0.001 mm.) in thickness. The diameter of a silkworm's thread is 140 times as great as this. Gold may be hammered into sheets 0.1 micron thick, 10,000 of which would have to be laid upon each other to give a sheet 1 mm. thick. The gold foil used to overlay the delicate silver threads of Lyonese lace has a thickness of less than 0.004 micron. One 100,000th of a cubic centimeter of vixen colours perceptibly a litre of alcohol. Five centigrams of musk will fill a room with its odour for years, notwithstanding frequent ventilation. One 3,000,000th part of a milligram of sodium colours appreciably the flame of a Bunsen burner; and so forth.

**48. Atoms—Molecules.**—Experience tells us that divisibility may be carried to the limits of perception, and, mathematically speaking, it may evidently be continued indefinitely. Nevertheless, the question may be raised, whether matter is really infinitely divisible, or whether it consists of individual particles incapable of farther subdivision in a physical sense. Or, in other words, we may inquire whether matter may be conceived

of as something which fills space continuously, or whether it must be thought of as an aggregation, or heap, of exceedingly small individual particles separated from one another by intervening spaces.

In this discussion we shall be compelled to rely upon the facts of chemical science. The well-known red colouring matter, called cinnabar, is a chemical compound of mercury and sulphur, which may be easily resolved into these two heterogeneous constituents, or formed again from them. On the contrary, neither sulphur nor mercury is capable of resolution into heterogeneous constituents, by any means at present known to chemistry. According to the view of modern science they are, therefore, to be regarded as simple *substances*, or *elements*. Cinnabar always contains for every 100 parts by weight of mercury, 16 parts of sulphur. If we take, for example, exactly 100 parts of mercury and 16 parts of sulphur, we shall obtain 116 parts by weight of cinnabar, without any of the mercury or the sulphur remaining unused. Had we taken, on the other hand, 100 parts of mercury and 17 parts of sulphur, or 16 parts of sulphur to 101 parts of mercury, we should have obtained 116 parts by weight of cinnabar as before; but in the former case, 1 part of sulphur, and in the latter, 1 part of mercury, would have remained uncombined. It could likewise be shown experimentally that 100 parts by weight of mercury combine chemically with exactly 8 parts of oxygen, 35.5 parts of chlorine, etc. The elements combine with one another, therefore, according to determinate, invariable weight-relations. Mercury and sulphur form another compound besides cinnabar, the black mercury sulphide, which contains to 16 parts by weight of sulphur, exactly 200 parts of mercury. Similarly, 35.5 parts of chlorine can combine with not only 8, but also with 16, or 24, or 32 parts of oxygen. It appears thus, that when one element can combine with another in more than one relation, the combining weights are all simple multiples of the least weight entering into combination. This important law, discovered by Dalton in 1803, is known to chemists as the *law of multiple combining relations, or proportions*. From these facts it is evident that 16 parts by weight of sulphur represent an

indivisible whole for 100 parts of mercury, and conversely, that 100 parts of mercury is an inseparable mass for 16 parts of sulphur, which may be multiplied (*e.g.* doubled), but can never be diminished. This behaviour is most simply explained by the assumption, that each element is composed of invariable, indivisible particles, or *atoms* (Greek, *ατομος*, "indivisible"). For the various elements, these atoms differ, in that their masses are as their corresponding combining weights. The numbers 100 for mercury and 16 for sulphur mean, then, that an atom of sulphur weighs 16 units, if the weight of an atom of mercury is 100 units. For this reason, these numbers are called *atomic weights*. Since the atom is not perceptible to the senses, its real, or absolute, weight is of course, and must remain, unknown to us. The atomic weights express only weight-ratios, and might therefore be replaced by other numbers having the same relations to one another, each being expressed in terms of the unit selected as the basis. If, as is customary, the weight of an atom of hydrogen equals 1, the atomic weight of mercury equals 200, that of sulphur 32, of oxygen 16, etc. When we say an atom is "indivisible," it is, of course, not meant in a mathematical sense; for, mathematically speaking, any definite quantity of matter, just as any definite number, is capable of indefinite subdivision. The idea of "indivisibility" as applied to the atom will be made more generally intelligible by calling to mind the word "individual," which has the same linguistic significance as the word "atom," both meaning an "indivisible something." As the single soldiers, "the individuals," are the ultimate constituents of an army, so are the atoms the final constituent parts of a body. In this physical sense, the division of the atom is no longer possible. The smallest possible division of cinnabar would accordingly be formed, if a single atom of mercury were combined with a single atom of sulphur, and the smallest possible particle of black mercury sulphide, if 2 atoms of mercury be combined with 1 of sulphur. The smallest particles of compound bodies are then groups of 2, or more, atoms, called *molecules*. Certain facts, whose discussion would lead us beyond the scope of this work, indicate that even in the elements, atoms do not exist

separately, but that here also 2 atoms are commonly combined into a molecule. All bodies, therefore, consist primarily of molecules, each of which is, in turn, composed of like or unlike atoms. The weight of a molecule, *i.e.* the *molecular weight*, is equal to the sum of the weights of its constituent atoms. The molecular weight of cinnabar is, then,  $200 + 32 = 232$ ; that of black mercury sulphide,  $200 + 200 + 32 = 432$ . A molecule of hydrogen consists of 2 atoms of hydrogen, and its weight is therefore equal to 2.

**49. The Elements.**—There are as many different kinds of atoms as there are *elements*. The elements thus far known, the symbols for their atoms, and their atomic weights, are the following seventy-four :

Aluminium	Al	27	Lead	Pb	206.5
Antimony	Sb	120	Lithium	Li	70
Argon	Ar	20 (?)	Magnesium	Mg	24
Arsenic	As	75	Manganese	Mn	55
Barium	Ba	137	Mercury	Hg	200
Beryllium	Be	9	Molybdenum	Mo	96
Bismuth	Bi	208	Neodymium	Nd	141
Boron	B	11	Nickel	Ni	58.5
Bromine	Br	80	Nitrogen	N	14
Cadmium	Cd	112	Osmium	Os	191
Cæsium	Cs	133	Oxygen	O	16
Calcium	Ca	40	Palladium	Pd	106
Carbon	C	12	Phosphorus	P	31
Cerium	Ce	141	Platinum	Pt	194.5
Chlorine	Cl	35.5	Potassium	K	39
Chromium	Cr	52.5	Præodymium	Pr	144
Cobalt	Co	59	Rhodium	Rh	104
Niobium	Nb	94	Rubidium	Rb	85
Copper	Cu	63	Ruthenium	Ru	103.5
Decipium	Dp	171	Samarium	Sa	150
Didymium	Di	142	Scandium	Sc	44
Erbium	E	166	Selenium	Se	79
Fluorine	F	19	Silicon	Si	28
Gallium	Ga	70	Silver	Ag	108
Germanium	Ge	73	Sodium	Na	23
Gold	Au	197	Strontium	Sr	87.5
Helium	He	4 (?)	Sulphur	S	32
Hydrogen	H	1	Tantalum	Ta	182
Indium	In	113.5	Tellurium	Tl	125
Iodine	I	127	Thallium	Tl	204
Iridium	Ir	192.5	Thorium	Th	232
Iron	Fe	56	Thulium	Tu	171
Lanthanum	La	138.5	Tin	Sa	117.5

Titanium	Ti	48	Ytterbium	Yb	173
Tungsten	W	184	Yttrium	Y	89
Uranium	U	239	Zinc	Zn	65
Vanadium	V	51	Zirconium	Zr	90.5

Those elements most widely distributed and of most importance in Nature's household, are emphasized by wider spaces between the letters of their names.

To express the fact that several atoms of an element are held in combination, the number of atoms is indicated by a small figure to the right and below the atomic sign, *e.g.* the "chemical formula"  $H_2O$ , of water, indicates that a molecule of water consists of 2 atoms of hydrogen and 1 atom of oxygen, and so, that it contains, then, 2 parts by weight of hydrogen and 16 parts of oxygen.

**50. Molecular Forces.**—From the assumption of invariable molecules, the idea necessarily follows that the molecules do not touch, but are separated from one another by empty interstices (not to be confused with perceptible pores). For, only on this basis does the general property of variability of volumes become intelligible. According to this view, when the volume of a body is increased, or decreased, the molecules are merely thrown farther apart, or crowded closer together, *i.e.* only the mutual distances of the molecules change, the molecules themselves remaining unaltered. If the molecules of a body do not cling together of their own accord, forces analogous to gravitation in the planetary system must be called into play between these molecules to cause them to coalesce. These forces are known as *molecular forces*. Their intensity increases rapidly with increasing distance, and becomes imperceptible even at extremely small distances. These latter distances are known as *radii* of the *sphere of action* of the molecular forces. That these radii are quite small is shown by the fact that if a body be broken, or torn, asunder, as a rule, it is impossible to reunite the fragments, since the particles of the ruptured surfaces cannot be brought closely enough together to make the molecular attraction sufficiently strong. If the surfaces of glass, or metal plates be polished so highly as to permit of extremely close contact of several points,

the surfaces cling together with great force—with much greater force than can be explained by atmospheric pressure alone. The molecular force of attraction which binds the molecules of a body together is called *cohesion*; and that which causes the particles of different bodies to cling together is called *adhesion*. These forces show themselves with unusual intensity, when one of the bodies is in a liquid state, and gradually solidifies by the evaporation of its liquid constituents, *e.g.* glue, cement, solders, etc. The attraction which combines atoms chemically into molecules is called *chemical affinity*, or *affinity*. Physics deals with those phenomena only in which the constitution, or composition, of the molecule is not affected, while chemistry concerns itself with those phenomena with which the composition of the molecule is altered.

**51. Cohesion.**—Rigidity and hardness are manifestations of the cohesive forces, which oppose themselves to any external force tending to separate the particles of a body. Rigidity may be defined as the force, which is just sufficient to separate the particles of the body. Different kinds of rigidity may be distinguished according to the manner in which the external force is applied. Rigidity in tension, or *absolute rigidity*, is the resistance of a body to *tearing*. It is proportional to the cross-section of the body, and independent of the length. It is measured by the number of kilograms required to tear apart a rod, or bar, of 1 mm. cross-section. For lead, it is 2.2; for tin, 2.6; for gold, 26; for silver, 29; platinum, 34; copper, 40; brass, 60; iron, 63; and for drawn steel, 83 kg. With most metals the absolute rigidity is diminished by tempering, or annealing. Transverse, or *relative rigidity* of a beam, or bar, is the resistance of the beam to transverse fracture and depends upon its length, the magnitude and form of its cross-section, the manner of applying the force, and the mode of supporting the beam. With equal masses, hollow beams and tubes have greatest transverse rigidity. Rigidity in *compression* is the resistance to crushing; rigidity in *shear* is the resistance of a body to the separation of its particles in a lateral direction; and *torsional rigidity* is the resistance of a body to rupture by a twisting force.

## SOLIDS.

*Hardness* is the resistance a body opposes to a force tending to wear or scratch its surface. To establish a criterion for the determination of hardness, Mohr constructed the following scale of hardness for the minerals occurring in nature in fixed and invariable forms, viz. (1) talc, (2) gypsum, (3) calc-spar, (4) fluor-spar, (5) apatite, (6) feld-spar, (7) quartz, (8) topaz, (9) sapphire, (10) diamond. In this list every mineral scratches the preceding, and is scratched by the following one.

With reference to the mode of separation of the particles of bodies, their cohesive forces may be classified. If the coherence of the particles of a body is not instantly destroyed, but a continuous and considerable alteration of form takes place before rupture, the body is said to be malleable, ductile, or flexible. If, on the contrary, the separation of parts be sudden, the body is called *brittle*. Hard bodies are usually brittle, and soft ones, *malleable*. The parts of malleable bodies may be reunited by merely pressing them together. For example, platinum utensils are made by properly compressing the platinum-sponge, and two glowing fragments of iron may be welded into one by crowding the white hot ends against each other.

These properties are due less to the material constitution of the particles than to their arrangement with reference to each other. Carbon, for example, when regularly crystallized in the form of diamond, is the hardest of known bodies; but when hexagonally crystallized as graphite, it is quite soft. By slight admixture of other materials, as also by change of temperature, cohesion is materially altered. One of the most familiar examples of this is the conversion of iron into steel by a slight increase in the quantity of carbon it contains. Highly hardened steel is produced by rapid cooling. In this process of cooling the surface of the body becomes cold and rigid, while the interior remains hot and distended. As the inner portions cool farther they experience resistance to their further contraction from the arch-like reaction of this vaulted surface. Consequently, the exterior portions of the body are in a state of high compression, while the internal parts are in an equally high state of tension. This strained condition of

the molecules of a body manifests itself in increased hardness and brittleness. By heating (or "tempering"), the hardened metal loses a portion of its brittleness, but also a portion of its hardness. Glass is also hardened by rapid cooling. Melted glass dropped into water solidifies into a very hard, slender-pointed globule, known as the Prince Rupert's drop, or the Batavian drop. The body of these drops will stand a smart blow; but if the tail be broken the whole flies into minute particles with considerable violence, just as a loaded arch tumbles to pieces when its key-stone is removed. The thick-bottomed *Bolognese phials*, formed by the rapid cooling of glass in the air, instantly fly into pieces when a sharp-cornered fragment of flint is thrown within them. The flint scratches the inner surface and thus destroys the resistance which, in its unimpaired condition, it opposes to the internal strain of the particles.

**52. Elasticity.**—A body is said to be *elastic* if, after deformation, it returns to its original form. All solids are elastic if the deformation does not exceed a certain limit, called the *limit of elasticity*. If this limit be exceeded the body undergoes a permanent change of form, or a *set*, and a weakening of the cohesive forces of its molecules results. If this limit be repeatedly transgressed the body will finally be torn asunder. With brittle bodies, however, rupture occurs suddenly. Within the limit of elasticity the change of form is gradual and, after the removal of the external force, the original form is also gradually regained. This behaviour of a body after the removal of all deforming forces is sometimes called the effect of *elastic reaction*.

If a silver thread 1 m. long and of 1 sq. mm. cross-section be suspended at one end, and loaded at the other with a weight of 1 kg., its length will be increased by 0.14 mm. Twice, thrice, etc., the weight will be found to produce twice, thrice, etc., the elongation. Consequently, *elongations are proportional to the tensile forces* (Hooke, 1675). If a thread 2 m. long be loaded with 1 kg., it will undergo a stretch of 0.28 mm. Since each meter of length stretches by 0.14 mm., the total stretch with a thread 2 m. long must, of course, be double that of the thread 1 m. in length, or *the elongation is proportional to the length of the thread*. A silver thread of 1 m. length and

2 sq. mm. cross-section is stretched by a weight of 1 kg., 0.07 mm. The thread of 2 sq. mm. cross-section may be regarded as a combination of two threads each of 1 sq. mm. cross-section. The tensile force is then divided, so to speak, into two equal parts by the two threads. Each has a cross-section of 1 sq. mm., and is stretched by a force of 0.5 kg. The length of each will therefore be increased by one-half of 0.14 mm., or by 0.07 mm. It appears, then, that with the same force, *elongations are inversely as areas of cross-section*. But these laws hold only within the limit of elasticity. For the silver thread (1 m., 1 sq. mm.) this limit is reached at an elongation of 1.4 mm., which would be produced by a load of 10 kg. The elongation cannot be carried farther without leaving a permanent *set* in the thread. This force may be used as a measure of the elasticity of the substance. By virtue of these laws the elastic behaviour of a body under a tensile force is completely known; if the fractional part of its length by which a wire or prism of 1 sq. mm. cross-section is stretched by a tensile force 1 kg. is known. This fraction is called the *coefficient of elasticity*. This coefficient is for silver, 0.00014, or more exactly  $\frac{1}{7100}$ ; that of gold is  $\frac{1}{5100}$ ; of platinum,  $\frac{1}{7800}$ ; of copper,  $\frac{1}{5100}$ ; of iron,  $\frac{1}{21000}$ ; of steel,  $\frac{1}{17000}$ ; of brass,  $\frac{1}{5000}$ ; of german silver,  $\frac{1}{12000}$ ; of lead,  $\frac{1}{15000}$ ; and of glass,  $\frac{1}{81000}$ .

The denominators of these fractions indicate the number of kilograms (e.g. for steel 19,000 kg.) which would be required to stretch a wire of the material in question, 1 sq. mm. in cross-section, to double its length, provided that, in so doing, the limit of elasticity be not exceeded. These denominators are called *moduli of elasticity*.

The elongation of a body in the direction of its length (*stretch, elongation*) is always accompanied by a contraction of its cross-section (*contraction*).

Denote by  $L$  the length of the wire (in m.); by  $q$ , its cross-section (in sq. mm.); by  $l$  (in m.), the elongation produced by the load  $P$  (in kg.); by  $\epsilon$ , the coefficient of elasticity of the substance; and by  $E = \frac{l}{\epsilon}$ , the modulus of elasticity, and the

above laws may all be condensed into the single equation

$$l = \epsilon \cdot P \frac{l}{q}, \text{ or } P = E \cdot q \cdot \frac{l}{l}.$$

If a prism of any substance be subjected to a pressure in the direction of its length, it will be shortened by just as much as a tensile force of equal intensity would stretch it. That the deformations of elastic bodies are directly proportional to the forces operating can be well illustrated by means of spirally wound metallic wires, called *spiral springs*, since with them relatively small forces, either in tension or compression, produce considerable changes in length without reaching the elastic limit. For this reason coiled springs may be advantageously used as *spring balances* to determine weights. These spring balances may be constructed in either of two ways. A slender coiled spring, fixed at its upper end, indicates by its elongation, read from a scale graduated to millimeters, the weight of a body placed in a scale-pan suspended to the lower end of the spring. This form is known as Jolly's spring balance. The second form of the apparatus is that in which a stiff spring, supported below and carrying a scale above, gives by its shortening, read directly either from a rectilinear scale or from a circular dial, graduated experimentally, over which a pointer plays, the weights of bodies placed upon a platform attached to the spring. The latter form is very extensively used for household purposes. Spring balances used for measuring large forces are called *dynamometers*. They consist usually of a strong strip of steel bent into the form of a spring which, by its deformation, sets a pointer in motion. If a dynamometer be placed between the traces of a horse and the plough, the force exerted by the horse to draw the plough may be read from the scale in kilograms. In all these apparatus the deformation consists almost entirely in the flexure of an elastic strip of metal.

*Torsional elasticity* is developed in a rod, or stretched wire, when the upper end is rigidly fixed and the lower turned or twisted by means of a horizontal lever-arm in a horizontal plane. The force with which the rod opposes this twist increases with the angle through which the lever-arm is turned.

The *torsion balance*, a contrivance for measuring small forces by balancing them against the twisting force of a wire, is an application of this law.

Elasticity finds numerous applications in practical life. In pocket watches and in clocks it acts as a driving force. A spiral spring, enclosed within a box, is twisted by a key into a state of high tension, and then, by virtue of its elasticity, slowly unwinds and sets the mechanism in motion. The coil of a crossbow, stretched by the hand and suddenly released, hurls the arrow with great velocity. The *ballistæ*, the siege guns of the ancients, depended likewise upon this principle of elasticity. Elasticity is also of great value in destroying the injurious effects of violent shocks. The springs under railway coaches, as also the stiff coiled springs holding the buffers of these coaches in place, serve this purpose. The spring balance, where elasticity is used for weighing and for measuring forces, has already been considered. With the ordinary balance the pressure of a body due to gravity is *compared* with that of the weights used to balance it. With this apparatus, therefore, the weight of a body would be the *same* at all points of the earth's surface. A spring balance, on the other hand, gives the *absolute* pull of the earth upon the body to be weighed, and, if graduated at any point of the earth's surface off the equator, and a weight of one kilogram be put upon its scale-pan at the equator, its scale would indicate less than one kilogram.

**53. Elastic Vibrations.** When the force tending to deform a body ceases to act, every particle of the body will be driven towards its original position by the force of elasticity, which is equal and opposite to the deforming force. The particles, however, do not suddenly come to rest in this position, but pass beyond it, thereby giving rise to a deformation opposite in kind to the former (*e.g.* compression instead of elongation), and then, under the influence of the elastic force due to this new deformation, they are again drawn toward their original position, and so on, until, after a series of *vibrations* (or *oscillations*), they finally come to rest in this position. Since the moving force is always proportional to the distance of

the moving point from the position of equilibrium, or to the distance this point must traverse to reach this position, these elastic vibrations are all of *equal duration*, or *isochronous*, as are those of a pendulum of small amplitude of vibration. The duration of vibration here means the time required to perform a complete *to and fro* vibration, and not, as in the case of the pendulum, the duration of a *to or fro* vibration. By means of a coiled metallic wire, suspended vertically, to whose lower end a weight is attached, elastic vibrations may be easily made apparent to the eye. If the weight be drawn downward with the fingers and then let loose, it will vibrate vertically upward and downward, the wire being alternately shortened and lengthened. The number of vibrations performed in a given length of time will always be the same, whether the weight be drawn downward slightly, or considerably.

If now the weight be drawn downward, 2 cm., for example, not only must the hand exercise against the gradually increasing elastic resistance of the wire an average pull twice as great as if it were drawn downward by only 1 cm., but it must also move through twice as great a distance. The work performed in overcoming the elastic force of the wire in the former case is therefore four times as great as in the latter. If, further, the weight be drawn downward with three times the force through three times the distance, nine times as much work must be performed. The instant the hand is removed the work performed is transferred to the weight, and reveals itself in the energy of the vibration. With twice the amplitude of swing, the vibrations are performed with four times the energy; with three times the amplitude, they are performed with nine times the energy, etc., or, generally, the *energy of vibration is proportional to the square of the amplitude*.

The duration of vibration of a particle under the action of the force of elasticity may be calculated by a method similar to that used with the pendulum. Like the pendulum, such a particle is capable of a circular motion resolvable into two vibrations of equal duration and perpendicular to each other. Denote by  $p$  the force at distance  $l$  from the position of equilibrium; the centripetal force at the circumference of a

circle with radius  $r$  is then  $pr$ . Again, if  $T$  be the period of revolution, and  $m$  the mass of the particle, this centripetal force also equals  $\frac{4\pi^2mr}{T^2}$ . Consequently  $pr = \frac{4\pi^2mr}{T^2}$ ;

whence

$$T = 2\pi\sqrt{\frac{m}{p}}.$$

An important application of the invariable equality of duration of elastic vibrations was made by Hooke, in 1658, in the regulation of watches. This he did by attaching to the balance wheel a delicate spirally coiled spring, which, by alternately winding and unwinding, checked, by the aid of the balance wheel, the movement of the escapement at equal intervals, and then released the advancing second-hand at precisely equal intervals of time, allowing it to advance one step.

**54. Crystallization.**—*Crystals* are solids of regular internal structure, which, when allowed to develop unmolested, are bounded by regular plane surfaces. If a piece of calc-spar be struck a smart blow with a hammer, it will fall apart into angular pieces, each of which is bounded by six surfaces, parallel to each other, two and two. This tendency of the body to cleavage along three definite planes betrays the regular internal structure of calc-spar. It is this peculiarity of structure which accounts for the external form of the crystals (Fig. 53) of calc-spar occurring in nature. In their simplest form these crystals are bounded by six surfaces, parallel, in pairs, to the three directions of internal cleavage. The bounding surfaces of crystals are not always, as in the adduced example, parallel to the directions of cleavage. Only pure chemical compounds, i.e. only such as are wholly free from foreign admixture, and elements are capable of crystallization. When the separation of a solid from its solution, or the solidification of a molten body takes place so slowly and quietly that the molecules of the body may dispose themselves regularly under the action of the molecular forces of attraction, crystallization may occur. All possible forms of crystals are reducible to six *fundamental forms*, or *types*, each of which, with its various derivatives, constitutes a *system of crystals*. The regular

eight-sided crystal (the regular octahedron, Fig. 49) may be regarded as the fundamental form of the *regular* or *isometric* system. This form is a double pyramid bounded by eight equal equilateral triangles. The three right lines connecting the opposite vertices of the octahedron, intersecting each other at right angles at the centre of the crystal, are called its *axes*. This system of three equal axes intersecting each other at right angles (Fig. 50) is of fundamental importance to the forms of



FIG. 49.—Octahedron.



FIG. 50.—Regular Intersecting Axes.

all crystals belonging to the isometric system. The other forms of crystals are also most simply discussed by reference to the same system of intersecting axes. The quadratic, tetragonal, or *orthometric* system is that in which all the forms are referable to a system of three rectangular axes, only two of which are equal, the third, or *principal* axis, being either shorter or longer than the others. The double quadratic pyramid (Fig. 51), enclosed by eight equal equilateral triangles, may be taken as



FIG. 51.—Double Pyramid or Tetrahedron.



FIG. 52.—Double Hexagonal Pyramid.

the fundamental form of this system. The crystals of the hexagonal system are referred to four axes, of which three are equal, lie in one plane and cross each other at angles of sixty degrees. The fourth axis, called the *principal* axis, is at right angles to the plane of the other three (the *secondary* axes), and passes through their common intersection. The fundamental form is the double hexagonal pyramid (Fig. 52) bounded by

twelve equilateral triangles. If the first, third, and fifth faces of the upper pyramid of this typical crystal, and the second, fourth, and sixth of the lower be extended until the remaining faces vanish, the rhombohedron of Fig. 53, bounded by six quadrangular surfaces, is formed. A crystalline form, obtained by extending the *alternate* surfaces until the other surfaces vanish, is called a *hemihedron*. This rhomboidal, or orthorhombic system is characterized by three rectangular axes of unequal length. The clinorhombic, monoclinic, or monosymmetric system has likewise three unequal axes, of which two intersect each other obliquely, while the third is at right angles to the plane of the other two. The relative lengths of the axes vary greatly for different crystals. In the clinorhombic, triclinic, or asymmetric system, the three axes are oblique to each other and unequal in length. As a rule, a definite form is peculiar to every simple or com-



FIG. 53.—Rhombohedron.

compound substance, or the various forms which its crystals are capable of assuming may be reduced to a single fundamental form, or, what is the same thing, they all belong to one and the same system of crystals. Many substances, however, are capable of crystallizing in two different systems. On this account they are termed *dimorphous*. *Calcium carbonate* crystallizes, for example, as *calc-spar* in the hexagonal system (i.e. as rhombohedra), and as *aragonite* in various forms of the rhomboidal system. Bodies which consist of a mass of imperfectly formed crystals (e.g. marble, white sugar) are called *crystalline*. Bodies, on the contrary, not exhibiting this crystalline structure are called *amorphous* (formless).

Amorphous bodies whose internal structure shows the same constitution throughout, exhibit the same physical characteristics in all directions (e.g. glass, all fluids). The same is true of all crystals of the regular system, whose internal constitution is characterized by three equal axes at right angles to one another. Such bodies are *isotropic*. The crystals of the five remaining systems, on the other hand, exhibit different properties in different directions, as regards their rigidity.

hardness, elasticity, heat-conductivity, propagation of light, and so forth. Such bodies are *anisotropic*, or *heterotropic*.

**55. Impact—Shocks.**—A shock occurs when a moving body collides with another body, either at rest, or in motion. For example, a moving railway car may strike against another at rest upon the track. As soon as the buffers come in contact with each other, the moving car exerts a pressure against the one at rest, and in turn suffers from the latter an equally great counter-pressure. The car which received the impact is set in motion and accelerated; the motion of the other car is, on the contrary, retarded. This pressure continues, however, only until both cars attain the same velocity. At the instant when this condition is reached, the first part of the effect of the shock is completed. But the buffers of cars are provided with spirally wound steel springs. While the cars are acting upon each other these springs are being compressed with a force equal to their reciprocal pressure. As soon as this pressure ceases, however, they recoil by virtue of their elasticity, with precisely the same force; so that in the second part of the effect of impact, the impinging car receives a backward pressure, and the other a forward pressure, equal in both cases to the pressure experienced during the first portion of the shock. Suppose, for example, the two cars to be of equal mass (*i.e.* equally heavy), then, at the end of the first part of the effect, the following car loses half its velocity, since it is compelled to impart it to the preceding car, and, in the second part, it again suffers an equal loss, and hence comes to rest. On the other hand, the car which was at rest at the outset receives, during the second part of the action, the same acceleration as during the first, and, consequently, moves forward with the entire original velocity of the colliding car. Precisely the same thing happens when two *elastic* bodies, *e.g.* two ivory balls, collide. During the first part of the shock their surfaces become flattened at the point of contact. During the second half, they resume their original form, the flattened portions of the surfaces mutually reacting against each other like a spring, so that each proceeds with the velocity formerly possessed by the other. That a flattening of the surfaces occurs at collision,

may be proved by dropping an ivory ball upon a smooth marble surface, covered with soot. The ball rebounds almost to its original height, and, though perfectly round, shows at the point of contact of its surface a circular black spot of considerably greater diameter than would be produced if it were quietly laid upon the blackened surface, thus showing that at the instant of collision, the entire surface of the ball covered by the spot was in contact with the plate. During the progress of the shock, no portion of the entire energy of the colliding bodies is lost, for that portion of the energy, which is consumed during the first portion of the shock to flatten the balls, i.e. to compress the springs, and converted from energy of motion into the potential energy of the compressed springs, is, during the second part of the shock, while the flattening is disappearing, or the spring is returning to its condition of equilibrium, wholly retransformed into energy of motion, or *kinetic energy*. But with *inelastic* bodies the matter is very different. If, for example, the springs of the buffers had been made of lead, instead of steel, they would have been compressed as before during the collision, but would not then have returned to their original state. The second part of the shock would consequently have no existence. The cars would continue to move each with half the velocity of the first, and would have lost completely that portion of the original energy which was consumed in compressing and heating the lead in the spring.

If two masses,  $m$  and  $m'$  (Fig. 54), move in the same straight line with the velocities  $c$  and  $c'$  ( $c' > c$ ), in the direction of the arrow ( $\rightarrow$ ), and at the close of the first part of the shock the masses have attained the common velocity  $u$ , the former will have gained the velocity  $u - c$ , and the latter will have lost the velocity  $c' - u$ . The forces which the two bodies exert upon each other, as expressed by the products of the masses into the corresponding changes of the velocities. Being merely an action and its corresponding reaction, they must be equal. We have, therefore,  $m(u - c) = m'(c' - u)$ , or  $(m + m')u = mc + m'c'$ , i.e. the sums of the quantities of motion before and after the shock are equal to each other. This holds both for *elastic* and for *inelastic* bodies.

Since with *inelastic* bodies, the second part of the shock disappears, the bodies move on together with the common velocity,

$$u = \frac{mc + m'c'}{m + m'}$$

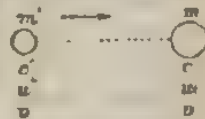


FIG. 54.—Shock

i.e. the velocity after the shock, therefore, is the weighted mean of the velocities before the shock. If, on the contrary, the colliding bodies are perfectly elastic, the mass  $m$  gains during the second act of the shock the velocity-increment  $u - c$ , the mass  $m'$  loses for a second time the velocity  $c' - u$ , and their respective velocities,  $v$  and  $v'$ , after the shock, are  $v = u + (u - c)$  and  $v' = u - (c' - u)$ , or  $v = 2u - c$  and  $v' = 2u - c'$ , in which equations, by merely substituting the above value of  $u$ , the values of  $v$  and  $v'$  are obtained in terms of  $m, m', c, c'$ .

If the masses are equal ( $m' = m$ ), then  $2u = c + c'$ , and, therefore,  $v = c'$  and  $v' = c$ , i.e. equal elastic masses proceed after shock with interchanged velocities. All these equations hold, not only when, as was assumed, the bodies move in the same direction, but they are also true for motions in opposite directions provided opposite velocities be regarded as negative.

It is, furthermore, easy to see that  $mv + m'v' = mc + m'c'$  on the one hand, and on the other,  $mu + m'u > mc + m'c'$ , i.e. with elastic masses the total kinetic energy before and after a shock is the same, but with inelastic bodies the kinetic energy is lost.

The laws of shock may be proved also by means of the *percussion machine*. In this apparatus, two or more spheres are suspended from the arm of a standard, in such way as to touch each other mutually when hanging at equilibrium. If one of these balls be raised to a certain height and then dropped, it will fall along the arc of a circle and strike against the other balls in a horizontal direction. With two elastic balls (e.g. of ivory) of equal mass, the falling ball comes to rest upon striking the stationary ball, which latter rises to the height from which the former fell. When several (e.g. seven) equal balls are hung in a straight line, and the first is dropped against the second, the seventh alone flies forward, each intermediate ball remaining quietly in its position of equilibrium, merely transfers the velocity of the ball next preceding it to the one next following. If the first two balls be dropped together against the third, the last two balls of the row fly forward; and if the first three impinge against the fourth, the last three balls move forward together, only the middle ball remaining at rest.

Two equal, inelastic spheres of moist clay, after collision, move forward together with half the velocity of the first, and both are permanently flattened.

If an elastic body strikes perpendicularly against an immovable elastic wall, it rebounds with unchanged velocity along the same line. If it strikes the wall obliquely, however, it is thrown backward with the same velocity and at the same

angle with the surface of the wall, as is easily observed in a game of billiards.

If the position of the masses  $m$  and  $m'$ , upon the line connecting their centres of gravity, at any instant before the shock, be given by their distances  $r$  and  $r'$ , from any point  $A$  (Fig. 55) upon this line, the distance  $s$  of their common centre of gravity from  $A$ , is by (28) given by means of the equation  $(m + m')s = mr + m'r'$ . If during the next infinitesimal interval of time  $\tau$ , the masses move through the infinitesimal distances  $\rho$  and  $\rho'$ , their centres of gravity during the same time will be displaced through the distance  $\sigma$ , in such way as always to fulfil the equation—

$$(m + m')(s + \sigma) = m(r + \rho) + m'(r' + \rho'),$$

from which, since the foregoing equation also holds, it follows that

$$(m' + m)\sigma = m\rho + m'\rho'.$$

If both sides of this equation be divided by  $\tau$ , and the velocities of the masses  $m$  and  $m'$ , that is,  $\frac{\rho}{\tau}$  and  $\frac{\rho'}{\tau}$ , be put equal to  $c$  and  $c'$  respectively, and the velocity of their common centre of gravity,  $\frac{\sigma}{\tau}$ , be designated by  $u$ , this latter velocity will then be determined by the equation—

$$(m + m')u = mc + m'c',$$

from which the velocity after impact was determined in a foregoing paragraph. The velocity of the common centre of gravity before the shock is, therefore, the same as after the shock, or the motion of the common centre of gravity is not changed by the shock. *This principle of the conservation of the centre of gravity holds not only for two, but for any number of masses, which may collide any number of times, because at each shock the compressional forces exerted are equal and opposite and, consequently, upon the motion of the centre of gravity of the whole, they can have no influence. After the bursting of a bombshell, for example, the common centre of gravity of all the fragments moves undisturbedly forward in its original parabolic orbit.*

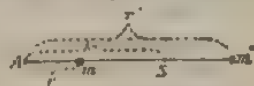


FIG. 55.—Conservation of Centre of Gravity.

**56. Friction.**—When two bodies, pressed against each other by forces, move relatively to one another, in consequence of the roughness of their surfaces their relative motion is opposed by a force called friction. The magnitude of the resistance of friction can be obtained by a simple apparatus invented by Coulomb, in 1781. This device consists of a little box with a smooth bottom in which known weights may be placed, and which slides upon a horizontal surface, or upon a track provided for the purpose. A cord attached to the box passes over a pulley and carries a scale-pan at its end. Weights are placed

upon this scale-pan until the box is just on the point of moving. The weight, inclusive of the weight of the scale-pan, required just to produce motion, gives then the frictional resistance which is to be overcome. In this way it is found that friction is independent of the areas of the surfaces in contact, for it is seen to be just as great upon the rails of a track as upon a smooth surface of the same material. It is found, furthermore, that frictional resistance is proportional to the load (the weight of the box itself being included in the load), or that the resistance of friction is proportional to the pressure exerted perpendicularly to the surfaces of contact.

The fraction which expresses for any given material the portion of this perpendicular pressure required to overcome friction, is called the *coefficient of friction*. This coefficient is, for cast-iron upon cast-iron, 0.16, or 16%; for cast-iron upon bronze, 0.15; for bronze upon bronze, 0.20. From this it appears that friction is greater between the surfaces of like than of unlike substances.

The following example will illustrate the manner of including friction in the work of machines. Let a heavy body be placed upon the surface of an adjustable inclined plane. Its weight  $Q$ , acting vertically downwards, may be resolved into two components (21), one of which,  $Q \cos \alpha$ , acts perpendicularly to the surface of the inclined plane, whose inclination is  $\alpha$ , and the other,  $Q \sin \alpha$ , acts parallel to this surface. The former represents the pressure with which the body is held against the plane, and gives rise to a frictional resistance, which, if  $f$  denote the coefficient of friction, equals  $f \cdot Q \cos \alpha$ . The latter component, on the other hand, tends to move the body down the plane. If now the inclination of the plane be increased continuously from zero, the body will remain at rest so long as friction is greater than this sliding force,  $Q \sin \alpha$ . At a certain angle, called the *angle of friction*, or the *angle of repose*, friction becomes equal to this sliding force, and, at this instant, where  $fQ \cos \alpha = Q \sin \alpha$ , the body begins to slide downward. From this equation we find  $f = \tan \alpha$ . The coefficient of friction may therefore be determined by measuring the angle of friction. The angular slope which loose materials,

such as sand, gravel, etc., assume when thrown into a heap, is the same as the angle of friction of the material.

Friction is called *sliding*, when, as in the cases above-mentioned, one body is pushed along upon the other, so that its surface irregularities are either torn off, or lifted away from those of the supporting surface. On the other hand, friction is termed *rolling*, when a round body, *e.g.* a cylinder, or a wheel, rolls over its supporting surface. Since, in this case, the elevations of the one body drop into the depressions of the other, and are then lifted out of these depressions, like the teeth of gear-wheels, rolling friction is much less than sliding. It is directly proportional to the pressure and inversely proportional to the radius of the cylinder, or wheel. In moving heavy loads, therefore, a great advantage is gained by converting sliding into rolling friction. This may be done by placing rollers under the load, or by supporting the load upon a frame-work provided with wheels.

With all machines, a portion of their work is consumed in the development of heat occasioned by the unavoidable frictional resistance, and is, therefore, lost to use. To diminish these resistances, lubricants are used. The hollows of the rubbing surfaces are filled up by the particles of the lubricant, and thus the degree of smoothness is enhanced. In many cases, on the contrary, friction is useful. The fastening of bodies together by means of nails, screws, strings, etc., is made possible by friction. The transmission of motion by means of belts, the retardation of motion by brakes, and so forth, owe their efficiency to friction. The foot would not adhere to the ground if it were not for friction, and the locomotive could not proceed upon its track, even when its drivers were rotated, if friction were wanting, as is sometimes illustrated when the rails are covered with ice.

With Prony's friction dynamometer (1822) friction is used to measure the work of machines. Two hollow wooden jaws, one of which carries a lever with an attached scale-pan, are pressed together about the shaft of an engine by means of a nut. The weight  $P$  is determined, which, with a definite number of revolutions, must be applied at the end of the

lever-arm  $l$ , to prevent the lever from being carried with the shaft by friction against the brakes, or jaws.  $Pl$  is then the moment of rotation which holds the moment of friction in equilibrium, and  $\omega Pl$  denotes the work per second, if  $\omega$  designates the angular velocity of the shaft. For  $n$  rotations per minute,  $\omega = \frac{2\pi n}{60}$ . If  $P$  is in *kg*, and  $l$  in *m*, the work in kilogram-meters  $= 2\pi n Pl : 60$ , and in horse-power it is obtained (17) by dividing this by seventy-five.

### III. LIQUIDS.

#### HYDROSTATICS.

**57. Liquids.**—Liquids are characterized by the ease with which their particles move upon each other, and by having an extremely slight compressibility. The very low degree in which liquids exhibit this property may be judged from the circumstance that they were long considered absolutely incompressible. Under ordinary conditions their compressibility is, indeed, so slight that they may, without material error, be so treated. While liquids offer no resistance to change of form, the latter depending wholly upon the form of the vessel containing them, they, nevertheless, resist a diminution of volume with tremendous force. Water is compressed by a pressure of 1 kg. per sq. cm. by only one fifty-millionth of its volume (71), and after the removal of the pressure it returns immediately to its original volume. The force with which the compressed liquid strives to expand holds the external pressure in equilibrium. Liquids are consequently perfectly elastic in respect of volume, but absolutely inelastic in respect of form.

**58. Transmission of Pressure.**—By virtue of the facility of liquid particles to move upon each other, liquids comport themselves with respect to external pressure quite differently from solids. If a solid be exposed to a pressure from above downward, this pressure will be transmitted in this same direction from layer to layer to the supporting surface without any appreciable lateral transmission. Indeed, forces both in tension and compression may be simultaneously applied to the body at the sides, and each will be transmitted likewise in but a single direction.

Imagine, on the contrary, a cylindrical vessel filled with a substance consisting of loose, movable particles, *e.g.* with fine shot, or with sand, and by means of a piston fitting closely

against the walls of the vessel, a pressure to be exerted upon the surface of the mass of sand, or shot. The particles against which the piston directly presses strive to wedge themselves between their neighbours, and these in turn strive in all directions to crowd themselves between those surrounding them, since all possible directions of deviation, not only forwards, but also sidewise, and even backwards, are open to them. The pressure is, consequently, transmitted throughout the mass in all directions, and is finally communicated to the walls of the vessel, against which it acts everywhere perpendicularly; for, if a hole be made through the wall of the vessel, either at the bottom, or in the side, the mass will always pass out through the orifice in a direction perpendicular to the wall. In consequence of the greater mobility of their particles, liquids show most perfectly this transmission of pressure in all directions. For them, therefore, this *fundamental hydrostatic law* holds: *Pressure applied to a liquid is transmitted with equal intensity in all directions throughout its entire mass.* The phrase "with equal intensity" signifies that every particle throughout the compressed liquid has, by virtue of the pressure upon it, the same tendency to motion in all directions. The pressure to which a given area of the wall of the vessel is exposed is consequently greater, the greater the number of liquid particles pressing against it, i.e. the greater the area considered. This "hydrostatic" pressure acts, not alone upon the walls of the vessel, but it is present everywhere throughout the liquid mass. For example, a sheet of tin, immersed in the liquid, suffers from both sides a pressure proportional to the area of its surfaces, directed perpendicularly to these surfaces. The pressure at any point of a liquid at rest is measured by the force acting upon the unit-surface at that point.

A useful application of the equal transmission of pressure of water in all directions is made in the *hydraulic press* (Bramah, 1795). It consists (Fig. 56) of a large cylinder (*cc*) and of a smaller one, filled with water and connected with the larger by a canal (*tt*). The large cylinder is closed by the piston (*pp*), while the smaller one contains the piston (*ss*) of a pump. The pressure exerted upon the latter is transmitted through the

water, and the piston, *pp*, is then pressed upward by a force which is to the pressure exerted by *s*, as the cross-section of piston, *pp*, is to that of piston, *s*. The piston, *pp*, carries at its top a plate (*nn*), which presses the object against a rigid plate (*c*), borne by heavy upright columns. When the piston, *s*, is raised, the valve, *d*, closes, the valve, *i*, opens and admits water from the reservoir, *bb*, through the screen, *r*. When the piston, *s*, is pressed downward in the barrel of the pump, the water is crowded into the cylinder, *cc*. The piston, *s*, is moved by means of a lever, the hand grasping the longer arm. Suppose the hand exert a pressure of 30 kg., and that the longer arm of the lever is six times the shorter, the latter being attached to the rod of the piston, *s*. The piston will then descend with a pressure of 180 kg. If

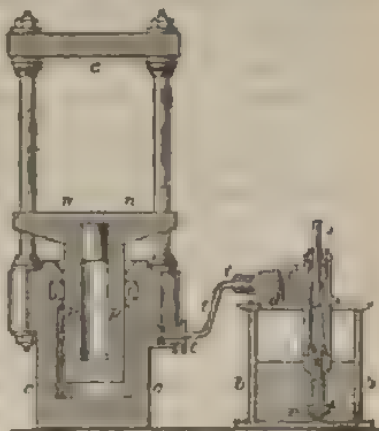


FIG. 56.—Hydraulic Press.

now, the lower surface of the compressing piston is one hundred times as great as that of the piston, *s*, it will be urged upward with a force of 18,000 kg. But the general principle of mechanics applies here; that what is gained in force is lost in velocity, or that the work of the moving forces is equal to the work of the resisting forces.

**59. The Effect of Gravity.**—We have hitherto considered only the transmission of an external pressure upon the liquid, and have left entirely out of consideration the effect of gravity upon it. It is at once clear that a liquid, contained in an open-topped vessel, can be in equilibrium only when its free surface is horizontal, i.e. when the direction of gravity is everywhere perpendicular to the surface; since, with any other form of the surface, there would necessarily occur a movement of the particles from the more elevated parts toward the less elevated, until finally a horizontal surface would result. It may also be

easily verified that with two or more vessels, communicating with each other by tubes, the surface of the liquid in all of them will stand at the same height, *i.e.* all the fluid surfaces will lie in the same horizontal plane, whatsoever form the vessels may have.

The *water-level*, used in determining horizontal lines, and consisting of a tin tube with ends turned vertically upward for the reception of glass tubes, depends upon this principle. Water, poured into these communicating tubes, rises in both of them to the same horizontal plane, and hence the line of sight for an eye looking along the two surfaces is necessarily horizontal. For the more accurate location of horizontal lines (*e.g.* of the optical axes of telescopes) and planes the level of Hooke, 1666, is used. It consists of a glass cylinder or tube, enclosed in a metal case, bowed slightly upward at its middle point, and nearly filled with alcohol, or ether. The unfilled part of the tube forms an air bubble, which always assumes the highest possible position along the tube. If the middle of the tube be designated by a suitable mark, and the bubble be made to play upon the mark, the base of the metal case will then lie horizontally.

Consider, for example, the watering-pot (Fig. 57), filled with water to MN. The surface of the water in the spout at N

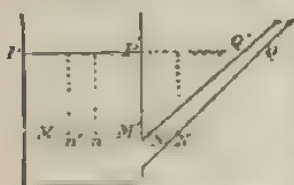


FIG. 57.—Watering-pot.

will be in exactly the same horizontal plane as is the surface of the water in the pot. If water be poured into the pot until it rises to the level PQ, then, since the water below MN, now as before, maintains its equilibrium, the oblique column of water, NN'Q'Q, in the spout must

hold in equilibrium the column, MM'P'P, in the pot, *i.e.* the pressure, which the column, NN'Q'Q, exerts upon the surface NN', and which is transmitted by the water underneath to the surface MM', must equal the pressure, which the water above MM' exerts downward upon an area,  $nn'$ , equal to the area NN'. The pressure to which the area  $nn'$  is exposed is merely the weight of the prism of water standing vertically above it, and extending to the plane PP'. Consequently

the pressure exerted by the oblique column, NQ, upon its base must equal the weight of a vertical column standing upon this base, and extending vertically upward to the plane of the surfaces of the water. The pressure which the gravity of a liquid exerts upon equal superficial areas, depends, then, only upon the vertical depth of the area below the surface of the water, and is proportional to this depth. Within the liquid mass all parts of any given horizontal plane are subject to the same pressure per unit-surface, and *this pressure increases downward proportionally to the depth*. Such surfaces of equal pressure are frequently called *equipotential surfaces*. In vessels of moderate size they appear to be horizontal planes, though they are really small portions of spherical surfaces concentric with the earth.

If the lower part of the U-shaped glass tube, *ac*, Fig. 58, contains a quantity of mercury, and water is poured into one leg of the tube, the mercury sinks in this and rises in the other leg until equilibrium is established. The horizontal plane, *ac*, drawn through the surface separating the two liquids, is then a surface of equal pressure, beneath which the mercury is of itself, in equilibrium, and upon the upper surface of which the column of water, *ab*, on the one hand, and the column of mercury, *cd*, on the other, exert equal pressures per superficial unit. It is found that the column of water measured vertically is 13·6 times as high as the column of mercury, and, consequently, that a given column of mercury weighs as much as a column of water 13·6 times as high. Volume for volume, therefore, mercury is 13·6 times as heavy as water. This number is called the *specific weight*, or, better, the *specific gravity* of mercury. It is also generally true that two different liquids, which do not mix, will be in equilibrium in communicating tubes when their heights above the surface separating them are inversely as their specific gravities.



FIG. 58 —  
U-shaped Tube.

**60. Vertical Downward Pressure.**—The pressure exerted by

a liquid upon the horizontal bottom of a vessel, is independent of its form and equal to the weight of a vertical column of the liquid standing directly above the bottom, and extending to its surface. In a vessel expanding upward, the pressure on the bottom is, accordingly, less than the weight of the liquid it contains, while, with a vessel contracting upward, it is greater than the weight. This conclusion, due to Stevin, 1586, so strange at first glance, has received the designation *hydrostatic paradox*. Its accuracy, however, is easily proved by means of a balance (Fig. 59), one of whose scale-pans is ground smooth, and may be made the bottom of variously shaped vessels, to be supported in succession upon a vertical standard. With

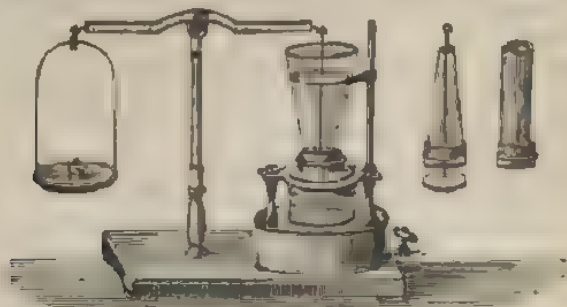


FIG. 59.—Hydrostatic Paradox

the same depth of water, precisely the same weight must be placed in the opposite scale-pan, to hold the liquid pressing against the bottom of the vessel in equilibrium, for all the different forms. If, however, the water were allowed to freeze in the vessels, a smaller weight would be needed to hold the liquid in the vessel tapering upward, in equilibrium, than in any of the other forms. This experimental proof was first performed by Pascal in 1653. The law of downward pressure can also be proved by means of the U-shaped tube of Fig. 58; for the surface of the mercury at *a* forms a movable bottom to the tube, *ab*, and, whatever form be given to it, the mercury column, *cd*, which holds the pressure at *a* in equilibrium, remains unchanged (Haldat). In accordance with this law, enormous pressures may be exerted by means of small quantities

of liquid. Pascal, in 1647, attached a slender tube, 10 m. high, to the upper end of a cask filled with water, and then, by merely filling the tube also with water, he succeeded in bursting the cask. The pressure on the bottom of the cask was, by the foregoing principle, equal to the weight of a column of water of cross-section equal to the area of the base of the cask, and extending to the top of the tube.

In the press, used for extracting the juices from vegetable substances, by the pressure of liquids, this principle is advantageously applied. The finely pulverized particles of the substance are placed between two sieve-like plates, enclosed within a large vessel, into whose cover a long, slender tube is fixed. This tube, as also the vessel, is filled with the liquid to be used in the extracting process. The substance is then submitted to a pressure equal to that of a column of the liquid whose cross-section equals that of the large vessel, and whose height equals the height of the liquid in the tube.

**61. Lateral Pressure.**—Upon every portion of the vertical wall of a vessel, filled with liquid, a pressure acts perpendicularly to the surface, and, therefore, in a horizontal direction, and with an intensity equal to the weight of a column of the liquid, whose base equals the area of the surface considered, and whose altitude equals its depth below the surface. In the drawing (Fig. 60), the pressure at any point ( $a$ ) of the wall of the vessel may be represented by

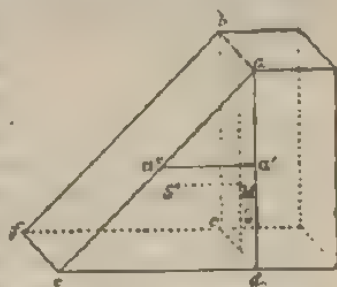


FIG. 60. — Lateral Pressure

a horizontal line,  $a'a''$ , of length equal to the depth,  $aa'$ , of this point below the surface. From this construction it appears that the rectangular wall,  $abcd$ , of a vessel is subject to a horizontal pressure, just as though the vessel were placed horizontally and loaded with the wedge-shaped prism of liquid,  $abcdef$ . A perpendicular dropped from the centre of gravity,  $S$ , upon the wall of the vessel, determines the point,  $M$ , at which this pressure, considered as the resultant of the

innumerable individual pressures acting at all the points of the surface, is applied. In the example just cited, this point, called the *centre of pressure*, lies upon the middle vertical line of the rectangle and at one-third of the depth from the base.

**62. Buoyancy.**—The pressure of a liquid, due to gravity, acts, not only downwards and sidewise, but also upwards. This upward action is called *buoyancy*. To prove the existence of this upward pressure, a large, open glass tube, with its lower end smoothly ground, against which a plane metallic disk is placed, is immersed in water, while the disk is held in place by a string attached to it, and extending upward through the tube. If, now, the end of the tube, closed by the disk, precede during the process of immersion, the disk will remain in place even though the thread be no longer held, because the upward pressure of the water holds it with considerable force against the end of the tube. But, if water be poured into the tube from above until it has attained nearly the same height within as without, the disk will fall off, because the downward pressure of the water in the tube, plus the weight of the disk, then exceeds the upward pressure of the water.

**63. Law of Archimedes.**—If a right cylinder with horizontal ends (ABCD, Fig. 61) be immersed in a liquid, every particle



FIG. 61 — Principle of Archimedes.

of its surfaces is exposed to a pressure depending upon its depth below the surface. The horizontal pressures against the lateral surfaces, being equal and opposite, mutually destroy each other. But the pressure acting upwards against the lower end, exceeds the downward pressure against the upper end, the former being equal to the weight of the liquid column (ABEF), and the latter to that of the column (CDEF). There remains, therefore, a pressure directed upward, equal to the excess of the former weight above the latter, or, what is the same thing, equal to the weight of a column (ABCD) of the liquid occupying the same space as the submerged body. This pressure, directed upwards, acts against the weight of the body, and reduces this weight correspondingly. We are thus led to the principle of Archimedes, viz. *a body immersed in a*

liquid, loses, by virtue of the pressure of the surrounding particles, a weight equal to that of the liquid displaced by it. To confirm this proposition experimentally, the hydrostatic balance (Fig. 62) may be used. One scale-pan of this balance carries below it a light hook, and this pan is also suspended close to the beam, so that vessels containing various liquids may be placed beneath it. Let a cylinder of metal be suspended to the hook by means of a slender cord, and a hollow cylinder, whose volume exactly equals that of the metal cylinder, be placed upon the scale-pan, and let the beam then be balanced by placing weights upon the opposite scale-pan. If, now, the cylinder be immersed in the water contained in the vessel below, its weight diminishes and the shorter scale-pan rises. Equilibrium is, however, completely restored if the hollow cylinder upon the scale-pan be filled with water. It is, therefore, apparent that the loss of weight of the immersed body is exactly equal to the weight of an equal volume of the liquid.

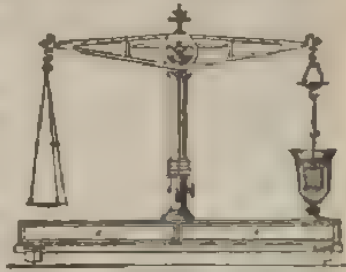


FIG. 62.—Hydrostatic Balance

Let a vessel filled with water be now placed upon the pan of an ordinary balance, and the empty hollow cylinder, together with weights sufficient to produce equilibrium, be placed upon the other pan. Furthermore, let a solid cylinder, suspended from a fixed standard, be immersed in the water contained in the vessel. Although the weight of the latter cylinder is entirely supported by the standard, the scale-pan on this side, nevertheless, sinks. Equilibrium will, however, be again restored, if the hollow cylinder on the other side be filled with water.

If, then, a body is immersed in a liquid, the latter apparently increases in weight by an amount equal to the weight of the displaced liquid (converse of the principle of Archimedes). The water rises in the vessel just as high as would have been the case if, before immersing the body,

a volume of water equal to that of the body had been poured into the vessel. The body opposes to the pressure of the liquid from all sides, an equal counter pressure, thereby acting as an equally large mass of water.

Archimedes' principle is applicable not only to cylinders and prisms, but also to bodies of any form whatever; for any body may be conceived as subdivided into slender vertical prisms, for each of which the principle holds good, and hence it must be true for the entire sum. Its validity may be shown more generally from the following considerations. If a body of weight,  $P$ , sink in a liquid to the depth,  $h$ , an equal volume of liquid of weight,  $Q$ , is at the same time raised through the same distance as that through which  $P$  sinks. Consequently, the work,  $Ph$ , of the freely falling body is diminished by the work,  $Qh$ , performed in raising  $Q$ . The total work performed is, therefore, the same as that performed by the weight,  $P - Q$ , falling freely through the distance  $h$ .

**64. Determination of Volume—Specific Gravity—Density.—**

If a body be suspended by means of a slender wire, or hair, to one of the pans of a hydrostatic balance, the known weight which, placed upon the other pan, produces equilibrium, gives directly the weight of the body in grammes.

Let the body be now immersed in water, and, by the addition of weights to the scale-pan, let equilibrium be established. The added weights will indicate the body's loss of weight, i.e. the weight of a volume of water equal to that of the body. Since a cubic centimeter of water weighs one gram, there will be as many cubic centimeters in the volume of the water displaced and also in that of the immersed body, as there are grams in the loss of weight (barring a small correction due to the temperature of the water). In this way the volume of any body, no matter how irregular, may be accurately found. By the same method, the *specific gravity* of bodies, i.e. the ratio of their weight to the weight of an equal volume of water, or the number which expresses how many times a body is heavier than an equal volume of water at 4° C., may also be readily obtained. It is only necessary to divide the weight of the body, which is indicated by the known weight lying in the lower scale-pan.

by the loss of weight in water as indicated by the weights placed upon the other pan.

The specific gravity of liquids may also be easily found by means of the hydrostatic balance. For this purpose, a body of any convenient form, *e.g.* a piece of glass, is suspended at one end of the beam and balanced by weights placed in a scale-pan at the other end. The loss of weight of this body is then determined, first in the liquid to be investigated and then in water. The former loss divided by the latter, gives the desired specific gravity. The loss of weight which bodies suffer in different liquids is, obviously, proportional to their specific gravities. Mohr's balance (Fig. 63) depends upon this principle. A small glass sinker, *A*, is suspended by a fine platinum thread to one arm of a scale-beam, and is partially filled with mercury and hermetically sealed, or it may be merely a glass tube containing a small thermometer, the balance being held in equilibrium by the scale-pan, *B*. The weights consist of bent brass wires, *P*, each of two of which weighs an amount equal to the loss of



FIG. 63. — Mohr's Balance.

weight of the sinker in water. A third weighs 0.1P, and a fourth, 0.01P. The arm of the beam to which the sinker is suspended is graduated into ten equal parts. To determine the specific gravity of a liquid, the sinker is immersed in a portion of it, contained in the vessel *CC*. If the liquid is sulphuric acid, for example, to establish equilibrium, one of the weights, *P*, must be placed at the end, *h*, of the beam, the other weight, *P*, at 8, the weight 0.1P at 4, and the weight 0.01P also at 8. Accordingly, the specific gravity of sulphuric acid is found to be 1.848. Jolly's spring balance (Fig. 52) may also be conveniently used to determine the specific gravity of small bodies. It is only necessary to suspend beneath the customary scale-pan, a second one in such way, that it may be immersed in a vessel of water. If a body be placed upon the upper scale-pan, the coiled spring will be stretched proportionally to

the weight of the body. If now, the body be placed upon the lower pan, the spring will then contract by an amount proportional to the body's loss of weight in water. The elongation divided by the latter contraction furnishes the desired specific gravity. To find the specific gravity of liquids, a glass sinker takes the place of the second scale-pan. The contraction of the spring on the immersion of the sinker in various liquids is proportional to their specific gravity.

The specific gravity of solids and liquids may also be accurately determined, without the use of Archimedes' principle, by means of the *pycnometer*. This is a small glass flask (Fig. 64), containing from 8 to 20 cubic cm., provided with a ground

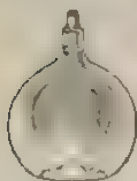


FIG. 64.—  
Pycnometer.

glass stopper, made from a piece of thermometer tube, so that if the liquid should become heated, a portion may pass out without loosening the stopper, or damaging the flask. The flask is first filled with the liquid, whose specific gravity is sought and weighed; then filled with water and weighed. The specific gravity is obtained by dividing the former weight by the latter.

To determine the specific gravity of solids, the flask is first filled with water and weighed; then a small particle of the body is placed upon the same scale-pan with the flask, and, equilibrium being established, the weight of the particle is obtained. The particle is now dropped inside the flask and the displaced water is allowed to flow out. After drying the flask carefully and weighing it again, the weight of a volume of water equal to that of the particle is obtained.

The following table contains the specific gravities of some of the most familiar solids and liquids—

#### A. Solids.

Iridium ... ..	22.40	Tin (cast ... ..)	7.29
Platinum (rolled) ... ..	21.50	Zinc (cast ... ..)	6.86
Gold (hammered) ... ..	19.36	Antimony ... ..	6.72
Lead (cast) ... ..	11.35	Heavy Spar ... ..	4.43
Silver (cast) ... ..	10.47	Diamond (highest ... ..)	3.53
Copper (cast) ... ..	8.79	Flint Glass ... ..	3.33
Brass ... ..	8.38	Fluor Spar ... ..	3.15
Iron (wrought) ... ..	7.79	Marble ... ..	2.84
Iron (cast) ... ..	7.21	Calc Spar ... ..	2.70

## A. Solids—continued.

Rock Crystal	..	268	Oak heart of	..	1.17
Aluminium ...	..	2.67	Amber ...	..	1.04
Bottle Glass	..	2.64	Wax-candle (white)	..	0.97
Mirror Glass	..	2.45	Sodium	..	0.97
Porcelain	...	2.40	Ice	...	0.92
Gypsum (crystallized)	..	2.31	Potassium	..	0.87
Sulphur (natural)	..	2.03	Beech-wood	..	0.80
Ivory	...	1.92	Linden-wood	..	0.60
Phosphorus	...	1.77	Lithium	...	0.59
Magnesium	...	1.74	Poplar-wood	..	0.38
Boxwood	...	1.33	Cork	...	0.24
Ebony	...	1.23			

## B. Liquids.

Mercury	...	13.596	Sea Water	...	1.02
Sulphuric Acid (con)	...	1.84	Linseed Oil	...	0.95
Nitric Acid	...	1.54	Olive Oil	...	0.91
Chloroform	...	1.48	Petroleum	...	0.89
Carbon Disulphide	...	1.27	Oil of Turpentine	...	0.87
Glycerine	...	1.26	Benzol	...	0.87
Hydrochloric Acid	...	1.21	Alcohol	...	0.79
Milk	...	1.03	Ether	...	0.74

The mass of a unit of volume of a body is called its *density*. In the *absolute* system of units, where the mass of a cubic centimeter of water is taken as the *mass-unit*, the above numbers also denote the densities. For the *terrestrial* system of units, however, these numbers must be divided by  $g = 981$ , to obtain the densities.

*Specific volume* is the volume of a unit of mass, and is accordingly the reciprocal of the density.

**65. Flotation.**—A submerged body, whose weight is exactly equal to that of the displaced liquid, loses its entire weight and floats, therefore, in the liquid, with no tendency either to sink or rise. If its weight be greater than that of the displaced liquid, it will sink; if smaller, it will rise and float upon the surface with only such a portion of its mass submerged as will displace a volume of the liquid whose weight equals that of the body. This proposition may be proved with the aid of the vessel of Fig. 65. When the vessel is filled with water to the mouth of the little tube, inserted in its side and a floating



FIG. 65.—For Floating Bodies.

body is placed carefully upon the surface of the water, a portion of the water will be led off through the tube and may be caught in a beaker placed beneath its outer end. If, now, the beaker with the water contained in it be placed upon one scale-pan of a balance and the floating body be thoroughly dried and placed in the other, the weight of the beaker itself having been previously balanced, equilibrium will then be established; consequently, the floating body is just as heavy as the water displaced by its submerged portion.

The point of application of the weight of the body is the centre of gravity,  $S$  (Fig. 66), while the buoyant effect of the



FIG. 66.—Metacentre.

liquid acts at the centre of gravity,  $A$ , of the mass displaced. These two oppositely equal forces destroy each other, and the body floats in equilibrium, provided these two

points lie in the same vertical line. If, then, the body be drawn out of this position of equilibrium, these forces form a couple, which tends to turn the body about its own centre of gravity, as shown in cross-section in Fig. 66. The centre of gravity of the displaced water is, for example, shifted sideways into the new position  $A'$ . If the vertical line through  $A'$  intersects the central line at a point  $M$  above the centre of gravity  $S$ , it is apparent that the couple tends to rotate the body back into its former position. The body is then said to float in *stable* equilibrium. But if  $M$  lies below  $S$ , the body will be overturned by the couple and will assume a new position of equilibrium. The point of intersection,  $M$ , of the direction of buoyancy, with this medial line, is called the *metacentre*. The equilibrium of a floating body is *stable* when its centre of gravity is *below* the metacentre, *unstable* when the centre of gravity is *above* the metacentre, and when, as with a homogeneous sphere, these two points *coincide* the equilibrium is *neutral*.

**66. Hydrometers.**—Floating bodies used to determine specific gravity, are called *hydrometers*. These instruments are of two kinds—those of *constant weight* and those of *constant volume*. That shown in Fig. 67 belongs to the first class. It

consists of a cylindrical glass bulb tapering downward and terminating below in a small globe filled with mercury, or other heavy substance, and continuing upward in a cylindrical stem (*x*). The instrument is then immersed in water, and the point on the stem to which it sinks is numbered 100. The stem is so graduated that the volume included between two consecutive divisions, equals 0.01 of the volume of water it displaces. If, now, the instrument, when immersed in liquid whose specific gravity is wanted, sinks only to the eightieth graduation, we know, then, that eighty parts by volume of this liquid weigh the same as 100 parts of water, i.e. as much as the entire hydrometer, and, consequently, that the liquid is heavier than an equal volume of water in the ratio of 100 : 80. The specific gravity of the unknown liquid is, therefore, inversely proportional to the submerged volume, and is found by dividing the number 100 by the number read from the graduated scale.

In the above example, the specific gravity =  $100 : 80 = 1.25$ . If, then, in a liquid which is specifically lighter than water, the instrument sinks to the graduation mark 110, its specific gravity would be  $100 : 110 = 0.909$ . To obviate the necessity of making the scale inconveniently long, it is preferable to have two hydrometers, one for liquids heavier than water with the graduation mark 100 (i.e. the water-point), at the upper end of the stem, and one for lighter liquids with the water-point at the lower end of the stem. Instruments of the form just described may be called *volumometers*. But the scale may also be so graduated as to indicate specific gravities immediately. Such forms of the instrument are called *densimeters*. Their consecutive graduations are unequally distant, those toward the lower end of the scale drawing closer and closer together continuously. In commerce, not only is the specific gravity of a liquid desired, but also the percentage of its various constituents, which determine its commercial value. Commercial alcohol is a mixture of alcohol and water, and its value depends upon the percentage of the former constituent. For testing



FIG. 67 —  
Graduated  
Hydrometer.

the mixture, special forms of instruments provided with scales indicating the percentage of alcohol directly are in use and are called *alcoholometers*. Various forms of these *percentage hydrometers* are known, and the names *alkalimeter*, *lactometer*, *salimeter*, etc., are applied to them. Each of these instruments can be used, of course, only with the liquid it is specially designed to test. Besides these forms of the instrument there are others with arbitrary scales, whose graduations are called "degrees." To this class belong the instruments of Beaumé, Beck, Cartier, and others, which give directly neither the specific gravity, nor the percentage of any constituent of the liquid. To determine the specific gravity of a liquid with their aid, a table must be used, though notwithstanding this fact, the latter forms of the instrument are more widely used than any other. Since the specific gravity of a liquid changes with the temperature, and, consequently, the data furnished by these instruments are correct only for the temperature at which they were graduated, which must always be given with the instrument. To obtain the temperature of the liquid investigated, to be used to correct the data of the hydrometer, a thermometer is often permanently attached in such way that its bulb is at the same time the bulb of the hydrometer.

Let us now consider the second class of *hydrometers*. These carry no scales, but are compelled by weights placed on a scale-pan to sink always to the same mark. They displace, therefore, always the same volume of liquid. The *Fahrenheit hydrometer* is made of glass, weighted with mercury, and has in place of the graduated scale, a slender neck provided with a mark, which carries at its top a scale-pan for the reception of weights. To sink the instrument in water to this mark, a certain weight must be placed upon the pan, and this weight, together with that of the entire instrument, gives the weight of the displaced water. To sink the instrument to the same mark in another liquid, different weights must be put upon the pan, which, added to that of the instrument, gives the weight of an equal volume of this liquid. Its specific gravity is then found by dividing the latter weight by the former. The *Nicholson hydrometer* (Fig. 68) serves to determine the specific

gravity of solids. It consists of a hollow cylinder, *a*, of brass, terminating both above and below in conical surfaces, the lower one carrying at its vertex a pan, *b*, and the upper one terminating in a slender stem also supporting a scale-pan, *d*. The stem is provided with a zero mark, *c*. To cause the instrument to sink to this mark in water, weights are placed upon the scale-pan, *d*. The body whose specific gravity is to be determined, and which must be lighter than the weight required to sink the instrument to the zero point, is placed upon the pan, *b*, and enough additional weights are then added to bring the zero point again to the surface. By subtracting these additional weights from the weights first used, the weight of the body is obtained. The body is now placed in the scale-pan *b*, under water, whereupon by Archimedes' principle, it will lose in weight an amount equal to the weight of the water it displaces. The weights, which must now be put upon the scale-pan, *d*, to depress the instrument to the zero point, give the weight of an equal volume of water, and the former weight needs only to be divided by this latter to give the specific gravity.



FIG. 68.—  
Nicholson's  
Hydrometer.

**67. Efflux of Liquids through Orifices.**—If an aperture be made in the bottom, or in the side, of a vessel filled with liquid, the action of gravity upon the liquid forces it through the aperture. If the surface of the liquid is kept constantly at the same height, *h*, and the efflux has become uniform, the mass of the liquid, *m*, flowing from the orifice, and which was formerly upon the surface, has fallen through the distance *h* before passing out, and the work *mgh* (*mg* being the weight of the liquid) has been performed by gravity. If the vessel is very large in comparison with the orifice, the velocities of the liquid particles in the vessel are very small in comparison with the velocity, *v*, in the orifice, and the above work, *mgh*, neglecting friction, is wholly consumed in imparting to the liquid mass, *m*, its energy =  $\frac{1}{2}mv^2$ . Consequently,

$$\frac{1}{2}mv^2 = mgh, \text{ or } v^2 = 2gh, \text{ or } v = \sqrt{2gh}.$$

The velocity of efflux is, therefore, equal to the velocity of

a body falling freely from the surface of the liquid to the orifice. This proposition, first enunciated by Torricelli in 1644, shows that the velocity of efflux is dependent only upon the height of the surface of the liquid above the orifice, being proportional to the square root of this height. It is independent of the constitution of the liquid, so that through orifices at equal distances below the surface, alcohol, water, mercury, etc., will flow out with the same velocities. These conclusions are confirmed by experience.

If a jet of liquid be made to flow through a tube fixed in an orifice and bent upward at its outer end, according to Torricelli's principle the jet should rise to the height from which the liquid fell. It actually attains only about nine-tenths of this altitude, since the friction of the tube walls, atmospheric resistance, and the falling drops of the liquid, greatly reduce the velocity.

The quantity of *efflux* per second is expressed in cubic centimeters, by dividing the cross-section of the aperture in sq. cm. by the velocity in cm., computed from the depth of the orifice below the surface (called the *head*), according to the above law. The actual quantity of efflux, since the jet does not have the same cross-section as that of the orifice, equals only about two-thirds (more exactly, 62 per cent.) of the computed quantity. In consequence of the pressure of the surrounding particles of the liquid as they flow toward the orifice, the prism of water flowing out is contracted, until its cross-section is only about two-thirds of that of the orifice.

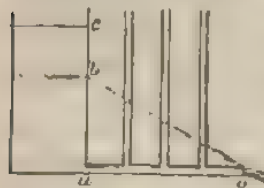


FIG. 69.—Efflux through Tubes.

This contracted portion of the escaping jet of liquid is known as the *vena contracta*.

**68. Efflux through Tubes.**—Torricelli's principle holds only for apertures in thin walls, whose edges oppose no appreciable resistance to the escaping jet. If, however, the liquid be compelled to flow through a horizontal tube (Fig. 69) communicating with a vessel at its bottom, after the flow has become uniform the pressure of the liquid on the walls of the

tube will be found to decrease gradually from its point of connection with the vessel, *a*, to its mouth, *c*. This is shown by the heights to which the liquid will rise in the separate vertical tubes placed at equal distances along a horizontal tube and communicating with it. The tops of the liquid columns lie in the straight line, *ob*, so that for equally long portions of the horizontal tube, equal differences of pressure occur. These differences of pressure would furnish the work of free fall, needed to overcome the frictional resistance in each consecutive segment of the tube. Consequently, the work of free fall through the distance, *ab* (*resistance head*), is wholly consumed in overcoming the frictional resistance through the entire tube, so that only the work due to the height, *bc* (called *velocity head*), remains to produce the energy of the escaping liquid.

**69. Reaction of the Jet.**—If water be confined in a vessel, and an aperture be made in one of the sides, the pressure is removed at this point from the portion of the surface of the wall equal to the cross-section of the orifice; but on the opposite side it remains in full force. There remains, therefore, upon the latter wall, an excess of pressure equal to the pressure, which forces the liquid through the orifice, and if the vessel were free to move in a horizontal direction, it would be driven in a direction opposite to that of the jet. *Segner's reaction wheel* (Fig. 70), generally known as the *hydraulic tourniquet*, or *Barker's mill*, depends upon this principle. The vessel (*A*), capable of turning about its vertical axis, is provided at its lower end with a horizontal tube with lateral openings near its ends. *Reaction turbines*, or *Scotch turbines*, extensively used as water-motors, are special applications of the general principle of reaction wheels. The usual turbine is a horizontal water-wheel lying below the surface of the water to be used. Fourneyron's

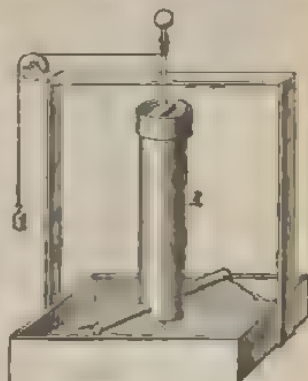


FIG. 70. — Segner's Reaction Wheel.

turbine (Fig. 71) consists of a pair of concentric disks, of which the inner and smaller, called the guide-wheel, is fixed, while



FIG. 71.—Fournreyron's Turbine.

the outer disk, A, turns about its vertical axis. The curved arms, or blades of the disk B, direct the outflowing water against a similar set of curved blades carried by the disk A. The latter wheel is driven forward in the direction of the arrow, both by the impact of the water flowing in against the blades of disk B, and also by the reaction against them of the outflowing water.

**70. Water Motors—Water Wheels.**—By means of vertical water-wheels turning on horizontal axes, the energy of flowing water is converted into that of *axial rotation*. The *undershot wheel* (Fig. 72) carries around its circumference a set of blades

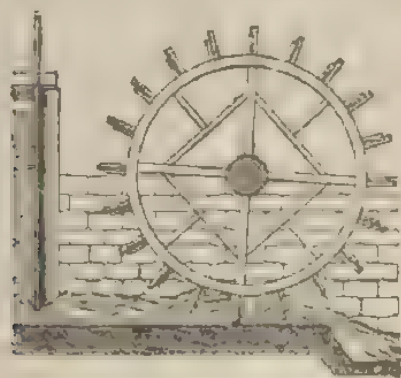


FIG. 72.—Undershot Wheel.

or buckets placed radially, that is, at right angles to the circumference of the wheel, and the wheel is so located that the current strikes against the buckets at the bottom of it. The wheel is turned by the impact of the running water against its buckets, the water at the same time giving up a portion of

its velocity to the wheel. Such wheels are applicable where the quantity of water is great, but the fall inconsiderable. The overshot wheel (Fig. 73) is used where the amount of water is small and the fall is great, as with small mountain streams. Here the blades are converted into so-called cells, or buckets, of appropriate form, and distributed about the circumference of the wheel. The water falls into these buckets on the upper part of the wheel, which is thus moved by the weight of the water chiefly, though a contribution to the energy of rotation also comes from the velocity of the current. As each bucket arrives at the lowest point of the revolution, it discharges its water and ascends empty. The work per second, called the *effect of a waterfall*, equals the work required to raise the quantity of water which falls during a second, to a height equal to the distance between the surfaces of the water below and above the fall. If the weight of this water in kg. be multiplied by the height of the fall in meters, the *effect of the fall* will be found in kgm. But this total theoretical effect of a fall of water is never realized, even when frictional resistances are considered; for the water, after acting on the wheel, still retains some velocity, and therefore does not impart the whole of its velocity to the wheel. The particles of water will always retain a velocity at least equal to that of the circumference of the wheel. The best overshot wheels furnish about 70 per cent. of the theoretical effect, while undershot wheels furnish a still smaller percentage.

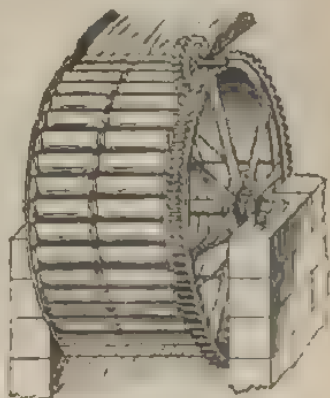


FIG. 73.—Overshot Wheel.

Other forms of water-motors are those due to Reichenbach, constructed on the principle of the steam-engine. Water, acting alternately at high and low pressure on either side of a piston, drives it backwards and forwards in a cylinder. The little water-motor constructed on this principle by A. Schmidt

is of great industrial value in cities where water at high pressure is available.

**71. Compressibility of Liquids.**—Liquids are so slightly compressible, when subjected to external forces, that they were for a long time considered incompressible. The Florentine Academy, in 1661, filled a silver sphere with water, hermetically sealed it, and subjected it to the blows of a hammer. Since the



FIG. 74. —Piezometer.

least change of form of the sphere diminished its volume, the water within it was necessarily compressed. It was found that at every blow of the hammer, water passed through the silver shell and collected as dew upon its surface; whence it was apparent that water would penetrate the metallic walls of the sphere rather than be compressed. Canton (1761) was the first to prove that liquids are compressible. If a vessel exposed to a pressure from within yields by virtue of its elasticity and expands, to prove the fact, and measure the amount of the compressibility of liquids, it is necessary to apply to the external walls of the vessel a pressure precisely equal to that within, otherwise the volume of the vessel will not remain constant. Oersted (1822) made use of the following method. A pear-shaped vessel (*a*, Fig. 74) with a permanently attached glass tube, was filled with liquid. A graduated scale is permanently fixed beside the tube which indicates exactly the ratio of the volume of the portion of the tube between two successive graduations, to the volume of the entire vessel. After

the vessel, *a*, called the *piezometer*, had been filled with the liquid to be investigated, *e.g.* with distilled water free from air, it was placed with the mouth of the tube downward in the basin of mercury, *b*, and by slightly changing the temperature, a little water was expelled through the tube, and a small

quantity of mercury took its place. By the side of the piezometer was placed a uniformly large glass tube filled with air, closed at its upper end, and open below, and also provided with a graduated scale. The latter tube and scale were to act as a *manometer* to measure the pressure. The entire apparatus was then placed within a strong glass cylinder, *cc*, which was filled with water, and closed with an air-tight cover. A force-pump was then rigidly attached to this cover, by means of which water could be drawn from the reservoir at the side, and forced into the cylinder. Since the pressure exerted upon the water by the piston of the pump was transmitted with equal intensity in all directions, the piezometer was compressed from without by the water around it, and from within, with the same intensity, by the column of mercury rising within the tube. The mercury also rose in the *manometer* tube. The height of the mercury in the *piezometer* indicated, then, the diminution in volume of the water contained in it, and the height in the *manometer* showed the pressure. When in the *manometer* tube, the air had been compressed to  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ , etc., of its original volume, it was known that the pressure was two, three, four, etc., times as great as the initial atmospheric pressure, i.e. that it was two, three, four, etc., "atmospheres," the normal pressure of the air (1 kg. per 1 sq. cm.) being designated "one atmosphere." From experiments such as this, mercury has been found to compress by 3 millionths of its original volume; water, by 50; commercial alcohol, by 80; and ether, by 111. When the pressure is removed, the liquid returns instantly to its original volume.

**72. Cohesion and Adhesion of Liquids.**—Notwithstanding the facility with which the particles of a liquid move upon each other, a force of attraction, called cohesion, acts between the constituent molecules. This force reveals its existence in the formation of drops and bubbles. If a mass of oil be brought into a mixture of alcohol and water of the same specific gravity as the oil, the buoyant effect of the mixture destroys the entire weight of the globule of oil, which, being no longer subject to the force of gravity, is drawn by its own molecular forces into the form of a sphere. By virtue of the mutual attractions of its

own particles, the globule assumes the form in which the surface is a minimum. Two drops combine the instant they touch each other, into a single drop, whose surface is obviously less than the sum of the surfaces of the two separate drops. The effect of cohesion is, therefore, to diminish the surface of a mass of liquid.

The molecular attraction of two particles of matter decreases very rapidly with increasing distance, and becomes wholly inappreciable even at comparatively small distances. If a sphere of radius equal to the greatest distance at which these forces are perceptible be conceived as described about any given point as centre, this sphere, called the *sphere of action* of the point, includes all those particles which act upon the particle at the centre. If the particle is situated within the mass of a liquid, every pair of particles of the liquid symmetrically situated with respect to the one in question, mutually destroy each other's effect, and the particle considered suffers from the surrounding particles of the liquid no effect whatever. If, on the contrary, the particle lies upon the surface of the liquid, only the lower half of the sphere of action is filled with attracting particles, and these will combine their forces of attraction into a resultant directed perpendicularly downward into the mass of liquid. Such a resultant, though somewhat smaller than the former, will also act upon every particle which lies nearer the surface of the liquid than the radius of the sphere of action. All those particles of the liquid, therefore, situated at a depth below the surface less than the radius of the sphere of action, are subject to a pressure directed downward, and called *cohesive pressure*. They form, so to speak, a tough, tense, elastic skin over the surface, whose constitution is different from that of the interior mass of the liquid. The tendency of this film to contract is called *surface tension*.

With curved surfaces, therefore, in addition to the cohesive pressure, the component of surface tension perpendicular to the surface must also be considered. This component is always directed toward the concave side of the surface; with convex surfaces, therefore, toward the interior of the liquid, but with concave it is directed outward. In the former case it

augments, in the latter it diminishes, the cohesive pressure. It is also greater, the greater the curvature of the surface, or the less, the radius of curvature.

Molecular attraction acts also between liquids and solids, in which case it is termed adhesion. Water-drops cling to a surface of window glass by virtue of cohesion. It is the latter force, also, which causes the particles of water to cling to the surface, when water is poured from a drinking glass. Water, sprinkled upon a clean surface of glass, spreads out over the surface and wets it; mercury, on the contrary, instead of moistening the surface, forms a rounded drop upon it. Water comports itself similarly when poured upon a surface coated with oil. In the first case, the adhesion of the water to the glass is obviously greater than the cohesion of the particles of water; while in the second, the cohesion of the mercury, and in the third, that of the water, exceeds the adhesion, in the one instance of mercury to glass and in the other of water to oil. The adhesion of water, therefore, to the side of the glass may be prevented by coating the mouth of the glass with oil.

**73. Liquid Films—Bubbles.**—When wire frames so constructed that their edges resemble polyhedra of various forms (e.g. tetrahedra, cubes, etc.) are dipped into soapy water and carefully withdrawn, the soapy solution adhering to the wires forms thin, tough films, which intersect one another within the frame in sharp straight edges. These figures arise out of the tendency of the molecular forces to form the smallest possible surfaces. Plateau called them figures of equilibrium, because the surface tensions in them hold each other in equilibrium. A spherical bubble, on the other hand, is not a figure of equilibrium. If a soap bubble be filled with tobacco smoke, the pressure which the liquid film exerts upon the contents of the bubble forces through the tube, to which the bubble still adheres, the air, together with the smoke, with such violence as to disturb the flame of a candle held near the mouth of the tube. The bubble meanwhile gradually contracts. If, on the contrary, the mouth of the tube be kept closed, the bubble neither contracts nor expands, since, in this case, the excess of

pressure of the air compressed on the interior holds the surface tension in equilibrium.

If  $P$  denote the pressure upon a superficial unit, and if the radius,  $R$ , of the bubble is shortened by the small quantity  $r$ , the work performed by the surface tension =  $4\pi R^2 P r$ . But both the interior and the exterior surfaces of the film are diminished by the common quantity—

$$4\pi R^2 - 4\pi (R-r)^2 = 4\pi (R^2 - R^2 + 2Rr - r^2) = 4\pi (2Rr - r^2)$$

which, when  $r$  is small in comparison with  $2R$ , may, without appreciable error, be put equal to  $8\pi Rr$ . The contraction for both surfaces accordingly

$16\pi Rr$ . If now  $a$  denote the work of the molecular forces required to diminish the surfaces by one unit, the total work performed by them, therefore, equals  $16\pi Rra$ , which must be equal to the surface tension found above. We have then  $4\pi R^2 P r = 16\pi Rra$ , from which, for both surfaces, it follows that

$$P = \frac{4a}{R}, \text{ and for each surface separately, } p = \frac{2a}{R}.$$

A spherically curved surface of liquid exerts therefore upon a superficial unit a pressure directed toward the concave side, and inversely proportional to the radius of curvature.

**74. Angle of Contact.**—Around the circumference of the wall of a glass tumbler, water rises a little above, while mercury sinks a little below, the surface of the liquid in the middle of the vessel. At the point  $P$  (Fig. 75), beside the wall



FIG. 75.—Contact Angle.

of a vessel, a particle is acted on by gravity directed vertically downward, by the force of adhesion,  $A$ , directed outward perpendicularly to the wall, and by the force of cohesion,  $C$ , directed toward the interior of the liquid.

The particle,  $P$ , therefore, can be in equilibrium only when the surface of the liquid has assumed the direction,  $PQ$ , perpendicular to the resultant,  $R$ , of these forces. The angle,  $QPW$ , between the surfaces of the liquid and of the wall, is called the *angle of contact*, or the *contact angle*. Since its magnitude depends only upon the ratio of the forces of cohesion and adhesion, it will be constant for the same liquid and the same vessel. According as the resultant,  $R$ , is directed outward through the vessel wall, or inward through the liquid, will that portion of the liquid around the edge rise above, or drop below the general level at the middle, where the surface has a direction exactly perpendicular to gravity and the cohesive pressure. The former will

occur when the wall of the vessel is wetted by the liquid, and the latter when it is not.

**75. Capillarity.**—If a slender glass tube (*capillary tube*) be immersed in water, the liquid will rise in the same to a level higher than that of the surface of the liquid outside of the tube. With two communicating tubes, also, water rises higher in the smaller than in the larger, which is contrary to the laws of hydrostatics.

An elevation of the liquid in the tube above the level of the liquid in the vessel always occurs when the sides of the tube are moistened by the liquid. This phenomenon is called *capillary attraction*, or *capillarity*. If, on the contrary, a glass tube be inserted in a basin of mercury, the glass will not be moistened and the mercury in the tube will sink below the general level. The latter phenomenon is frequently called *capillary depression*. In both cases the difference of level of the liquid within and without the tube is inversely as the diameter of the tube. These phenomena, which are comprised under the general term capillarity, find their explanation in the joint effect of cohesion and adhesion. We shall now proceed to the discussion of the causes of these phenomena. Since the effect of the vessel wall extends only to a very small distance, the surface of the liquid in the middle of a wide vessel remains horizontal. In a narrow tube, however, where the effect of the wall extends to the middle, or even beyond, the surface of the liquid must assume the form of a concave shell, if the wall is wetted, and of a convex shell, if not wetted. Such a curved surface exhibited by liquids in narrow tubes is called a "meniscus." Any curved liquid surface has been shown to exert a pressure always directed toward its concave side. It is this pressure which, in a moistened capillary tube, where the concave side is always directed upward, elevates the liquid above the outside level to a point, where the pressure of the column holds the pressure due to surface tension in equilibrium. It is likewise easy to see that, in an unmoistened tube, the pressure due to surface tension must depress the column. Since this molecular pressure increases with the curvature of the surface, which is greater the narrower the tube, it is evident that the elevation

or depression of the liquid must be inversely proportional to the diameter of the tube. The height attained by the column in a moistened tube depends upon the nature of the liquid, but not upon the material of the tube. In a tube of 1 mm. diameter water attains a height of 30 mm., sulphuric acid 17, alcohol 12, and ether 10.

In a moistened tube, where the contact-angle is zero, the surface forms a hemisphere concave upward, of radius  $R$ , equal to the radius of the tube. This tube exerts an upward pull equal to  $\frac{2a}{R}$  per superficial unit, which holds in equilibrium the hydrostatic pressure  $hs$ , acting downward upon the liquid column of altitude  $h$ , and of specific gravity  $s$ . We have, therefore,  $hs = \frac{2a}{R}$ , or  $h = \frac{2a}{Rs}$ . The work,  $a$ , required to diminish the surface of the liquid by one unit is called the constant of capillarity. From the above equation, it may be computed when  $h$ ,  $R$ , and  $s$  have been measured. The height of ascent between two parallel plane surfaces is inversely proportional to the distance  $R$  between them, and we have  $hs = \frac{a}{R}$ . If the plates touch along their vertical edges at an angle so that their distance at any point increases proportionally to the distance of this point from their common edge, the upper surface of the elevated liquid will take the form of an equilateral hyperbola.

The effects of capillarity are familiar to us in many forms in daily life. If a lump of white sugar be touched to the surface of coffee, the brown liquid will be seen to rise rapidly through the lump. The numerous small interstices between the little crystals of the sugar constitute an elaborate network of capillary tubes. A heap of sand thrown upon moist ground becomes moistened to its apex from the same cause. The absorption of liquids by blotting-paper, sponges, and other porous bodies, as also the rising of oil in the wicks of lamps, all depend likewise upon capillarity.

**76. Solution.**—The process by which a body passes from a solid to a liquid state in consequence of the attraction between its molecules and those of a liquid is called *solution*. The liquid used to dissolve the solid is called a *solvent*. It merely takes up the molecules of the solid without chemical change, and forms from them an homogeneous liquid mass. This liquid is also called a *solution* of the solid. Sugar, salt, saltpetre, dissolve in

water; shellac, in alcohol; gold, in mercury; and from their solutions solids are separated, unchanged in their constitution, on vaporating the solvent. With higher temperatures, the liquids are, as a rule, capable of taking up larger quantities of the soluble bodies than at lower. Hot water, for example, dissolves more saltpetre than cold, but salt dissolves as readily in cold as in hot water. A solution is said to be saturated, when it contains as much of the body as it can hold in suspension at the temperature in question. On cooling a hot saturated solution of saltpetre a portion of the dissolved substance is deposited in a solid condition, while the liquid at the lower temperature still remains saturated.

It may, however, occur that when the solution is cooled below the *saturation point*, no portion of the solid is separated. The solution is then *super-saturated*. A slight disturbance of the solution, produced either by agitation, or by dropping into it a crystal of the dissolved substance, will, however, cause a sudden deposition of a portion of the solid. Only bodies capable of crystallization, i.e. *crystalloids*, such as the salts mentioned above, have definite solubilities. Amorphous bodies, such as glue, gum-arabic, albumen, etc. (*colloids*), are soluble in all proportions, but in small quantities of the solvent they become only gelatinous.

A solid dissolves in a liquid, whenever the adhesion of the particles of the two bodies is greater than the cohesion of the particles of the solid.

**77. Diffusion.**—If alcohol be poured carefully over the surface of water, the particles of the two bodies will soon be found to be homogeneously mixed together. This cannot be due to gravity, since the alcohol, being lighter than the water, must float upon the surface. This process of mixing of the two liquids in contact with each other, is analogous to solution. It takes place by a gradual interchange from layer to layer, caused by the adhesion of the particles of the two liquids, which for liquids capable of exhibiting this phenomenon is greater than the cohesion of the particles of either. During the process, a change of volume frequently occurs. For example, alcohol and water, when mixed together, contract, and

thereby develop heat. With liquids not capable of admixture, cohesion is greater than adhesion, and diffusion does not occur. They merely arrange themselves above one another in the order of their specific gravities (*e.g.* oil and water).

**78. Osmose.**—When two different liquids, capable of inter-mixing, are separated by a thin porous partition, an interchange of their molecules will take place through the membrane. This process is called *osmose* (endosmose, exosmose, Dutrochet, 1826). In the neck of a flask, the bottom of which has been broken away, let a glass tube be fitted by means of a perforated cork, and a bladder be tied over the bottom. Let this vessel, filled with a liquid, *e.g.* alcohol, be immersed in a larger vessel containing water. The alcohol will be observed to rise gradually in the tube, to a height of 40 or 50 centimeters. Water has, therefore, passed through the bladder into the alcohol against gravity. This is called *endosmose*. A portion of the alcohol has, however, passed out of the vessel into the water, as is readily detected from the colour of the water, if the alcohol has been first coloured. This is *exosmose*. The rising of the liquid in the tube shows that more water than alcohol has passed through the membrane, and the height finally reached by the liquid column is a measure of the excess of pressure which drives the water through the partition into the alcohol (osmotic pressure). If the bladder be now replaced by a caoutchouc membrane, it will be found that more alcohol than water passes through. The nature of the membrane, therefore, plays an important rôle in the interchange. The passage of the two liquids in unequal proportions is due to the fact that the membrane can absorb different quantities of different liquids into its pores. According to Liebig, 1000 parts by weight of dry ox-bladder, in 24 hours absorbed 268 parts by weight of water, 133 of saline solution, 38 of alcohol, 17 of neat's-foot oil. If, then, alcohol and water be separated by an ox-bladder, the latter will take up from the one side water, and from the other alcohol, in the ratio 268 : 38. The water which has been drawn into the membrane, is now drawn out by the adhesion of the particles of alcohol to those of water, and similarly those of the alcohol are drawn out toward the water; but for every 268 parts

of water passing through the membrane only 38 parts of alcohol pass.

Numerous examples of osmotic action meet us in everyday life. Beans and peas soaked in water swell up because more water passes inward through their membranous coatings than passes outward. If radishes, cut in slices, be sprinkled with salt, the slices "draw water," i.e. the liquid contained in the cells of the vegetable passes outward in considerable quantities to the concentrated salt-solution which is formed by contact of the salt with the moist surfaces. Osmose plays also an important part in animal and plant life, for the circulation of the sap and the blood through the walls of the cells and blood-vessels depends upon osmotic action. Graham has shown that bodies which are crystalline when solid, and called by him *crystalloid* substances, e.g. sugar, salt, etc., pass much more readily through porous membranes than do non-crystalline substances, such as glue, albumen, caoutchouc, etc., which are more or less gelatinous, and called by him *colloid* substances. This behaviour of substances makes possible the separation, by osmose, of these two classes of substances from their mixtures. The process of separation is called *dialysis*, and the apparatus for executing it is called a *dialyzer*, which is merely a large flat vessel of hard rubber, with a bottom consisting of parchment. The dialyzer is allowed merely to float in a vessel containing a considerable quantity of water. If now a mixture containing a solution of gum-arabic and sugar be poured into the floating vessel, after a time the sugar will have passed almost completely through the porous parchment into the water, while, in the *dialyzer*, almost a pure solution of gum-arabic will remain.

## IV. GASES.

(AEROSTATICS.)

**79. Force of Expansion—Tension.**—Gaseous bodies, or *gases*, have, in common with liquids, the power of ready displacement and great mobility of their particles. They differ from liquids, however, very essentially, in that they are easily compressible, and also in that their particles tend to separate and their volumes to expand indefinitely. To prevent the escape of gases the vessels containing them must be closed on all sides. By virtue of this expansive tendency, called also *expansive force*, or *tension*, the enclosed gas exerts upon the walls of the vessel a pressure which is everywhere perpendicular to the wall and proportional to the area of the surface under pressure. Within the interior of the gaseous mass this pressure is also exerted in all directions with equal intensity, so that upon both sides of a smooth plate immersed in the gas, equal pressures perpendicular to the surfaces will be exerted, whatever position the plate may assume. The tension of air is easily proved by means of a tightly-closed bladder containing a small quantity of air. The tension of the confined air cannot reveal itself, because the surrounding air presses against the external surface of the bladder with a force equal to this tension. If, however, the bladder be placed under the receiver of an air-pump and the air exhausted, the expansive tendency of the air in the bladder stretches it until its walls become quite tense. If the air be again admitted into the receiver, the bladder contracts, and its walls finally shrink up into their former condition.

**80. Weight of the Air.**—If it were not for the attractive

force of the earth—that is, *gravity*—the expansive force of the air which surrounds the earth as an enveloping layer, would carry our atmosphere off into space and deprive us of this necessary means of subsistence. To show that the air has weight, let a glass globe, whose neck is provided with an air-tight stop-cock, be exhausted of air and suspended to one arm of a balance. Let equilibrium be now established by means of weights suspended to the other arm. If now the cock be opened and the air allowed to flow in, the balance inclines toward the side of the globe. It will be found that if the globe contains one litre, a little more than one g. must be placed upon the scale-pan to restore equilibrium. Water is accordingly not quite 1000 times (exactly 773 times) as heavy as air. Hence, in addition to the pressure due to the expansive force of gases, the bottom of a vessel filled with air, or other gas, is also exposed to a pressure due to gravity. The latter equals the weight of the column of gas standing vertically over the bottom. With gas, as with liquids, the difference in pressure on two equal surfaces at different heights will be equal to the weight of a column of gas whose altitude is the difference of the heights of these surfaces, and whose base is their common area, the density of the column being the mean density of the gas between the surfaces. The pressure due to the weight of the gas is, of course, very small in comparison with that due to its expansion. It is indeed so small that with small quantities of gas it may be entirely neglected; but with very tall vessels, and especially with our atmosphere, it plays an important part. Since air is compressible, each of its layers is condensed by the weight of those superposed upon it. The density of the air, therefore, increases continuously from above downward. On the surface of the earth, at the bottom of the sea of air surrounding the earth, this pressure is a very considerable quantity.

To convince one's self of the intensity of atmospheric pressure, it is only necessary to withdraw it from the side of a body. If, for example, a short lead pipe is placed above the opening into the receiver of an air-pump with its upper end tightly closed by a piece of bladder, after a few strokes of the

pump, the bladder will be pressed into the cylinder, and will finally burst with a deafening report. Mercury poured into a wooden bowl, glued to a receiver, is driven through the pores of the wood and falls in droplets from the inner surface of the bowl (mercury shower), by reason of the atmospheric pressure when the air is partially exhausted from the receiver. The experiment of the inventor of the air-pump, Otto von Guericke, Mayor of Magdeburg, performed before the Imperial Diet of Regensburg (1654), in the presence of Emperor Ferdinand III., to prove the existence of atmospheric pressure, has become world-renowned. Two hollow metallic hemispheres, two feet in diameter, were fitted together air-tight (*the Magdeburg hemispheres*), and were exhausted of air as completely as possible. They were held together so firmly by the pressure of the air, that sixteen powerful horses were scarcely able to tear them asunder.

**81. Barometers.**—To determine the intensity of atmospheric pressure, let a straight glass tube, 80 to 90 cm. long, and hermetically sealed at one end, be completely filled with mercury. The tube should be filled by pouring into it small portions of mercury, and then boiled, so as to expel all the air which may be mixed through the mass of the mercury. Let now the open end be closed with the finger and immersed in a basin of mercury, the finger removed, and the tube placed in a vertical position (Fig. 76, A). A portion of the mercury now flows out of the tube, and the column within the tube stands at a height of 76 cm. above the surface of the mercury

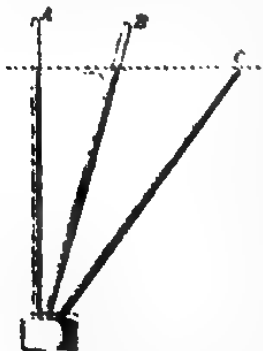


FIG. 76.—Torricelli's Experiment.

in the vessel. The space within the tube above the mercury is completely free from air, since any atmospheric particles clinging to the tube-walls were expelled by the boiling process. That this is true can be experimentally proved by inclining the tube into a new position. When the tube is inclined the top of the mercurial column stands always at the same height

above the level of the mercury in the basin. The column fills a larger and larger portion of the empty space (Fig. 76, B), and in the position (Fig. 76, C), where the tube has been inclined until its upper end is less than 76 cm. above the surface of the mercury in the vessel, the empty space is completely closed, no trace of an atmospheric bubble remaining. In honour of Torricelli (1644), who was the first to perform this experiment, the vacant space above the mercurial column is called *Torricelli's vacuum*. But why does the mercurial column, whose lower end is in direct communication with the mercury below, refuse to flow out into the vessel? Obviously, it is because the column is sustained by the atmospheric pressure, acting downward upon the surface of the mercury, and transmitted by this liquid in all directions. Just at the mouth of the tube this pressure manifests itself as a force directed upward. It follows, then, that a column of mercury 76 cm. or 760 mm. high holds the atmospheric pressure in equilibrium, and furnishes, consequently, a measure of this pressure. The atmospheric pressure per superficial unit may now be expressed in units of weight. If the cross section of the tube equals 1 sq. cm., the column of mercury 76 cm. high contains 76 sq. cm. of mercury. Since now a cub. cm. of mercury weighs 13.6 g., the weight of the column equals  $76 \times 13.6 \times 1000$  g., or a little more than 1 kg. This is, therefore, the weight with which the mercurial column strives to sink. To hold the mercury within the tube and to balance it, atmospheric pressure must oppose to the column an equal force. Consequently, the atmosphere exerts upon each square centimeter of surface a pressure of 1 kg., or a column of air whose base is 1 sq. cm., and which extends from the surface of the earth, vertically to the top of the atmosphere, weighs 1 kg., i.e. as much as a column of water 10 m. high. A sheet of writing paper 20 cm. long by 15 cm. wide, has an area of 300 sq. cm., and carries, therefore, an atmospheric pressure of 300 kg. Since, however, this pressure acts as strongly upward against the lower side of the sheet as downward against the upper side, the paper may be moved about just as readily as though no pressure at all were acting upon it. Assuming the surface of the human body to equal 1 sq. m., it is exposed

to the enormous pressure of 10,000 kg. We are not conscious of this pressure, because it acts from all directions, downward and upward, forward and backward, outward and inward, upon equal surfaces with equal intensities. The pressure of air, or, in fact, of any gas, is not, as a rule, expressed in kilograms. It is more convenient to state it in terms of the height of the mercury column which holds this pressure in equilibrium. An adjustable metal scale, graduated to millimeters and placed



FIG. 77.—Siphon Barometer

vertically beside the tube, has its zero-point lying at the vertex of a conical pin at the surface of the mercury in the vessel. Only a small portion of the scale near the top of the mercury column need be actually graduated. Such an apparatus is called a *barometer*, and since the mercury below is held in a cistern, this particular form is called a *cistern barometer*. When the mercury sinks in the tube it must rise in the cistern, and conversely. These oscillations of the surface of the mercury are smaller, the larger the cistern in comparison with the diameter of the tube, and for very large cisterns they may be neglected. With the ordinary barometers for household purposes, tube and cistern are made in a single piece by soldering to the curved end of the tube a pear-shaped vessel (*phial barometer*). Since the diameter of the cistern is here quite small, the oscillations of the surface of the mercury in the vessel are appreciable, so that an accurate reading is, with this instrument, neither possible nor intended. In the *siphon barometer* (Fig. 77), both branches of the bent glass tube are of equal diameter, so that the mercury rises in one branch by just as much as it falls in the other. To find the height of the barometric column the tube is shifted by means of a screw below, until the surface of the mercury in the short branch stands at the zero of the scale, or, when the scale is fixed with respect to the tube, both ends of the mercurial column are read. The mercurial barometer possesses the

disadvantage of being inconveniently heavy and long. This inconvenience is of no small consequence when instruments have to be transported from place to place. *Metallic barometers*, in which the atmospheric pressure is held in equilibrium by the elasticity of a solid, are free from these objectionable features.

In Vidi's *aneroid barometer* (*spring barometer*), the pressure of the air acts against the thin corrugated top of a metallic box, from which the air has been partially exhausted, and bends it inward more or less, according to the intensity of the external atmospheric pressure. By a suitable system of multiplying levers, the motion of the top of the box is transmitted to a pointer, which plays above a dial face. When the dial has been graduated by comparison with a mercurial barometer, the position of the pointer indicates directly the intensity of the atmospheric pressure.

The barometric height of 760 mm., given above and called *normal barometric height*, corresponds to the pressure of the atmosphere at the surface of the sea. At higher altitudes where the pressure of the atmosphere is less, the barometric column is correspondingly lower. In Potosi, for example, altitude 4300 m. above the sea, the height of the barometer is only 471 mm. From the sea-level one must rise 10.5 m. to shorten the mercurial column by one millimetre. At higher altitudes, where the density of the air is less, a greater change of altitude is needed to produce the same diminution. From Potosi, for example, a rise of 16.8 m. is needed to produce a fall of 1 mm. in the barometer. Since the law of diminution of atmospheric pressure with increasing altitudes is known, the altitude of any place above the sea may be computed from the barometric height at that place. This, of course, implies that the barometer maintains at every place a definite height. This is, however, not true. The atmospheric pressure at any given place changes incessantly. When the barometric height of a place is spoken of, the mean height, determined as a mean value from numerous observations extending over long intervals of time, is meant. By the altitude of a place above the sea, we mean the altitude computed from this mean barometric height.

Experience has shown that the mean value for any given station, lies about midway between the least and greatest observed values.

The last-mentioned fluctuations of the barometric height, oftentimes of considerable magnitude, arise from disturbances of the equilibrium of the atmosphere, which latter precede or accompany changes in the weather. To this connection between the weather and the atmospheric pressure, the barometer owes its reputation as a weather prognosticator and its claims for a place among household utensils. Accordingly barometers are frequently supplied with *weather scales* which carry from below upward the words: "Storm, Much Rain, Rain, or Wind, Variable, Fair, Dry, Very Dry."

The designation *Variable*, which occupies the middle of the scale, must be specially determined for the place for which the barometer is destined to be used. Since atmospheric pressure is transmitted in all directions with equal intensity, the same atmospheric pressure obtains within the rooms of our dwellings as without, even though the doors and windows be kept closed. The barometer, therefore, need not be suspended outdoors, but may stand under cover in a well-protected place. Wherever the barometer is used it must be remembered that mercury expands with heat and in consequence thereof becomes lighter. As a result, even when the atmospheric pressure is constant, the height of the barometer will vary when the temperature varies. It has, however, been agreed that the height of the mercurial column at zero degrees shall be used as a standard for atmospheric pressure. Since the law of expansion of mercury is known (it is  $\frac{1}{10000}$  for  $1^{\circ}$  C.), the small correction, which must be applied to the observed height of the barometer to reduce it to 0 is easily computed, provided that, when the barometer is read, the temperature of the mercury, as indicated by an attached thermometer, be also read. The expansion of the scale must also be considered. The mercurial column is furthermore somewhat lowered by reason of the surface tension of its meniscus. This capillary depression, which is smaller the larger the tube (with cistern barometers), must be added to the observed height.

**62. Boyle's, or Mariotte's, Law.**—Every one is familiar from childhood with the pop-gun and its effects. The ends of a tube are closed with close-fitting corks, between which is contained a column of air having the same density and pressure as the outside air. When one of the corks is driven inward by a piston, the pressure on the column of air increases as the piston is driven inward, until the other cork, unable longer to resist this pressure, suddenly flies out with a report. Every one has also observed that when he opens a close-fitting pen-case a resistance is perceptible which increases the farther he draws the lid off, until finally, after exerting considerable force, the case opens with a smart crack. The air enclosed within the case possesses at first the same pressure and the same density as the surrounding air. But while the cover is being drawn off, the air within expands into a larger and larger space, and in a corresponding degree does its pressure become less than that of the outside air, so that the excess of pressure of the outside over the inside air resists the withdrawal of the cover. Boyle (1662) and Mariotte (1679) were the first to discover the exact connection between pressure and volume of a gas. This connection may be shown by the following method. Two glass tubes, one of which is closed by a glass stop-cock at one end while the other is left open at both ends, are connected by means of a rubber tube. The latter, as also a portion of the glass tubes, is filled with mercury. The tubes are clamped in blocks which may be raised and lowered along a vertical standard, graduated in cm. and mm. When the cock is open the mercury stands at the same height in both tubes. This will still remain true even if the cock be closed, since the air included in the closed tube exerts the same pressure as that acting on the



FIG. 78.—  
Mariotte's Law

mercury in the open tube. If now the open tube is raised, the mercury rises in both tubes, though quite slowly in the closed tube, since the air confined above the mercury must be compressed. When this air is compressed to exactly one-half its initial volume, it will be found that the mercurial column in the open tube, reckoned from the surface of the mercury in the closed tube, is exactly equal to the height of the barometric column at the same instant. The pressure of the confined air now holds in equilibrium, besides the pressure of the atmosphere acting in the open tube, the pressure of this column of mercury, which exactly equals the pressure of the air. The confined air, now compressed to one-half its original volume, exerts a pressure equal to double its former pressure, *i.e.* it exerts a pressure equal to twice that of the atmosphere, or, more briefly, a pressure of *two atmospheres*. If the confined air be compressed to one-third its initial volume by raising the open end of the tube still higher, it must support, in addition to atmospheric pressure, a column of mercury equal to twice the height of the barometer, *i.e.* in all, a pressure of three atmospheres, etc. If, finally, the open tube be depressed below the point at which the height of the mercury is the same in both tubes, the imprisoned air will expand and the height of the mercury in the closed tube will be higher by  $\frac{1}{2}$ , or by  $\frac{2}{3}$ , of the barometric height, than the surface of the mercury in the open tube, when the air has attained twice, three times, its initial volume. Its pressure can now hold in equilibrium the pressure of the air acting in the open tube, only with the aid of a column of mercury equal to  $\frac{1}{2}$ ,  $\frac{2}{3}$ , of an atmosphere. Its pressure, therefore, when its volume has been doubled, or tripled, is only  $\frac{1}{2}$ , or  $\frac{1}{3}$ , of its initial pressure. We find thus the law of Boyle and Mariotte commonly known on the Continent as Mariotte's law and in England as Boyle's law: *The pressure of a given quantity of gas is inversely as the volume, or directly as the specific gravity, or density, provided the temperature remains the same.*

If  $p_1$  and  $v_1$  denote the pressure and volume of a gaseous mass in its initial condition,  $p$  and  $v$  the same magnitudes in any other condition, we shall have from this law.  $p : p_1 = v_1 : v$ .

or,  $pv = p_0v_0$ . Since  $p_0v_0$  is a given constant quantity, Mariotte's law may also be expressed thus: *the temperature remaining the same, the product of volume and pressure of any gas is constant.*

Arago and Dulong have tested Mariotte's law for the atmosphere, to pressures as high as twenty-seven atmospheres, by using a tube attached to a ship's mast, raised beside the tower of the College of Henry IV., in Paris. Later investigations by Regnault (1847) have shown that Mariotte's law, though nearly, is not absolutely exact. He showed that with increasing pressure oxygen is compressed somewhat less, and other gases somewhat more, than Mariotte's law requires.

**83. Barometric Formula.**—From Mariotte's law, we may now derive the law according to which the atmospheric pressure diminishes with increased elevation. If  $h$  denote the height of the barometer in millimeters at any altitude, and the barometer be carried from this point one meter higher, the column of mercury will fall by a small portion whose weight would exactly equal the weight of the atmospheric column of the same cross-section as the mercury column and one m. high. Since water is 773 times as heavy as air at  $17^\circ$ , and 760 mm. pressure, and mercury is 13.6 times as heavy as water, and since, farther, the densities of the air, from Mariotte's law, are as  $b : 760$ , the height of the little mercurial column, whose weight equals that of the atmospheric column 1000 mm. high, is only  $\frac{1000}{773 \cdot 13.6} \cdot \frac{b}{760}$  mm. The height of the barometric column  $b$ , therefore, in the latter position is given by the formula

$$b_1 = b - \frac{1000}{773 \cdot 13.6} \cdot \frac{b}{760} = \left(1 - \frac{1000}{773 \cdot 13.6 \cdot 760}\right)b = kb,$$

where the number in parenthesis, which is a proper fraction differing but little from unity, is designated by  $k$ . The barometric height, then, at a station 1 m. higher than a given station, is found by multiplying the height of the barometer at the lower station by the number  $k$ . If then, the altitude of the station be conceived to change by 1 m. increments, at an altitude 2 m. above the initial station, the barometric height  $b_2 = kb_1 = k^2b$ , at 3 m. above  $b_3 = kb_2 = k^3b$ , etc., and finally, the barometric height  $b'$ ,  $h$  meters above the initial station is  $b' = k^hb$ , which is the law of diminution of barometric height with increasing altitudes. If  $b$  and  $b'$  denote the barometric height at a lower and a higher station respectively, by means of this "barometric formula," the difference in altitude of the two stations is readily computed. We find easily

$$h = -\frac{1}{\log k} (\log b - \log b').$$

or, if the above numerical value for  $k$  be substituted in meters—

$$h = 18400 (\log b - \log b').$$

In these formulas, of course, differences of temperature, of moisture, and of

gravitational effects at the two stations have been neglected. To get more accurate formulas, small corrections would have to be introduced depending upon these circumstances.

**84. Manometers** are instruments for measuring the tension of gases and vapours. The *open-air manometer*, used ordinarily for measuring pressures which differ but little from atmospheric pressure, in its simplest form consists of a bent glass tube, one branch of which communicates with the body containing the gas, while the other is open to the admission of air. A small quantity of mercury is contained in the bend of the tube, and when the mercury rises to the same height, in both branches, the internal pressure is the same as the external pressure of the atmosphere. If the internal pressure exceeds the external, mercury rises in the open branch until the pressure of the elevated mercurial column, plus atmospheric pressure, holds the internal gas-pressure in equilibrium. A millimeter scale, from which the height of the column in the open branch above the surface of the mercury in the closed branch is read, gives a measure of the excess of the pressure of the gas above that of the air, and, if the pressure of the gas is desired in terms of the height of a mercurial column, the indication of the scale must be added to the height of the barometer at the instant of observation. With very small differences of pressure, such, for example, as is involved in determining the pressure of gas in a lead-pipe, the manometer may be filled with water, which, being  $\frac{1}{13.6}$  as heavy as mercury, will give 13.6 times as great a variation for the same fluctuation in pressure. With very heavy pressures the open-air manometer would be very inconvenient, from the great height needed for the second branch. For such pressures the *closed manometer* is used (conf. 71). In this form of the instrument the second branch is closed above and contains a quantity of air, confined within it by the mercury. According to Mariotte's law, as the mercurial column rises the pressure of the imprisoned air increases inversely as its volume. This pressure is read in atmospheres from a scale, etched on the tube and graduated in accordance with Mariotte's law. The indication of this scale, added to the pressure of the elevated column of mercury, gives the pressure of the enclosed gas, or

steam. The *metal manometer*, frequently applied to the determination of steam pressures, consists of a bent tube of thin elastic metal, which, on the admission of steam into it, straightens out by an amount depending on the pressure of the steam. The deformations of this tube are transferred by a suitable system of levers to a pointer which indicates by its position with respect to an experimentally graduated scale, the pressure of the steam.

85. The *Air-pump* is an instrument by which the air contained within a closed space may be rarefied. The instrument invented in 1650, by Otto von Guericke, accomplished the rarefaction by means of a piston moving within a hollow cylinder, or *barrel*. Its essential parts can be most readily understood by an explanation of the hand air-pump (Fig. 79).

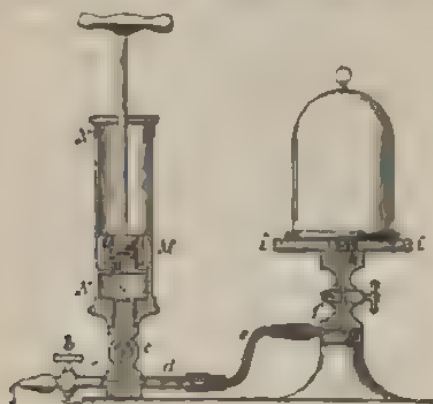


FIG. 79.—Hand Air-pump.

The piston, *M*, can be moved upward and downward in the barrel, *NN*, by means of a handle attached to the piston-rod. The channel, *kdefgh*, leads to the barrel from the space from which the air is to be exhausted. This so-called *receiver* consists usually of a glass jar with carefully ground edges fitting air-tight to the oiled surface of the plate, *ii*. The perforated piston, *OP*, is provided with a valve constructed by binding over the upper opening of the portion *P*, a piece of bladder in which at the sides of the opening two slits are cut. A similar valve lies at the bottom of the barrel at *k*. Both valves open

by pressure from below, and close by pressure from above. If the piston is drawn upward while the stop-cock, or tap, *c*, is open, the air in the receiver and the canal expands into the larger space opened to it, lifting the valve at *k* by its pressure, and flowing into the barrel behind the piston. Meanwhile the piston-valve, *P*, remains closed by reason of the pressure of the external air upon it. If the piston is now depressed, the air within the barrel is again condensed, its pressure closing the valve, *k*, and cutting off the channel leading to the receiver. The enclosed air soon attains a pressure strong enough to open the valve of the piston and pass out through the opening, *O*, while the air in the receiver and in the channel remains rarefied. When the piston has reached the bottom, and the air in the barrel has passed entirely through the piston valve, the same process repeats itself, the rarefied air being still further rarefied in the same manner as before. It is apparent from this that after a sufficiently great number of repetitions of the motion of the piston back and forth in the barrel the receiver can be exhausted to any degree of rarefaction desired, though, of course, the exhaustion can never be made complete. The rarefaction must, of course, cease when the tension of the air beneath the piston is no longer able to lift the valve, *P*. Suppose now, the receiver to be cut off during the ascent of the piston in the barrel. The air contained in the barrel below the piston, when the piston is down (within which small space the tension of the air can never sink below external atmospheric pressure), will expand and fill the entire barrel, and the density of this air will be to that of the atmosphere as the little volume between the valves when the piston is down is to the volume of the barrel. If the air in the receiver has already been rarefied to this same degree, no more air will pass through the valve, *k*, and all further pumping is useless. The alternate connection of the barrel with the receiver, and with the air, can be accomplished either by valves, or by stop-cocks. The degree of rarefaction attained is determined by barometric tests. A glass tube 76 cm. long has its lower end immersed in a vessel of mercury. This tube is bent at the top, and by means of a rubber tube may be connected with a side-tube of the air-pump by means of the stop-cock, *b*. When

the cock is open the mercury rises in the tube until its elevated column, plus the tension of the internal rarefied air, holds in equilibrium the tension of the external atmosphere. The internal tension is then obtained by subtracting the height of the column of mercury in this tube from the simultaneously observed barometric height. If, for example, the column of mercury have a height of 740 mm. when the barometric height

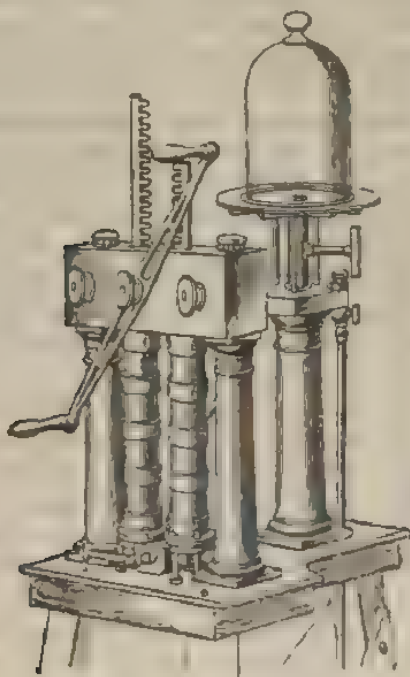


FIG. 80. --Double-barrelled Air-pump.

is 750 mm., the inner tension equals that of a column of mercury 10 mm. high, and is therefore only  $\frac{1}{75}$ , or  $\frac{1}{7.5}$ , of the original atmospheric pressure. Since, according to Mariotte's law, the density of air is as its tension, it follows from this that the air in the receiver is reduced to  $\frac{1}{7.5}$  of its initial density. For experimental purposes, air-pumps with two barrels (Fig. 80, *double-barrelled air-pump*) are ordinarily used. The piston rises

in one barrel, while it sinks in the other. This motion is accomplished by a toothed wheel, engaging on both sides in a rack in which the upper end of the piston-rod terminates. The



FIG. 81.—  
Shortened  
Barometer.

harmful influence of the little space referred to above, between the valves, is, in the double-barrelled pump, reduced by cutting the right barrel off from the receiver with a three-way stop-cock (Grassman's, or Babinet's stop-cock), and at the same time connecting it with the left barrel, after the limit of rarefaction has been reached in the ordinary way. The left piston sinks, driving the rarefied air without condensation into the right barrel, so that the injurious space can be filled only with rarefied air. To test the degree of rarefaction barometrically, with these more perfect air-pumps, the *shortened barometer* (Fig 81) is used.

This consists of a U-shaped glass tube, with one open and one closed branch. Mercury fills the bend and the closed branch, which latter is much shorter

than in the usual barometer. The mercury does not begin to sink until the tension of the rarefied air in the open branch has been reduced to less than one-fourth of atmospheric tension. The difference of the heights of the mercury in the two branches gives, then, the tension, and accordingly, also, the density of the air in the receiver.



FIG. 82.—Mercurial  
Barometer.

The air in a receiver may also be rarefied by putting it in connection with a Torricellian vacuum (*conf.* 81). The mercurial air-pump (Geissler, 1857) is an apparatus resembling a barometer, which, by a repetition of this process of connecting a receiver with a vacuum, permits a considerable degree of rarefaction. A glass tube, C (Fig. 82), about 76 cm. long, carries at its upper end a wide glass vessel, A, while its lower end

is connected by the rubber tube, D, with the glass vessel, B.

In an enlargement of the tube, *tr*, into which the vessel, *A*, is continued upward, a three-way tap, *o*, is fitted, by whose means the vessel, *A*, may be connected, either through *r* with the space from which the air is to be exhausted, or through *t* with the glass sphere, *p*, opening into the air. While *A* is open toward *p*, the vessel, *B*, is raised until *A* is completely, and *p* is partially, filled with mercury. If now *A* is closed above by the tap, *o*, and the vessel, *B*, is gradually lowered, the mercury sinks correspondingly. In the tube, *C*, however, the mercury remains at the instantaneous height of the barometer above the surface of the mercury in the vessel, *B*. The apparatus is merely a barometer with a very extensive vacuum in the vessel, *A*, with which, by turning the tap properly, the receiver may be connected. After the air has filled the entire space accessible to it, the receiver is again cut off, and, by raising the vessel, *B*, the air drawn into *A* from the receiver is compressed, and after a second setting of the tap it is driven outward through *p*. This same process may be repeated indefinitely. Mercurial air-pumps work slower than piston pumps, but they permit of a far higher degree of rarefaction.

The *water air-pump*, or *filter-pump* of *Bunsen* (1868) is based upon a wholly different principle. From a reservoir, or conduit, water is brought through the tube, *ac* (Fig. 83) into

the larger tube, *d*, which must not be completely filled. The water, falling vertically through the lead-tube, *f*, extending to a distance of 10 m. downward, carries along with it any air held between its drops. Consequently, the air contained in the tube stem, which is fitted air-tight by means of a rubber cork into

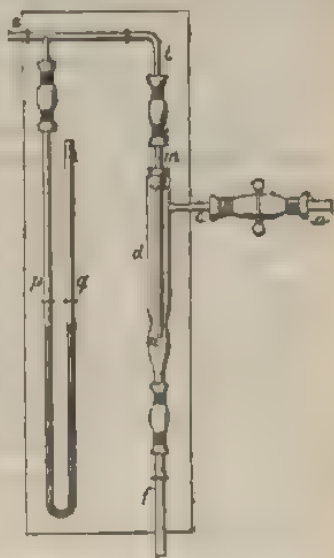


FIG. 83.—Bunsen's Pump.

the tube, *b*, will be drawn out of this tube by the falling water. If the tube, *st*, be connected with a closed space, air will be drawn out through *st*, and within the enclosure a rarefaction will ensue, whose extent can be determined by means of the open-air manometer (*pq*) connected with *st* from the rising of the mercury in the branch, *p*, and its falling in *q*. Bunsen's pump is extensively used in chemical laboratories for filtering and drying precipitates. The tube, *st*, is then connected with the interior of a vessel upon which the funnel and filter are placed. The external atmospheric tension drives first the liquid and then air through the precipitate, and thus rapidly dries it. Upon this same effect of falling drops of liquid, Sprengel's *mercurial air-pump* is based. It consists of a glass tube terminating above in a funnel and dipping below into a basin of mercury. If mercury is poured into the funnel it carries the air in the tube before it and through a tube opening into the side beneath the funnel, the air may be drawn from any enclosure with which this tube is connected.

**86. Condensing Pumps, or Condensers.**—Any air-pump working by taps operated in such manner as to draw in the atmosphere when the piston rises, and to force it into a receiver when the piston sinks, may be called a *condensing pump*, or a *condenser*. The apparatus, when used in this way, condenses in the same proportion in which it rarefies when used conversely. Simpler apparatus, specially designed for the purpose, are ordinarily used in condensing gases.

These instruments are usually constructed as follows:—In a hollow cylinder, to which the receiver to be exhausted is screwed, an air-tight piston is made to move back and forth. When the piston is pushed in, the air confined beneath it is compressed and crowded through a valve which opens under the increasing pressure into the receiver. When the piston is withdrawn, the inner tension holds this valve closed. The barrel is filled through a lateral opening with air from without, which, at the next descent of the piston, is crowded into the receiver, and so on. With such a pump the air may be compressed in the hollow shaft of an air-gun to ten times its normal tension. If now a trigger open for an instant a valve

leading from the air-chamber, the pressure of the escaping air will drive a close-fitting projectile from the barrel with great velocity.

**87. Diving-bell.**—If a vessel be immersed, mouth downward, in water so that the air is held within it, the water rises within the vessel, as it is crowded downward, until the tension of the compressed air becomes equal to the pressure of the outside air increased by the pressure of a column of water whose altitude equals the distance between the surfaces of the water within and without the vessel. In diving-bells constructed on this principle, workmen are able to descend to great depths and to remain there for a considerable time, breathing the air contained in the bell. The air is continually renewed from above by condensing pumps.

**88. Transmission of Pressure—Buoyancy.**—From the facility of the relative motion of gaseous particles, the same laws hold for the transmission of pressure in gases as in liquids. The pressure exerted upon a gas is transmitted by it in all directions with equal intensity. Under the action of gravity, a gaseous mass, *e.g.* our atmosphere, will be in equilibrium only when one and the same pressure obtains throughout any given horizontal layer. It is also true that, even within a mass of water whose surface is exposed to atmospheric pressure, this pressure is always added to the hydrostatic pressure at any point throughout the mass. This is well illustrated by the *Cartesian diver*, so called from its inventor, Cartesius (Descartes). These little, hollow glass figures, frequently executed in fantastic forms, filled partly with water and partly with air, with openings at their sides, are made to float in a glass cylinder filled with water and closed at the top air-tight with a rubber membrane. By a pressure of the hand upon the membrane, the air at the top of the cylinder is compressed, its increased pressure is transmitted to the water, and crowds the liquid into the hollow figures, which thereby become heavier and sink. If the pressure be released, the compressed air in the figure drives the water out, and the figure, becoming lighter, rises again. The divers may thus be made to rise, to sink, or to float quietly at a fixed level.

The principle of Archimedes also holds for gases as well as for liquids: *any body immersed in air loses a weight equal to that of the air it displaces.* To prove this, suspend from one end of the beam of a small balance a hollow sphere, and from the other, a small lead weight of such size that, when surrounded by air the combination will be in equilibrium. In reality, the sphere is heavier than the lead, and equilibrium apparently obtains because the buoyant effect of the air is larger for the sphere than for the weight. This is shown by placing the apparatus under the receiver of the air-pump. With increasing exhaustion the hollow sphere sinks lower and lower; but, on admission of the air, apparent equilibrium is immediately restored. This apparatus, devised by Otto von Guericke, was used by him to determine the degree of rarefaction of the air in the receiver, the barometer-test being then unknown, and it is therefore called *Guericke's manometer* (Dasyometer).

If the apparatus just described be immersed in other gases, the globe rises, or falls, according as the gas is specifically heavier, or lighter, than air, because the buoyant effect of the surrounding gas is, in the first case, larger, and in the second smaller, than that of the air. If, in each case, equilibrium be restored by the addition of weights, the added weights will indicate how much more, or less, a volume of the gas, equal to that of the globe, weighs than the same volume of air, and from this results immediately the specific gravities of the gases (Lommel, 1886). In accurate weighing the buoyant effect of the air must be considered, the apparent weight being increased by the small effect due to atmospheric buoyancy, to give the true weight as it would be obtained in a perfect vacuum.

If the weight of a body is less than that of an equal volume of air, it will rise with a force equal to the excess of the latter weight over the former, if left to float freely in the atmosphere. It will continue to rise until it reaches a layer of the atmosphere whose weight, volume for volume, is equal to its own. *Balloons* are such bodies. They are composed of covers of light material, filled either with hot air (Montgolfier, 1782), or some other gas lighter than air, such as hydrogen, or illuminating gas (Charles, 1783).

**89. Some Applications of Atmospheric Pressure.**—The *siphon* is usually a bent tube with two unequal branches, and is used for transferring liquids from one vessel to another by the aid of atmospheric pressure. If the short branch, *bs*, of the tube, *ash* (Fig. 84), filled with liquid, be immersed in a mass of liquid, the pressure of the air at both ends of the tube acts upward with the same intensity. In the short branch of the tube, however, the pressure of a column of liquid whose height equals the vertical distance from the surface of the liquid in the vessel to the highest point, *s*, of the bend of the tube, acts against atmospheric pressure, while in the longer branch the higher column, extending from the mouth, *a*, vertically to the horizontal plane



FIG. 84.—Siphon.



FIG. 85.—Automatic Washbottle.

through *s*, reacts against this pressure. The excess of atmospheric pressure in the shorter branch of the tube is then greater than in the longer, and since this excess is directed upward, the liquid must rise in the shorter tube and flow out at the mouth of the longer, until either the mouth *b* no longer dips beneath the surface, or until the surface of the liquid in the vessel lies in the same horizontal plane with *a*. The absolute magnitude of the atmospheric pressure is of no consequence. The work of moving the liquid is done by a force depending only on the *difference in level* between the surface of the liquid and the mouth of the siphon. In order that the siphon may work, however, its highest point must not lie higher above the surface of the liquid than the length of the column of liquid which will hold the pressure of the atmosphere

in equilibrium. For mercury, therefore, the bend must not be higher than 760 mm., and for water not higher than 10 m., above the liquid level. Under the receiver of an air-pump the siphon refuses to act as soon as the pressure of the air becomes less than that of the liquid column in the shorter branch. That the pressure of the air, *a*, acts thus may be proved by the apparatus of Fig. 85. The siphon, *ab*, whose longer branch dips beneath the surface of the water in the funnel, *d*, communicates by means of a perforated cork with the neck of a flask filled with water. Through another opening of the cork a tube, *cc*, passes, which terminates just beneath the cork. If now the tube, *cc*, be kept closed with the finger after the siphon has begun to flow, a small quantity of water will still continue to pass, and since air cannot enter the vessel through the closed tube, the air within the tube must expand. Its tension will therefore decrease until the excess of pressure of the air outside, over that inside, holds in equilibrium the difference of pressure of the longer column of water and the shorter. At this point the siphon will cease to act, because the water it contains is thus held in equilibrium.

This device is used in practice as an *automatic washing apparatus*, where, in washing precipitates, it is desired to keep the filter filled to a fixed height with water. If the tube *cc*, having its end bent downward, be brought exactly to the surface of the water in the funnel, when the water flows through the siphon toward the funnel and the level of the water rises slightly, the mouth of the tube *cc* will be gradually closed by the rising water and the flow retarded. As soon as the gradually sinking water-level in the funnel opens the mouth of the tube, *c*, for a moment, and thereby admits a quantity of air into the flask, the siphon, *ab*, will again begin to act. The siphon is ordinarily filled by immersing the shorter branch in the liquid and drawing with the mouth at the end *a* (Fig. 84) of the longer branch. This rarefies the air in the tube so that its pressure becomes less than that of the air outside, which is also the pressure on the surface of the liquid in the vessel, and the latter, consequently, drives the liquid up into the tube. With the apparatus of Fig. 85 the siphon may be

started by blowing in the tube, *cc*. The air in the flask is thereby condensed, its pressure becomes greater than that of the external air, and drives the water into the siphon. To fill the siphon conveniently by suction without danger of drawing into the mouth corrosive, or poisonous liquids, a lateral tube, *t* (Fig. 86), is attached to the longer branch, at the upper end of which, after closing the tube at *b'*, the experimenter draws with his mouth until the liquid begins to rise in the little bulb (*poison siphon*). Any rubber tube may be used as a siphon.

If a glass tumbler, filled with water, be covered with a sheet of paper and inverted, the water does not flow out. The



FIG. 86.—Poison Siphon



FIG. 87.—Pipette

pressure of the air acting upward against the paper with sufficient intensity to sustain a column of water 10 m. high, prevents the water from flowing out. The paper merely prevents the water and air from exchanging places while the tumbler is being reversed. If the mouth of the vessel is quite narrow, so that a well-rounded drop of water may form over it, no paper will be needed. The surface tension of the meniscus will be sufficient to exclude the air. The *pipette* depends upon this principle. It consists of a tube open at both ends, contracting both ways from the middle, as shown in Fig. 87. It is used to procure from casks and other closed vessels small portions of liquids for tests. If one end of the pipette be inserted into the liquid and the other end be left open, it fills to the level of the outside liquid. If now the upper end be closed air-tight with the thumb and the pipette raised out of

the liquid, a small portion flows out, the air confined in the upper portion of the tube expands and its tension falls below that of the air, until the latter becomes equal to the internal pressure, plus the pressure due to the liquid column, whereupon equilibrium is established and the liquid remains in the tube. By raising the thumb and admitting the air above, any desired amount of the liquid may be dropped into a vessel, and by again closing the tube with the thumb the outflow will again cease. To fill the pipette, the lower end may be immersed in the liquid, when, by drawing with the mouth at the upper end, the air within is rarefied and its pressure correspondingly diminished, until the pressure of the air on the surface of the liquid in the vessel fills the tube. To be able to determine accurately the quantity of the liquid used for the test, these pipettes are frequently graduated to cubic centimeters.



FIG. 88 —  
Mariotte's Flask.

*Mariotte's flask* (1686) is a large bottle, or flask, with a lateral opening below and provided with a close fitting cork, through which a glass tube passes air-tight (Fig. 88). If a portion of the water contained in the vessel be allowed to flow out, the air above the water expands, and its pressure diminishes until the atmospheric pressure in the tube exceeds the internal pressure plus the pressure of a water-column from the lower end of the tube to the water level, and bubbles will then rise from the lower end of the tube. In the plane of the lower end of the tube, *b*, as long as the surface of the water, *c*, is not below *b*, external atmospheric pressure obtains, and the water flows out only under the pressure of a column of water extending from the orifice to the lower end of the tube. By means of Mariotte's flask it is thus possible, notwithstanding the gradual sinking of the water level in the flask, to maintain a constant pressure, and, consequently, a constant velocity of efflux at the orifice. The deeper the tube is inserted, the slower the outflow, which ceases entirely when the lower end of the tube reaches the level of the orifice.

*Pumps* are used to raise water by the aid of atmospheric pressure. The most familiar form of this apparatus is the *suction pump*. In the pump barrel, C (Fig. 89), a perforated piston, K, fitted as closely as possible and provided with a valve, works up and down. The valve opens by pressure from below, and closes by pressure from above. A valve in every way similar to the piston-valve works at the bottom of the barrel, V, where the suction-tube, R, extending below the surface, U, of the water, connects with the barrel, K. When the piston is raised the air beneath it expands, filling the larger volume, and its pressure diminishes correspondingly. The piston-valve is held shut by the greater external pressure, while the valve, V, yielding to the pressure of the air in the suction-tube, opens, and causes the air in this tube also to expand. Since, behind the piston, rarefied air of low tension fills the tube, the pressure of the air on the surface of the water in the *well*, or *spring*, raises in the tube, R, a column

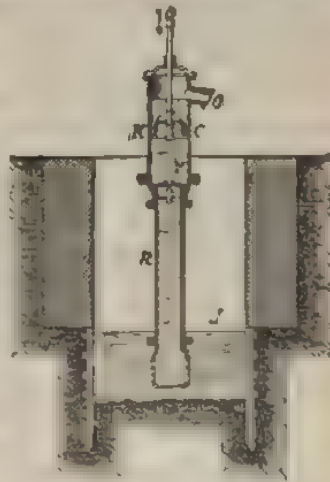


FIG. 89.—Suction Pump.

of water to such height that its pressure, combined with that of the internal air, equals the external atmospheric pressure. If now the piston descends, it compresses the air behind it, which, by virtue of its increased tension, closes the valve, V, and prevents the column of water from sinking. The tension within the barrel, K, then opens the piston-valve, and crowds the water upward through it. If, at the first stroke of the piston, we assume that the water has risen to the valve, V, and that, when the piston has descended to the bottom of the barrel, no air remains behind it, during the second stroke, the valve, V, opening, the water raised by the pressure of the air follows the piston immediately and passes, therefore, directly into the pump-barrel. If the piston descends again, the valve,

V, closes immediately, and the water in the barrel must, by virtue of the pressure of the piston itself, open the piston-valve and pass through the bore of the piston into the barrel chamber above. At the next stroke, the piston raises this water with it and carries it to the opening O, above the highest point of the travel of the piston, where it flows out through the *spout*. The height to which water may be raised by such a suction-pump is not unlimited. The pressure which holds in equilibrium a column of mercury 76 cm. high, can carry a column of water 13.6 times higher than this, or about 10 m. high, and no higher. If, therefore, the valve, V, is higher than 10 m. above the surface of the water in the well, no water can rise into the pump-barrel though the pump work perfectly and the piston produce an absolute vacuum. With the imperfect pumps found in the market it is never possible to carry the valve, V, more than 7 m. or 8 m. above the water level. From the observations of the Florentine pump-makers, that water will not rise higher than this, Torricelli was led to the proof of the fact, and measurement of the intensity of atmospheric pressure by the barometer. Prior to that time the

rising of water in pumps had been explained as a horror of nature for a vacuum (*horror vacui*).

In the force-pump the piston is not perforated. The water in the pump-barrel is forced by the pressure of the descending piston through an orifice opening from the barrel into an upright tube provided with a valve at its top.

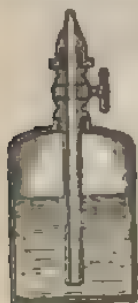


FIG. 90 — Hero's Ball.

**90. Hero's Ball** (Hero, 150 B.C.).—This apparatus consists of a vessel (Fig. 90) partially filled with water, in the mouth of which a tube, open at both ends, fits air-tight. If the pressure of the air is greater within the vessel than without,

water will rise in the tube and flow out as a jet from its mouth. To increase the internal tension above the external, air may be blown in through the tube, provided with a tap for confining it, either by the mouth or by a force-pump, or the outside air may be rarefied by placing the apparatus under the receiver of an air-pump. The simplest form of Hero's ball is an ordinary

*wash-bottle*, consisting of a flask provided with a doubly perforated stopper, through which two tubes are fitted air-tight, one of which reaches almost to the bottom of the flask, curves outward near its top end, and terminates in a slender point, while the other extends just through the stopper. If air is blown through the latter tube, water rises in the former and passes through the open end in a slender stream. The so-called *soda-fountain*, used in the production of effervescent drinks, is a Hero's ball. Its tube reaches almost to the bottom of the vessel, is bent aside near its upper end, and closes with a tap, or cock. When the tap is opened the liquid is driven by the pressure of carbonic acid gas through the mouth of the tube with considerable force. The *air-chamber* of a fire-engine is only a large Hero's ball, in which by two force-pumps working alternately, water is compressed and the air within the chamber condensed. If the cock be opened, the compressed air within forces out a continuous and powerful stream of water. *Hero's fountain* is a Hero's ball in which the air is condensed by a column of water.

91. The **Hydraulic Ram** is an apparatus for raising water which was invented by Montgolfier in 1797. By means of the energy of running water, this machine raises a part of the water to a higher level than that from which it descends. It consists of an air-chamber (Hero's ball), *r*, into which the vertical tube, *d*, opening at *e*, into a vessel for the reception of the water which has been raised, fits by an air-tight connection (Fig.

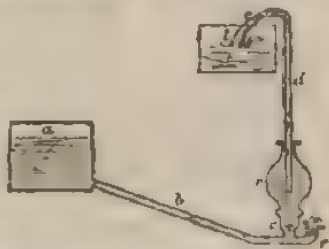


FIG. 91.—Hydraulic Ram

91). The air-chamber is provided below with a valve, *c*, opening upward. At the mouth of the conduit, *b*, which leads from a reservoir, *a* (e.g. a pond, or river), is a valve, *v*, which opens downward by its own weight. The water flowing downward through the tube, *b*, closes by its impulse, the valve, *v*, raises the valve, *c*, flows into the air-chamber and compresses the air within until the valve closes by the excess of pressure from

above. So soon as the water has come to rest, the valve, *v*, opens by its own weight and allows water to flow out. The water being again set in motion closes the valve, *v*, with a shock, at the same time lifting the valve, *c*, and by a continued repetition of this play of the valves the water compresses the air in the chamber until the inner tension, still less than the tension of the inflowing water below, is able to raise a quantity of water through the tube, *b*, into the vessel, *l*.

**92. Efflux of Gases.**—A gas flows outward through an aperture in the thin walls of a vessel containing it, whenever the tension inside the vessel is greater than that on the outside.

If *p* denote the excess of the pressure in the vessel, *ω*, the cross-section of the orifice, and *s*, the small distance traversed by the escaping cylindrical jet of gas during a short interval of time, *pωs* represents the work performed by the excess of the internal pressure. If the orifice is small and the vessel large, so that the gas contained within it has no perceptible motion, this work is almost completely expended in producing the energy  $\frac{1}{2}mv^2$  (*v* denoting the velocity of efflux of the gas) of the escaping gaseous mass *m*. We have, therefore, almost exactly,  $\frac{1}{2}mv^2 = p\omega s$ . If *s* denote the specific gravity of the gas with respect to water, and *g* the acceleration of gravity, *ωs* will denote the weight, and  $\frac{\omega s}{g}$ , the mass of the gas. We have, then,

$$\frac{1}{2}mv^2 = p\omega s, \text{ or } v^2 = \frac{2gp}{s},$$

or also

$$v = \sqrt{\frac{2gp}{s}}.$$

Under the foregoing assumptions, therefore, we have the following law which was first proved experimentally by Graham: *The velocity of efflux of gases is directly proportional to the square roots of the excesses of internal pressure and inversely proportional to the specific gravities of the gases.*

If, therefore, different gases flow under the same pressure, the squares of their velocities of efflux are inversely as their specific gravities, or, what is the same thing, these specific gravities are as the squares of the times required for equal volumes of the gas to pass through the aperture. Bunsen has devised an ingenious method for the determination of the specific gravity of gases based upon these laws. Let a quantity

of gas be put in the cylinder, AA. Fig. 92, which contracts above into a small tube, B, within which, at *o*, a thin perforated platinum plate is held, through the small opening in which gas passes outward on the removal of the stopper at *a*. The cylinder, AA, with the stopper in place, is immersed in mercury until the point, *r*, of the glass float, DD, is exactly at the level, C, of the mercury in the vessel, CC. If the stopper be now removed the excess of the external over the internal pressure forces the gas outward through B, and it is only necessary to note the time which elapses from the removal of the stopper until the mark, *t*, on the float reaches the level of the mercury. It, for example, it has been found that equal volumes of air and of oxyhydrogen gas consume, respectively, 117.6 and 75.6 seconds to flow through the orifice, the specific gravity of oxyhydrogen gas referred to air =  $(75.6) : (117.6) = 0.413$ .

**93. The Pneumatic Trough** is used for procuring gases unmixed with the atmosphere, and collecting them in vessels. If, for illustration, gas should be led directly from the apparatus in which it is generated into a so-called empty vessel, i.e. a vessel filled with air, the gas would be contaminated by admixture with the air. If, on the contrary, the vessel which is to contain the gas, a test-tube, for example, be first filled with some liquid (water or mercury), and while its mouth is kept closed, it be immersed in the same liquid, mouth downward, the gas may then be collected without difficulty. The mouth of the immersed tube is opened, and through it the end of a glass tube, bent upward and leading from the developing apparatus, is inserted. The bubbles of the gas, now rising through the liquid in the tube, crowd the liquid downward, and collect above its surface in a pure unmixed condition. To enable the bubbles of gas to rise, sufficient pressure must

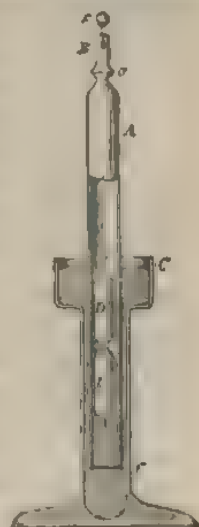


FIG. 92.—Bunsen's Apparatus for Specific Gravity of Gases.

obtain in the developing apparatus to overcome the pressure of the liquid column in the receiving vessel.

**94. Gasometers** are apparatus for preserving gases and permitting the withdrawal of any desired quantity of them. The form of the instrument used in laboratories consists of a cylindrical tin vessel, A (Fig. 93), closed air-tight, communicating with the open vessel, B, by tubes,

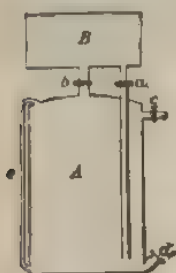


FIG. 93.—Laboratory Gasometer.

which may be closed by taps at *a* and *b*. The tube, *a*, extends from the bottom of the upper vessel to the bottom of the lower; the tube, *b*, from the bottom of the upper to the top of the lower. The lower vessel has near its top a lateral opening provided with a tap, *c*. near its bottom is another, closing with a cork, and finally, the tube, *e*, is used as a *water-gauge*. The lower vessel is first filled completely with water poured in through the

vessel, B, while the orifice, *d*, is closed, and the taps, *a*, *b*, and *c*, are kept open. If now the taps all be closed the water will not flow out, even when the orifice, *d*, is open, since it is held in by the atmospheric pressure at *d*. The tube leading from the developing apparatus is inserted through the opening, *d*, and the bubbles of gas rise from its mouth filling the vessel, A, by crowding out the water through the orifice, *d*, past the

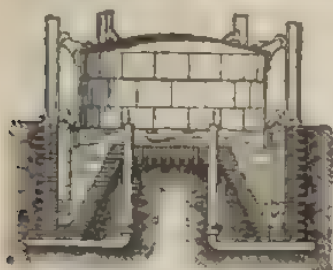


FIG. 94.—Gasometer for Factory.

sides of the tube leading from the developer. When the vessel, A, is almost filled with gas, the orifice, *d*, is closed. The tap, *a*, being now opened, water flows from the vessel, B, which is always to be kept full of water, into the vessel, A, and compresses the gas contained in A with a force corresponding to

the difference of level of the water in the upper and the lower vessels. By reason of this internal tension the gas may be forced either through the tube, *c*, or into a vessel inverted in the reservoir, B, serving as a pneumatic trough, through the

tube, *b*. The large *gasometer* (Fig. 94) used in the manufacture of illuminating gas consists of a cylindrical cistern excavated below the surface of the ground, with walled sides, within which a large cylinder, made of boiler-iron, closed above and open below, is partially balanced by counterpoises. The cistern is filled almost to its surface with water. Beneath the inverted cylinder, or tank, two bent tubes, or pipes, provided with taps, open upward. Through the one gas enters from the developer and, by its tension, raises the tank, which was previously immersed, flush with the surface of the water. The tube from the developer is now closed, the other tube, called the *main*, is opened, and the cylinder sinking by its own weight drives the gas through the main for use.

**95. Gas-meters** are instruments for measuring the volume of the gas passing through a main. The so-called *wet gas-meter* consists of a cylindrical box of tin, or cast-iron (Fig. 95), within which a drum may be rotated about a horizontal axis. The drum is divided into four sections, each of which communicates with the interior of the box through the openings, *a*. The box is half filled with water, mixed ordinarily with glycerine, or alcohol, to prevent freezing. The gas from the main flows in at the centre, *b*. If it be allowed to flow out through the orifice, *c*, to a lamp, or burner, the pressure of the air in the section above the surface of the liquid against the side-walls diminishes. This inequality of pressure produces a rotation of the drum so that one after another of the sections rises above the surface of the liquid and discharges its gaseous contents through *c*, at a rate corresponding to the consumption by the flame. When the four sections have been emptied once each, the drum has made one complete revolution. By means of a toothed wheel mechanism the number of revolutions and, consequently, the volume of gas consumed within a given time, is indicated by a pointer and graduated dial-plate.



FIG. 95. Gas-meters.

**96. Diffusion of Gases.**—If two communicating vessels be placed one above the other, the upper containing hydrogen

and the other carbonic acid gas, after a time the two gases will be uniformly distributed throughout the vessels, forming a perfectly homogeneous gaseous mass, despite the fact that carbonic acid gas is twenty-two times heavier than hydrogen. All gases gradually distribute themselves throughout space as though no other gases were present, provided that they do not react upon each other chemically. Therefore the pressure of a gaseous mixture is equal to the sum of the pressures of the constituent gases (Dalton's law, 1803). This process of distribution is called *diffusion*. This property of gases explains the intermixture of oxygen and nitrogen in the same proportions throughout all parts of our atmosphere.

If two gases are separated by a porous partition, e.g. by a thin plate of unglazed earthenware, or gypsum, an exchange of the two gases through the pores of the partition occurs, the lighter gas percolating through the porous wall more rapidly than the heavier. Over a porous earthen jar, such as is used for galvanic elements, let a glass receiver, provided with a tube for admitting gas, be inverted. Through an opening in the bottom of the jar let a U-shaped glass tube (manometer), with its bend filled with water, be fitted air-tight by means of a cork. If illuminating gas, which is lighter than air, be admitted into the receiver, it will diffuse more rapidly through the porous wall into the jar than will the air in the opposite direction. The pressure within the jar must, consequently, rise. The water will then sink in the inner branch of the manometer and rise in the outer. If, now, the receiver be removed, the illuminating gas, with which the cell is now filled, will pass outward into the surrounding air more rapidly than the latter passes inward. The pressure on the interior decreases then, and the water-column rises in the inner branch and falls in the outer. The reverse occurs when, instead of illuminating gas, the heavier carbonic acid gas is admitted into the receiver. If a tube, connecting the porous jar with a closely corked air-chamber, which is partially filled with water, and terminates just beneath the cork through which it passes air-tight into the chamber, while, through a second perforation of the cork, another tube is fitted, also air-tight, and inserted to the bottom

of the chamber, forming thus a Hero's ball, water will be seen to spout from the latter tube to a considerable height the instant illuminating gas is admitted into the receiver.

Attempts have been made to utilize this principle for the detection of the presence of marsh-gas in the air of coal-mines. If a vessel closed with a porous earthen lid, and connected with the branch of a U-shaped tube filled with mercury, be brought into the air of a mine contaminated with this gas, the pressure within the vessel will be increased in consequence of the more rapid diffusion of the lighter marsh-gas. The mercurial column in the other branch will rise therefore, and, by closing a galvanic circuit, may be made to ring an electric signal-bell, announcing impending danger.

According to Graham, the rates of diffusion of two gases are inversely proportional to the square roots of their specific gravities. Hydrogen, for example, which is  $\frac{1}{16}$  as heavy as oxygen, passes through a partition separating these gases four times as rapidly as oxygen.

**87. Absorption of Gases.**—Liquids possess the property of absorbing gases when brought into contact with them. The process is termed *absorption*.

Soda-water is water which has absorbed carbonic acid and holds this gas, as it were, in solution. One liter of water, at 15° C., always absorbs one liter of carbonic acid, regardless of the pressure to which the gas is subjected. Since, according to Mariotte's law, with double, triple, four-fold, and so forth, pressure, twice, thrice, four times, and so forth, as much gas will be contained in the same space, it follows that, with constant temperature, the weight of the quantity of gas absorbed by a definite liquid varies as the pressure under which the absorption takes place (Henry's law, 1803). In the manufacture of soda-water, the pressure necessary to saturate the water with a sufficient quantity of carbonic acid, is obtained either by developing the gas within a very small vessel, or by the use of suitable condensing pumps. The preparation of champagne consists in confining within the liquid, the carbonic acid formed during fermentation under the high pressure developed within the tightly-corked flask.

It is this pressure which drives out the loosened cork with a loud report. The carbonic acid, which was confined by the pressure within the closed flask, escapes from the liquid which, after the cork is drawn, is exposed only to the pressure of the air *a*. The champagne then effervesces. Under the receiver of an air-pump, stale beer, from which, when exposed to the air, carbonic acid no longer passes, may be made to effervesce violently.

The rapidity of absorption varies with the nature of the liquid and the gas used. Ammonia is absorbed by water with great rapidity. If a glass tube is inserted through a cork into an air-chamber filled with this gas, and its outer end is closed with a small stopper, the closed end being immersed in water, as soon as the tube is opened the water rushes into the air-chamber, filling it completely. The gas has been entirely absorbed by the water. If the water in the vessel has been coloured red with acidulated tincture of litmus, the alkaline action of the ammoniacal liquid in the vessel will colour it immediately blue. Commercial spirits of sal ammoniac is water saturated with ammonium, and commercial hydrochloric acid is water saturated with hydrochloric acid gas.

One part by volume of water at 15° C. absorbs 7.27 volumes of ammonium, 4.50 of hydrochloric acid, 43.5 of sulphuric acid,  $3\frac{1}{2}$  of hydrogen sulphide, 1 of carbonic acid,  $\frac{1}{4}$  of oxygen, and  $\frac{1}{6}$  of nitrogen. One volume of alcohol, on the other hand, absorbs 3.2 volumes of carbonic acid. The numbers which express the number of volumes of a gas absorbed by one volume of the liquid, are independent of the pressure, and are called coefficients of absorption. A liquid will absorb from a mixture of gases, quantities of each individual gas proportional to the tension of the respective gases in an unmixed state (Dalton's law). The quantity of carbonic acid absorbed, therefore, is not increased if the vessel containing the acid contain, at the same time, another gas—air, for example. The atmosphere is, as we know, a mixture of 21 volumes of oxygen with 79 of nitrogen. If the coefficients of absorption of these two gases were equal, air, when absorbed in water, would retain these gases in the same proportions. is

absorbed in greater quantities than nitrogen, air, after absorption in water, is relatively richer in oxygen than is ordinary air. The absorbed air contains 35 per cent. instead of 21 per cent. of oxygen, the element necessary to respiration, as against 65 per cent. of nitrogen, which is not capable of sustaining respiration. This property of these gases is of great importance for aquatic animals, provided with gills, and breathing this absorbed air. Absorption diminishes, as a rule, with increasing temperature. Water, for example, at 0°, absorbs 1.8 volumes of carbonic acid; at 15°, 1 volume; at 20°, 0.9 volume. Heating, therefore, will drive a portion of the gas out of the liquid in which it is dissolved, and, on boiling, most absorbed gases are completely expelled. On the contrary, many metals, *e.g.* silver and copper, which in a molten condition, absorb oxygen, give off the absorbed gas on cooling, and the process of escaping occurs so violently from the liquid portion of the metal, as to hurl droplets of the molten mass hither and thither. This phenomenon is called *sparking*. Solid bodies are also capable of absorbing gases and holding them within their mass. Metallic palladium, for example, used for a long time in dilute sulphuric acid as the negative pole of a galvanic pile, is capable of absorbing a volume of hydrogen 936 times as great as its own. This process is called *occlusion*. When heated to glowing, platinum and iron absorb hydrogen, and the latter also carbonic oxide with extreme facility, and hold these gases also even after returning to their normal temperature.

Moreover, all solids possess the property of condensing upon their surfaces gases in which they are immersed. Any body which has been exposed for a time to the air, or any other gas, collects about its surface a condensed layer of gas which, from its adhesion, clings so closely to the surface that it can be removed only by heating, or by thoroughly washing the surface with alcohol, pumice stone, graphite, etc. Since this peculiar mode of absorption, more appropriately called *adsorption*, depends only upon superficial area, porous bodies, such as charcoal, manifest it in a high degree. The interior walls of countless little pores augment the surface out of all proportion to the volume of the body. If a fragment of freshly

burned charcoal be passed through mercury in a pneumatic trough, over which a glass tube has been filled with carbonic acid, under the mouth of the tube, and allowed to rise into the carbonic acid, the mercury, together with the floating fragment of charcoal, will rise in the tube. In this way it is found that beechwood charcoal, from which the absorbed air has been removed by heat, is capable of adsorbing 35 times its volume of carbonic acid, and 90 times its volume of ammonium. Since the adsorbed gas is condensed, and condensation is always accompanied by the development of heat, adsorption always produces heat which, under some circumstances, may rise to glowing. For this reason, spontaneous combustion sometimes occurs when finely-ground charcoal, to be used in the manufacture of gunpowder, has been piled into large heaps. The finely pulverized iron used by apothecaries for medicinal purposes, when heaped up in the air, adsorbs oxygen so violently as to take fire and burn up. Such bodies as these are called *pyrophorous*. If platinum sponge (i.e. finely porous platinum, obtained by burning platinum sal ammoniac), which has adsorbed oxygen from the air and condensed it in its pores, be exposed to the action of hydrogen, the latter gas will be also adsorbed with the development of sufficient heat to bring the sponge to glowing and to ignite the hydrogen. Doebereiner's fire-kindler is manufactured upon this principle. Many solids have the power to adsorb water-vapour from the air and condense it into water, e.g. table salt, potassium, etc. Ordinary glass condenses upon itself a thin layer of water. Such bodies are called *hygroscopic*. Many bodies of the animal and vegetable kingdom, for example, hair, fish-bone, catgut, etc., are likewise shown to be hygroscopic, by the fact that they adsorb water from moist air and swell. Adsorption, without doubt, has its cause in the adhesion of the molecules of the body to the gaseous particles in their immediate vicinity.

## V. HEAT.

**98. Heat** is the cause of that condition of a body, in which when the body is touched, effects are produced on the nerves of the skin, which we distinguish as cold, cool, tepid, warm and hot. To this series of terms correspond gradual differences in the thermal condition, or *temperature*, of the body. Estimates of temperature based on the sense of touch are very uncertain, for the sensation of heat depends not alone upon the condition of the body, but upon the condition of the sense-organ as well. If the one hand be immersed in warm and the other in cold water, and then both be immersed at once in tepid water, the latter will seem warm to one hand and cold to the other.

A reliable estimate of a body's thermal condition may be obtained from the fact that change of temperature is always accompanied by change of volume. When two bodies of unequal temperatures touch each other, the warmer cools, and the cooler warms, until both reach the same temperature. The truth of this is shown by the fact that from the instant both have reached this condition, no change of volume occurs. The rapidity of change of temperature is nearly proportional to the difference of temperature of the bodies between which the change takes place. Most bodies expand when heated. If a metal ball fit a metal ring exactly, so as just to pass through the ring when cool, it will not pass through after heating. Alcohol enclosed in a glass tube terminating in a hollow bulb, rises in the tube on heating. If the glass bulb is filled with air, and a drop of mercury is placed in a horizontal section of the tube, so as to prevent the influx of outside air, when the globe is heated, the enclosed air will expand and push the drop forward in the tube. Apparatus of this nature were used by Galileo (1592) and Drebbel (1621) as *thermoscopes*

for estimating variations of temperature. Each particular instrument of the sort just described, always furnishes the same reading for the same temperature, but readings of different instruments are not thus comparable with each other.

99. **Thermometers** are instruments which indicate the thermal condition of bodies in contact with them. They are provided with scales, graduated conventionally, from which the temperature may be read.

To prepare a mercurial thermometer, a spherical, or cylindrical, bulb is blown on the end of a tube of uniform bore, the bulb together with a portion of the tube being filled with pure mercury. The mercury is then heated until, by its expansion, it fills the entire tube, driving out all the air. When the mercury is just on the point of flowing out, the hollow of the tube is fused shut at the top. The presence of air in the tube would of course interfere with the expansion of the mercury, since, by virtue of the oxygen it contains, the mercury would be partially oxydized, and hence contaminated, or air-bubbles might rise in the tube and render the instrument wholly useless.

If now the instrument be surrounded with melting ice, the mercury will soon sink to a definite point and remain there, no matter how the melting of the ice takes place. This point is marked upon the tube. It is called the *freezing point*, and corresponds to the invariable temperature at which ice melts, or water freezes. A second fixed point is obtained by surrounding the instrument with the steam of boiling water. The mercury rises for a time, but finally comes to a fixed position, and remains at it. Water cannot be heated in an open vessel higher than to the temperature at which its steam has attained the tension of the air. The end of the mercurial column corresponding to this temperature is also marked, and is called the *boiling-point*. The boiling-point, thus obtained, will be correct only if, during the determination, the barometric height is 760 mm. If the atmospheric pressure is different, a small correction will be necessary. The space between these two fixed points is sometimes called the *fundamental distance*. Howsoever its length may differ for different thermometers,

it has, nevertheless, with all of them, the same relation to the temperatures to be measured, and upon this important fact the comparison of the data of different instruments, in whatever way they may be subdivided, is made possible. Celsius (1742) divided this fundamental distance into 100 equal parts, called degrees ( $^{\circ}$ ). This is called the *centesimal*, or *centigrade*, scale. This same subdivision is carried beyond the boiling and freezing points, throughout the length of the tube. At the freezing point, 0 (zero) is written, and at the boiling point, 100. This was first done by Stroemer. Celsius numbered the graduations in the reverse order. Degrees above zero are reckoned upwards (positive), and are written with the  $+$  sign, or with no sign at all, while degrees below zero are counted downward (negatively), and are written with the  $-$  sign. Réaumur (1730) subdivided the fundamental distance into 80. The freezing point was called zero as before, but the boiling point was designated 80. With Fahrenheit's (1724) thermometer, the number 32 is written beside the freezing point, and by the boiling point, 212. The fundamental distance is accordingly divided here into 180 parts. Fahrenheit thought he had found in the low temperature of the winter of 1709, the point of absence of heat, or the "absolute zero-point." He produced this temperature also artificially by means of a mixture of ice, water, and sal ammoniac, and took it as the zero of his scale. As a second fixed point, he chose the temperature of the human body, which almost coincided with the point 100 of his scale. Fahrenheit's (F.) scale is generally used in England and America; while in Germany, in daily life, the scale of Réaumur (R.); and in France, the centesimal scale of Celsius (C.) is used. In scientific investigations, the centesimal scale is in general use to-day. It is, moreover, not difficult to convert the reading of any of these scales into that of either of the other two. We have  $100^{\circ}\text{C.} = 80^{\circ}\text{R.} = 180^{\circ}\text{F.}$  To convert degrees of the Réaumur scale into those of the Celsius, it is only necessary to multiply by 10 and divide by 8. Celsius degrees are converted into Réaumur by multiplying by 8 and dividing by 10. To convert Fahrenheit into Celsius, or Réaumur degrees, it is necessary first to subtract 32, to find

the number of degrees above zero, and then to multiply the remainder by  $\frac{1}{3}$  and  $\frac{2}{3}$  respectively. Besides mercury, alcohol is also extensively used in filling thermometers. On account of its more uniform rate of expansion, mercury is, however, to be preferred. On the other hand, alcohol thermometers are still in use for very low temperatures, where mercury freezes and is rendered no longer serviceable (mercury freezes at  $-38^{\circ}2$  C.). Alcohol thermometers are graduated by comparison with the mercurial.

Mercurial thermometers become useless also above the temperature at which mercury boils (about  $350^{\circ}$  C.). If, however, the boiling be prevented by the pressure of a gas in the upper part of the tube, the mercurial thermometer may be used to temperatures as high as  $550^{\circ}$  C.

The alcohol thermometer is used for the maximum and minimum thermometer (*thermometrograph*) of Rutherford (Fig. 96). The purpose of the apparatus is to show at a single

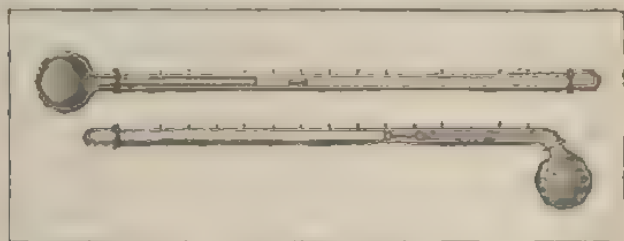


FIG. 96.—Rutherford's Thermometrograph.

reading, the highest and lowest temperatures which have occurred during any given interval of time, e.g. within 24 hours. A mercurial and an alcohol thermometer are attached horizontally to the same metallic plate with their bulbs in opposite directions. The former shows the highest temperature reached, for the thermometer being placed horizontally when the temperature rises, the mercury pushes a small piece of iron wire (*the index*) before it, which, when the liquid contracts, remains in the part of the tube to which it has been carried, since there is no adhesion between the iron and the mercury. The alcohol thermometer indicates the lowest temperature reached. In this, a small hollow glass tube serves as an index.

When it is at the end of the column of liquid, as the temperature falls the column contracts, and carries the index with it in consequence of adhesion, until it has attained its greatest contraction. When the alcohol expands with rising temperature, it passes between the sides of the tube and the index, and does not displace the index. The position of the index then indicates the lowest point reached by the column, and hence the lowest temperature. The instrument is again made ready for use, by tilting the plate one way or the other until the indices come again in contact with the liquid surfaces. The maximum and minimum thermometer of Six (Fig. 97) consists of a U-shaped glass tube, *nop*, whose bend contains a quantity of mercury. The vessel, *d*, and the left branch are filled above the mercury with alcohol, which acts as a thermometric liquid. In the right branch, terminating in the bulb, *p*, exhausted of air, there is likewise a column of alcohol, extending to *q*. Each branch contains a bit of steel wire, *a* and *b*, enclosed within the portion filled with alcohol. The wire, *b*, is elevated with rising, and the wire, *a*, with falling temperature, by means of the mercury, and, on the return of the mercury to its previous state, the wires are left in their extreme positions, since they work with very slight friction against the inner walls of the tube. The friction is frequently produced by means of slender threads of glass, such as are shown in the cut. The index, *a*, indicates the lowest, and *b*, the highest temperatures, since the last setting. The indices are set by means of a small magnet, held just outside of the tube, by whose attraction the bits of steel are drawn back to the surfaces

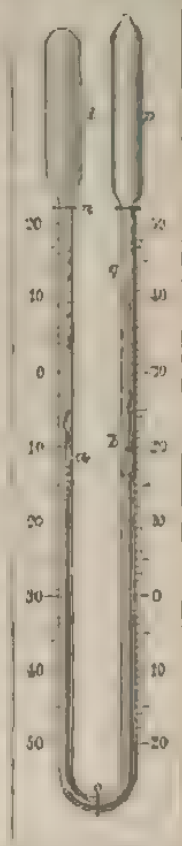


FIG. 97. — Maximum and Minimum Thermometer of Six.

of the mercury-columns. To measure the temperature of the blood in the human body, physicians use a small maximum thermometer, called a *fever thermometer* (Fig. 98, full size), in which the upper thread of mercury is separated by a small bubble of air from the mercury below. The upper part of the



FIG. 98.—Fever Thermometer.

thread rises with the expansion of the mercury below, but does not fall as this portion of the mercury cools and sinks. Before every temperature test, the upper thread must be brought back into place by swinging the thermometer about, a double bend in the tube preventing the two portions of mercury from uniting. To use the instrument, it is inserted into a hollow of the body of the patient, *e.g.* under the tongue, and allowed to remain for at least six minutes before reading. With a healthy man, the temperature  $37^{\circ}2$  C. would be indicated. The graduation is carried far enough to permit reading to tenths of a degree, and needs only to be carried through the portion of the tube over which the temperature of the blood varies.

This method of graduating the fundamental distance into equal parts in the construction of thermometers is, of course, wholly arbitrary. The mercurial thermometer, however, is of the highest value in precise estimates of thermal phenomena, and the apparatus could be illy dispensed with.

**100. Expansion of Solids.**—Increase of volume, or expansion, of bodies with rise of temperature is so slight as to make special appliances necessary to render it perceptible and measurable.

Let one end of a metal bar, lying horizontally in a tin trough, be held by a fixed support, and allow the other to press against one arm of a lever, carrying at its axis of rotation a small mirror, *s* (Fig. 99), seen in plan. Let also a ray of light fall upon the mirror, which after reflection will produce a bright spot upon a graduated scale placed some distance from the mirror. If, then, the bar be heated, the mirror will turn and

the motion of the illuminated spot will make the expansion of the bar easily perceptible. If the trough were filled, first with melting ice, or snow, and then with boiling water, the distance traversed by the spot of light, read from the scale, would furnish an estimate of the amount of expansion of the bar due to a rise in temperature from the freezing to the boiling point of water, or from  $0^{\circ}$  to  $100^{\circ}$  of the centigrade scale. To obtain a precise determination, it is only necessary to consider the known lengths of the lever-arms and the distance of the mirror from the scale, in connection with the extent of shift of the spot of light. For very precise work, a telescope is directed towards



FIG. 99.—Expansion of Bar.

the mirror, which produces an image of the scale placed horizontally just above the telescope. The apparatus is so adjusted that the zero, 0, of the scale is seen at first in the centre of the field of the telescope (*i.e.* on the cross hairs of a reticle); but after the expansion of the bar and the consequent rotation of the mirror, the graduation mark, *m*, is brought back to the middle of the field.

By means of apparatus similar to this, it has been found that a bar 1 m., or 1000 mm. long, composed of the substances named below, expands, while the temperature is being raised from  $0^{\circ}$  to  $100^{\circ}$  C., by the amounts set opposite the substance in the following table :—

Glass	...	...	0.8 mm.	German Silver	...	1.8 mm.
Platinum	...	...	0.9 "	Brass	...	1.9 "
Steel	...	...	1.1 "	Silver	...	1.9 "
Iron	...	...	1.2 "	Tin	...	2.0 "
Gold	...	...	1.4 "	Lead	...	2.8 "
Copper	...	...	1.7 "	Zinc	...	3.0 "

If what is known to be nearly true, be assumed exactly correct, *viz.* that the expansion takes place uniformly between

$0^\circ$  and  $100^\circ$ , i.e. for equal rises in temperature equal expansion occur, the expansion for  $1^\circ \text{ C.}$  is found by dividing the expansion for  $100^\circ$  by 100. A zinc bar, for instance, 1 m. long expands, when its temperature is raised  $1^\circ \text{ C.}$ , by 0.03 mm., or by 0.00003 m., i.e. by  $\frac{3}{100000}$  of its original length. This numerical value expressing the fractional part of the length of a body at  $0^\circ$ , by which it expands when its temperature is raised  $1^\circ$ , is termed its *coefficient of linear expansion*.

If the coefficient of linear expansion of a body be denoted by  $\alpha$ , when the body is heated from  $0^\circ$  to  $t^\circ$ , each of its linear units increases by  $\alpha t$  and becomes equal to  $1 + \alpha t$ . If  $l_0$  is the length of the body at  $0^\circ$ , its length  $l_t$  at the temperature  $t^\circ$ , is consequently—

$$l_t = l_0(1 + \alpha t).$$

Many useful applications are made of the different rates of expansion of bodies. Since the time of vibration of a pendulum



FIG. 100.—  
Gridiron  
Pendulum.

increases with its length, a clock provided with an ordinary pendulum must lose when the temperature rises and gain when it falls. In the gridiron pendulum, called also the *compensation pendulum* (Fig. 100), this disturbing influence of the heat upon the regularity of the rate of running of the clock is equalized, or *compensated*. The shorter and more expansible zinc rods, *zz*, tend to carry the bob of the pendulum upward by just as much as the longer and less extensible iron rods, *eee*, seek to draw it downward. Pocket watches, in which fluctuations of temperature are compensated, are called *chronometers*. With these, the expansions are equalized by strips of metal soldered together, so that, as the temperature rises, the more expansible metal lies on the convex side of the curve produced. Such strips, bent to semicircular form, with the more expansible metal on the outside, and carrying small weights at their ends, are attached to the circumference of the balance-wheel. When the temperature rises, the weights approach the centre of motion, and thereby equalize the outward expansion of the metallic parts of the balance-wheel. Such metallic

strips are also used in the construction of metallic thermometers. With the maximum and minimum thermometers of Hermann and Pfister (Fig. 101), the spirally wound strips of metal—*ss*, the outer portion consisting of steel and the inner of brass—have their inner ends screwed fast to the pin, *a*, and their outer ends free. With increasing temperature, the brass expands more strongly than the steel, and the spiral opens somewhat: its free end moves toward the left, carrying before it a light movable pointer, *cd*, by means of the pin, *p*. On cooling the spiral contracts and closes, its free end moves toward the

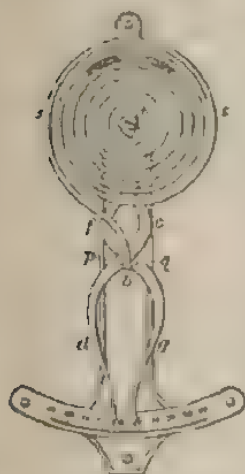


FIG. 101.—Metal Maximum and Minimum Thermometer.

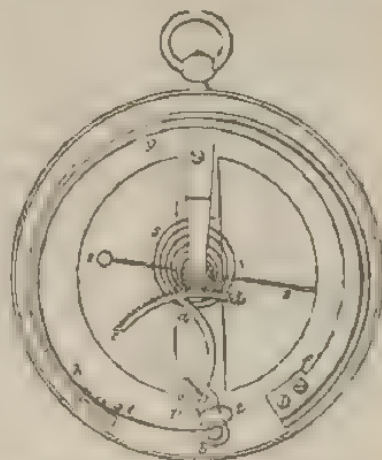


FIG. 102.—Quadrant Thermometer

right, leaving the pointer, *cd*, at the highest point reached, the pointer, *fg*, being carried by means of the pin, *g*, toward the right. When the temperature again rises, the free end of the spring returns toward the left, leaving the pointer, *fg*, over the highest graduation reached by it on the right, thus indicating the lowest temperature. The curved scale is graduated by comparison with a mercurial thermometer.

The *quadrant thermometer* (Fig. 102), made in the form and size of an ordinary pocket watch, contains a circular band, *fgh*, consisting of copper within and steel without, having the end

*f* fixed, while the other end works by means of the hook, *u*, against the projection, *p*, of a lever, *boa*, which turns about the point, *a*. The lever communicates through the toothed arc, *od*, and the pinion engaging with it, a motion to the pointer, *z*, in the direction from *f* toward *g*, when the circular strip, *fgh*, is expanded by a rise of temperature. With falling temperature, the pointer is returned by the coiled spring, *st*. The graduated dial over which the pointer moves is not shown in the figure. The *metallic thermometer* of Breguet (Fig. 103), the most sensitive thus far constructed, consists of a slender metal ribbon, *AB*, coiled like the threads on a screw, and composed of platinum, gold and silver. The three metals, the gold between the less expansible platinum on the one side, and the more extensible silver on the other, are rolled into an extremely thin ribbon. The lower end of the coil carries a pointer, *od*, suspended just above a graduated dial, *NN*.

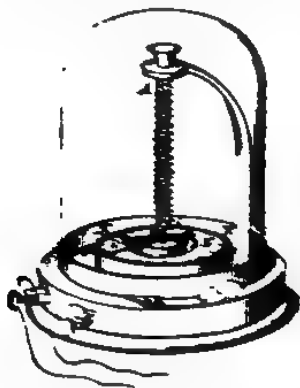


FIG. 103.—Breguet's Metallic Thermometer.

The instrument figured above is so constructed, as to be adapted to testing the delicate thermal effects of weak galvanic currents. A platinum wire dips downward from the needle into the basin, *HH*, of mercury, which is in circuit with one of the two binding screws shown in the cut. A current entering by this binding screw, passes by means of the mercury and the platinum wire, through the metal ribbon, and returns by way of the brass standard, *AN*, which is connected with the other binding screw. The ribbon is warmed by the passage of the current, and turns the pointer through a number of degrees corresponding to the thermal effect of the current.

The expansion of bodies on heating and their subsequent contraction on cooling take place with great force. In constructing iron bridges, in laying the steel rails on railroads etc., spaces large enough to permit of the expansion of the several pieces must be left between all metallic parts, otherwise the structure would be twisted or crushed by the tremendous force of expansion. The smith always fits the hot tire to the wheel, so that, after cooling, the force of compression of the

contracting tire gives a rigidity to the wheel scarcely attainable by any other known means.

With those solids from which bars, or rods, could be made, it was, of course, most natural to ascertain first their linear expansion. Since, however, bodies expand in breadth and thickness at the same rate as in length, the expansion in the volume, or the *volumetric expansion* of bodies, may be obtained from the linear expansion. By the coefficient of *volumetric expansion*, or *cubical expansion*, is meant the numerical value which expresses what fractional part of its volume at  $0^\circ$  a body expands when its temperature is raised  $1^\circ$ . It is very nearly three times the coefficient of linear expansion.

If  $l = l_0(1 + \alpha t)$  denote the length of the edge of a cube at  $t^\circ$ , where  $l_0$  is its length at  $0^\circ$ , the volume  $v_t$  of the cube at  $t^\circ$ , is—

$$v_t = l^3 = l_0^3(1 + \alpha t)^3 = v_0(1 + \alpha t)^3 = v_0(1 + 3\alpha t + 3\alpha^2 t^2 + \alpha^3 t^3)$$

Since  $\alpha t$  is a small quantity, the last two terms may be neglected, each being much smaller than  $\alpha t$  itself, and we may write, with close approximation—

$$v_t = v_0(1 + 3\alpha t) = v_0(1 + \beta t)$$

where  $\beta = 3\alpha$ , is the coefficient of cubical expansion.

Since specific gravity is inversely as the volume, denoting the specific gravities of the body at  $0^\circ$  and  $t^\circ$ , by  $s_0$  and  $s_t$  respectively, and the coefficient of cubical expansion by  $\beta$ , we may write—

$$s_t : s_0 = v_0 : v_t(1 + \beta t)$$

$$\text{or, } s_t = \frac{s_0}{1 + \beta t} = s_0(1 - \beta t + \beta^2 t^2 - \beta^3 t^3 + \dots)$$

or, disregarding the very small terms,  $\beta^2 t^2$ ,  $\beta^3 t^3$ , etc., we obtain with sufficient exactness—

$$s_t = s_0(1 - \beta t).$$

**101. Expansion of Liquids.**—Only the coefficient of cubical expansion can be considered with liquids. To prove the fact, and at the same time to determine the magnitude of the volumetric expansion of a liquid, a glass flask with a contraction indicated by a mark,  $a$ , in the neck may be used (*dilatometer*, Fig. 104). If the vessel be filled with alcohol to the mark  $a$ , and brought to  $0^\circ$  by surrounding it with melting ice, and if it be then exposed to the temperature of the surrounding air by



FIG. 104.—Dilatometer.

withdrawing it from the ice, alcohol will immediately rise in the neck of the vessel above the point *a* into the funnel-shaped space at the top. If the vessel be weighed together with its original contents and then again after the liquid above the mark *a* has been removed, by deducting the weight of the empty flask, the ratio of the weight of the alcohol passing through *a* to the weight of the liquid at  $0^{\circ}$  is obtained, and this ratio will be the number which expresses the fractional part of its original volume by which the liquid has been expanded between the limits of temperature used. But the number thus found only gives the apparent, or *relative*, expansion of the liquid with respect to glass. The capacity of the glass vessel also increases with the temperature, precisely as though it were a material body. To obtain the true, or *absolute*, expansion of the liquid, it is necessary to add the expansion of the vessel itself to that of the liquid as found above. Since it is known that the coefficient of cubical expansion for ordinary glass is about 0.00025, this latter expansion is easily computed.

As different varieties of glass expand unequally, for very delicate work it is not sufficiently accurate to use the coefficient of cubical expansion of glass, computed from that of the linear expansion. For each dilatometer, this coefficient of cubical expansion must be specially determined. This is readily done when the absolute expansion of any liquid has been found. Dulong and Petit have, for this reason, determined directly the absolute expansion of mercury (1818). The liquid was put into two glass cylinders, connected by a slender tube. One of the cylinders was cooled to  $0^{\circ}$  and the other was heated to  $100^{\circ}$ . From the heights of the two mercury columns measured by a *cathetometer* (an instrument devised for this special purpose), the ratio of the specific gravities of mercury at  $0^{\circ}$  and at  $100^{\circ}$  was indicated, and, accordingly, also the ratio of the volumes, since, in two communicating vessels of liquid in equilibrium, the specific gravities are inversely as the densities. It was found that while mercury was heated from  $0^{\circ}$  to  $100^{\circ}$ , it expanded by 0.0181 of its original volume. If the definition of a degree of temperature as given above, is to have any definite significance, the assumption must be made that

both mercury and glass expand uniformly between  $0^{\circ}$  and  $100^{\circ}$ , *i.e.* for each degree they must expand by the same amount. The absolute coefficient of expansion of mercury for  $1^{\circ}$  C. is, therefore,  $0.00018$ , or  $\frac{1}{5555.6}$ . In order to ascertain the expansion of the dilatometer, the relative expansion of mercury is obtained by it, between known limits of temperature as above explained. This, subtracted from the known absolute expansion of mercury, furnishes the expansion of glass. If the instrument is used afterwards for other liquids, this difference must always be added to the observed relative expansion, to obtain the true expansion. In this way the following liquids have been found to have the expansions given here:—

From $0^{\circ}$ to $100^{\circ}$	{	Water	0.013
		Olive Oil	0.000
		Petroleum	0.100
.. $0^{\circ}$ .. $80^{\circ}$		Alcohol	0.097
.. $0^{\circ}$ .. $33^{\circ}$		Ether	0.054

Most liquids do not expand uniformly (*i.e.* their expansion does not agree with the assumed uniform expansion of mercury). They expand, as a rule, more rapidly with higher than with lower temperatures, or their coefficients of expansion do not remain constant, but change with the temperature. With the alcohol thermometer, then, which is graduated by comparison with the mercurial thermometer, the graduations increase in value continuously upward.

Great force also accompanies the expansion of liquids. For this reason, on filling a cask with oil, or petroleum, it is customary to leave a small quantity of air-filled space to prevent the bursting of the cask in case of a rise in temperature of the contents.

A dilatometer may be used conversely as a constant weight-thermometer in determining temperatures. The best form of the instrument for this purpose is a glass vessel with a neck drawn to a point. It is used by filling the vessel with mercury at  $0^{\circ}$ , and weighing it. Then, after subjecting it to the temperature to be determined, and a portion of the mercury has passed out, the vessel is again weighed. The ratio of the mercury which has passed out to that remaining within the vessel

is the desired relative expansion. If the coefficient of mercury relatively to the glass used has been previously determined (it is 0.000154 for ordinary glass), the temperature attained is then readily computed.

Suppose the contents of the instrument at temperature  $0^\circ$  weighs  $P$ , and at  $t^\circ$  it weighs  $P'$ . If  $v_0$  denote the volume at  $0^\circ$ ,  $\beta$  the coefficient of expansion of mercury,  $\gamma$  that of glass, and  $a$ , the specific gravity of mercury at  $0^\circ$ , then at  $t^\circ$ , the volume of the vessel is  $v_0(1 + \gamma t)$  and the specific gravity of the mercury is equal to

$$\frac{a}{1 + \beta t}$$

Also

$$P = v_0 a, \quad P' = v_0 a \frac{1 + \gamma t}{1 + \beta t}$$

whence

$$\frac{P - P'}{P'} = \frac{(\beta - \gamma)t}{1 + \gamma t},$$

or, since  $\gamma$  is very small, with sufficient accuracy—

$$\frac{P - P'}{P'} = (\beta - \gamma)t.$$

where  $\beta - \gamma$  is the *relative*, or *apparent*, coefficient of expansion of mercury.

**103. Anomaly of Water.**—Water comports itself in a peculiarly interesting way. Let a glass flask, in which, through a



FIG. 105.—Water Thermometer.

tightly-fitting cork, a glass tube, open at both ends, and a thermometer, are inserted (Fig. 105), be filled to a point some distance up the tube with water at  $0^\circ$  (i.e. surrounded with melting ice). Moreover, let enough mercury be poured into the flask so that the expansion of its volume shall be just neutralized by the expansion of the mercury, and that the volume of the water filling the remaining space in the flask shall exhibit its absolute volumetric expansion. If, now, the vessel is taken from the ice so that its contents become gradually warmer by absorption of heat from surrounding objects, the column of water in the tube will sink until the thermometer indicates  $4^\circ$ . The water will, thereafter, gradually rise to the original height at  $0^\circ$ , which is attained at about  $8^\circ$ , after which it rises continuously with the temperature, though with increasing rapidity. We see from this that water contracts from

0° C. to 4° C., and then expands as the temperature increases; that a mass of water has a smaller volume at 4° than at any other temperature; and, consequently, that at 4° it has its maximum density. Hence, its specific gravity also has its greatest value at this temperature. One liter (or 1000 cub. cm.) of water expands on heating from 4°—

to	6°	by	$\frac{1}{10}$	cub. cm.
"	10°	"	1	"
"	30°	"	5	"
"	60°	"	17	"
"	100°	"	43	"

It is due to this remarkable property that the water of our large lakes never freezes to the bottom. In winter the superficial layers of water cool by convection and radiation through the air and sink, yielding their places to warmer water from below. This continues until the temperature of maximum density has been reached, or, in other words, until the whole mass has attained the temperature of 4°. If the surface layers become still colder they cannot sink, since they are rarer than the layers below. When the surface layers have reached the temperature of freezing they will be converted into ice. The ice must also remain upon the surface, and thus the surface will become covered with a protecting mantle, which, retarding radiation from beneath, permits only a very gradual thickening of the ice. At great depths, a lake maintains the temperature of 4° the whole year through, even when it is completely frozen over, thus rendering possible the perennial existence of aquatic animals.

This process may be imitated in miniature. A tall, wide, cylindrical glass vessel, in which two thermometers are fitted, one with its bulb near the bottom, and the other having its bulb near the top, is filled with water at the temperature of an ordinary dwelling-room. If the water is cooled from above, by dropping pieces of ice upon its surface, the lower thermometer for a time sinks, but stops at 4°, and remains at this point while the upper instrument stands continually at 0°.

**103. Expansion of Aeriform Bodies, or Gases.**—In measuring the expansion of gases, it must be remembered that the volume

of a gas depends not only upon its temperature, but, from Mariotte's law, also upon the pressure to which it is exposed. In measuring the volumes of the original and of the expanded gas, therefore, care must be taken that the measurements are executed with the gases under equal pressures. The apparatus of Fig. 106 can be advantageously used to accomplish this purpose. A hollow glass globe, A, is connected by a slender glass tube, B, with the shorter branch, C, of a wider glass-tube, CD (manometer), into which mercury may be admitted through

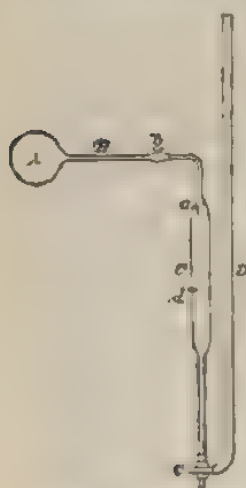


FIG. 106.—Expansion of Gases.

the open branch, D, and let out through the tap at *c*. The mercury column in D may thus be made to stand at any desired height. In the shorter branch, a pointer is attached near the top of the cylindrical enlargement at *a*, to be used as an index. The globe, A, whose volume, together with that of the tube, B, to the index, *a*, has been previously determined, is filled with dry air and surrounded with melting ice, or snow. By means of the tap, *b*, still in connection with the outer air, the surface of the mercury in the shorter branch is brought to the index, *a*, and in the longer to the same level. The tension of the air inside the globe is then equal to that of the external air, the latter being indicated by a barometer. If the tap, *b*, is now closed and the globe, A, is surrounded with steam (vapour of boiling water), the air in the globe expands and drives the mercurial column of the shorter branch downward, and that of the longer branch upward. By aid of the cock, *c*, enough of the mercury is now drawn off to make the mercury stand at the same height in both branches, when the internal and external tension will again be equal. If the surface of the mercury in the shorter branch is at *d*, the enclosed air has expanded by the volume between *a* and *d*, for a rise in the temperature from 0° to 100°. The space between *a* and *d* must

be determined, and to this end the tube, *c*, is filled, after the experiment is finished, to *a* with mercury, while enough of the liquid is then drawn off through *c* to bring the surface to *d*, whereupon, by weighing the mercury drawn off, the desired volume is readily obtained. It is found in this way, that 1000 cub. cm. (1 l) of atmospheric air expands when heated from the freezing to the boiling point of water, by 367 cub. cm., or by  $\frac{1}{273}$  of its original volume. For all other gases the same expansion is found. If the expansion of gases be assumed uniform, their common coefficient of expansion is  $\frac{1}{273}$ , or 0.00367. We arrive thus at the law of Gay-Lussac (Charles, 1787; Dalton, 1801; Gay-Lussac, 1802). With equal pressures, all gases expand by equal amounts on heating, i.e. by  $\frac{1}{273}$  of their volume at 0°, or, all gases have the same coefficient of expansion as air. This law, in conjunction with Mariotte's, that the temperature of a gas remaining the same, the tension varies inversely as the volume, furnishes complete information with regard to the relations which connect the temperature, pressure, and volume of a gas. In particular, we see that if a gas be heated while its volume remains constant, its tension, for each degree rise in temperature, increases by  $\frac{1}{273}$  of its tension at 0°. For if, in the apparatus, Fig. 106, the volume of air at 100° reaching to *d*, be compressed by pouring mercury into the tube, *D*, to its original magnitude (at *a*), or in the ratio of 1367:1000, according to Mariotte's law the tension must increase in the inverse ratio of 1367:1000. The tension would in this case, of course, be the same as if, during the rise in temperature from 0° to 100°, the expansion had been prevented by pouring mercury into the open branch, *D*. That an increase of tension, in the ratio mentioned, has actually occurred, may be seen from the heights of the mercurial column in the larger branch above the point, *a*. It is found to be  $\frac{367}{1000}$ , or  $\frac{1}{273}$  of the simultaneous barometric height. We see, thus, that the coefficient of expansion of gases is at the same time also their coefficient of tension, which latter indicates the increment of pressure and of tension respectively for each degree of temperature, the volume remaining the same. It is, then, possible to obtain this coefficient of tension also by observing in the

branch, D, the height of the mercurial column above the index,  $a$ .

**104. Air Thermometers.**—After the expansion of gases has been found, the instrument shown in Fig. 106, may also be used as an air-thermometer to ascertain the temperature of the air within any space. The globe, A, is brought into the



FIG. 107.—Jolly's Air Thermometer.

space, and either the expansion under constant pressure, or the increase in tension with constant volume is observed, and from the observed datum the temperature is ascertained. The second method is to be preferred as being more convenient, and, at the same time, more accurate than the first. The air thermometer (Fig. 107) devised by Jolly (1874) is especially convenient. The two branches of the manometer are connected by a rubber tube. By the aid of sliders on the vertical column and a graduated scale etched on the surface of a mirror and fixed along one side of the column, the tube may be moved upward and downward by measurable amounts. If the eye be so held that the image of the pupil coincides with the meniscus, the line of sight over the meniscus strikes the scale perpendicularly, and thus the error introduced by oblique vision (parallax) is eliminated.

Experience shows that air and gas thermometers generally agree much better with one another than do mercurial thermometers constructed of different sorts of glass. The difference of expansion of the different sorts of glass has a far smaller influence upon the data of gas thermometers than upon those of mercurial thermometers, since a gas expands 146 times, and mercury only seven times, as much as glass. Furthermore, a gas thermometer may be used within much wider limits of

temperature than a mercurial. The utility of the latter is, of course, comprised between the limits set by the freezing and boiling points of mercury. The air thermometer, or, still better, the oxygen thermometer, has, therefore, been selected as the normal thermometer to whose data in precise measurements all readings of the mercurial thermometer are referred. The assumption is made here also that equal changes of volume, or of tension, correspond to equal changes of temperature, or that gases expand uniformly. From the circumstance that between  $0^\circ$  and  $100^\circ$ , the mercurial thermometer agrees very closely with the air thermometer, we infer that within these limits, the expansion of mercury is also uniform.

**105. Mariotte-Gay-Lussac's Law—Absolute Temperature.**—If at  $0^\circ$  and under a pressure of  $p_0$ , the volume of a gas be denoted by  $v_0$ , according to Gay-Lussac's law the volume,  $v$ , at temperature,  $t$ , the pressure remaining unaltered, is given by—

$$v = v_0(1 + at),$$

where  $a = \frac{1}{273}$ . If now, with constant temperature, the pressure is changed from  $p_0$  to  $p$ , according to Mariotte's law, the new volume,  $v$ , will be obtained from the equation—

$$pv = p_0v_0$$

or

$$pv = p_0v_0(1 + at).$$

This equation is a symbolic representation of the result of combining Mariotte's and Gay-Lussac's laws. It is sometimes called the conditional equation of a gas, because it expresses the inter-relationship of the three magnitudes, pressure, volume, and tension,  $p, v, t$ , upon which the condition of a gas depends.

If  $t$  remains unchanged,  $p_0v_0(1 + at)$  is also constant, and the general equation then expresses Mariotte's law, viz.  $pv = \text{a constant}$ .

If the pressure remains constant ( $p = p_0$ ) from  $0^\circ$  to  $t^\circ$ , Gay-Lussac's law results,  $v = v_0(1 + at)$ .

If, finally, the temperature is changed, and the volume

remains constant ( $v = v_0$ ) from  $0^\circ$  to  $t^\circ$ , from the general equation, we obtain for the pressure, or the tension—

$$p = p_0(1 + \alpha t).$$

Consequently, for every degree of rise, or fall, in temperature, the pressure, or the tension, increases or decreases by  $\alpha = \frac{1}{273}$  of its value at  $0^\circ$ , and, as was found above, the coefficient of expansion becomes the coefficient of tension.

Assuming these equations to hold generally, it follows that for  $t = -\frac{1}{\alpha} = -273$ , the tension of a gas vanishes. This temperature is called the absolute zero, and the temperature reckoned from it, obtained by adding to the instantaneous temperature  $t$  of the Centigrade scale, the number 273, is called the *absolute temperature*—

$$T = 273 + t.$$

Many simplifications are made possible by the introduction and use of absolute temperature. The Mariotte-Gay-Lussac law, for instance, may be put in the form—

$$pv = p_0 v_0 \left(1 + \frac{t}{273}\right),$$

or, what is the same thing,

$$pv = \frac{p_0 v_0}{273} (273 + t) = \left(\frac{p_0 v_0}{273}\right) T,$$

and, by denoting the constant quantity  $\frac{p_0 v_0}{273}$  by  $R$ ;

$$pv = RT.$$

The Mariotte-Gay-Lussac law may then be thus expressed: *For all gases, the product of pressure into volume is proportional to the absolute temperature.*

The number  $R$ , which is the same for all gases, is called "the constant of the Mariotte-Gay-Lussac law (the M. G. L. law)."

This law also holds for the osmotic pressure (78), with which the pure solvent (water) passes through the "semi-permeous" partition toward a dissolved substance (e.g. sugar). A partition wall which is permeable to the

solvent alone is called "semi-pervious." Such a membrane composed of copper ferro-cyanide, forms upon the inner surface of a porous earthenware cell by the interaction of calcium ferro-cyanide and copper sulphate. If the cell, filled with a solution of sugar, be surrounded by water, a mercurial manometer, communicating with the interior of the cell, shows that the osmotic pressure is proportional to the degree of concentration, i.e. to the density, of the dissolved substance and, with equal densities, it increases by  $\frac{1}{273}$  for  $1^{\circ}$  C. (Pfeffer, 1877). Osmotic pressure is, therefore, the same as the pressure which the dissolved substance would exert if it were distributed in a gaseous form throughout the space occupied by the solution (Van t'Hoff).

This law is more nearly correct the rarer the solution.

**106. Deviations from Mariotte-Gay-Lussac's Law.**—The behaviour of gases departs slightly from this law. Regnault (1847) found that air and carbonic acid diminish in volume somewhat more rapidly with increasing pressure than is required by Mariotte's law; but oxygen, on the contrary, diminishes somewhat less rapidly. It was also found that the coefficients of expansion of various gases are not exactly equal, and that, with one and the same gas, the coefficient of expansion is not exactly equal to the coefficient of tension. A gas which would exactly follow Mariotte-Gay-Lussac's law is termed an *ideal*, or a *perfect*, gas. The deviations of any particular gas from its ideal condition increase with increasing pressure and rising temperature, but all gases approximate the perfect gaseous condition.

The more perfect conditional equation of gases, given by Van der Waals (1873), which takes cognizance of the deviation from the ideal law, is—

$$(p + \frac{a}{v^2})(v - b) = RT,$$

where  $a$  and  $b$  are small numerical values to be determined by observation. For  $a = 0$  and  $b = 0$ , this reduces to the ideal law,  $pv = RT$ .

**107. Determination of Volumes of Gases.**—Since a quantity of gas may be made to occupy any desired volume according to the temperature and pressure to which it is exposed, there would evidently be no meaning in an attempt to determine its volume, unless at the same time the pressure and temperature of the gas were determined. If, however, these two circumstances are known, it is easy to find from Mariotte-Gay-Lussac's law,

the volume of the gaseous mass under a pressure equal to that of a column of mercury of 760 mm. and at the temperature  $0^{\circ}$ . It has been conventionally established that the condition of a gas under this pressure (the normal barometric pressure) and temperature, shall be designated its *normal condition*. To this condition all measurements of gaseous properties must be reduced to make them comparable with each other.

Suppose, for example, a quantity of gas has been collected over a pneumatic trough filled with mercury, in a glass tube graduated to centimetres, and that its volume is  $v$ . The pressure is found by subtracting from the simultaneous barometric height (in mm.) the height of the mercurial column in the tube. The temperature,  $t$ , is that of the surrounding air, read from a thermometer hanging near. The volume,  $v_n$ , in the normal condition is found now from the equation  $p_n v_n = p v (1 + \alpha t)$ , where all magnitudes are known, since  $p$ , is taken equal to 760 mm.

**103. Specific Gravity of Gases.**—To obtain the weight of the air in its normal condition, the air within a glass globe of volume  $v$  (in cub. cm.), is rarefied as much as possible, and the pressure  $h$  mm. of the remaining air is read from a testing barometer. The globe, which is provided with a tap, is then placed upon a balance and counterpoised (preferably by means of a second equally large globe, so that the buoyant effect of the enveloping air may be the same on both sides), and the tap is opened. The first globe, having grown heavier from the air admitted into it, sinks, and weights  $g$  grams, must be put in the other scale-pan to restore equilibrium. The air now contained in the globe has the same tension as the outside air, as is indicated by the barometric height  $z$  mm. at the time. Since the tension of the air in the globe before opening was  $h$  mm., the tension of the air admitted which has also been weighed, is  $z - h$  mm. It is now known that  $v$  cub. cm. of air of  $z - h$  mm. tension and at  $t^{\circ}$  C. the temperature of the experiment room, weigh  $g$  grams. The volume,  $v_n$ , of the air which has been weighed in its normal condition, is then found from the above equation: if  $p_n = 760$  mm., and the observed values of  $v$ ,  $z$ , and  $g$  =  $z - h$  are substituted in it and the

weight of a cubic centimetre of air is found to be equal to  $\frac{7}{8}$ . The weight of a cubic centimeter of air at 0 and under 760 mm. pressure is thus found to be 0.001293 g., or 1 litre of air weighs 1.293 g. In its normal condition, therefore, air is  $\frac{7}{8}$  as heavy as an equal volume of water at 4° C., and  $\frac{7}{8} = 0.001293$  expresses the specific gravity of air with respect to water.

Treating other gases in the same way, their specific gravities may likewise be obtained. Since the specific gravities of aeriform bodies referred to water are very small numbers, it is preferable to refer them to air, or, still better, to hydrogen, the lightest of all gases; and as all gases, according to Mariotte-Gay-Lussac's law, change their volumes according to the same law with variations of temperature and pressure, we define the specific gravity of a gas as the number which expresses how many times the gas is heavier than an equal volume of air, or hydrogen, of the same temperature and pressure.

The specific gravities of some of the gases most frequently met with, referred to both air and to hydrogen, are given in the accompanying table.

Gas	Referred to Air.	Referred to Hydrogen
Hydrogen . . . . .	0.0692	1.000
Ammonium . . . . .	0.5970	8.50
Nitrogen . . . . .	0.9714	14.00
Air . . . . .	1.0000	14.45
Oxygen . . . . .	1.1056	16.00
Hydrochloric acid gas . . . . .	1.2470	18.00
Carbonic acid gas . . . . .	1.5290	22.00
Cyanogen . . . . .	1.8000	26.00
Chlorine gas . . . . .	2.4700	35.50

**109. Melting—Melting Point—Melting Heat.**—When a body is exposed to heat, its temperature rises until it reaches a certain height, whereupon the body passes into the liquid condition, or it melts. For every substance, melting, as a rule, occurs at a fixed and definite temperature, called the

*melting point.* The following are the melting points of some bodies :—

	°C.		°C.
Mercury ... ..	-39.0	Zinc ... ..	412.0
Ice ... ..	0.0	Antimony ... ..	432.0
Benzol ... ..	+ 4.4	Silver ... ..	1000.0
Tallow ... ..	43.0	Copper ... ..	1100.0
Paraffin ... ..	46.0	Gold ... ..	1200.0
Wax ... ..	62.0	Cast iron ... ..	1200.0
Sulphur ... ..	115.0	Cast steel ... ..	1375.0
Tin ... ..	230.0	Wrought iron ... ..	1600.0
Bismuth ... ..	250.0	Platinum ... ..	1775.0
Cadmium ... ..	320.0	Iridium ... ..	1950.0
Lead ... ..	326.0		

It is a remarkable fact that the melting point of many metallic mixtures, called *alloys*, is below the melting point of either of the constituents. The quick solder of tinkers, consisting of 5 parts by weight of tin and 1 of lead, melts at 195° C.; Rose's alloy, 2 parts of bismuth, 1 of lead, and 1 of tin, melts below the boiling point of water, at 95°; Wood's metal, 1 to 2 parts of cadmium, 7 to 8 parts of bismuth, 2 parts of tin, and 4 of lead, fuses at 65 to 70°. All bodies melt with sufficient elevation of temperature, provided they are not chemically decomposed before reaching the melting point (*e.g.* wood). Coal alone has hitherto shown itself incapable of fusion by the means at our disposal.

So long as melting continues, the body maintains constantly the temperature of its melting point. If, on a cold winter day, a vessel filled with snow which has been cooled below the freezing point, to - 5, for instance, be placed upon a warm stove with a thermometer inserted in the snow, the mercury gradually rises and shows that the temperature of the snow becomes - 4, - 2, - 1, to 0°, successively. But at 0° the mercury remains stationary, until the snow has been completely melted and converted into water at 0°. The liquid then becomes warmer and the thermometer rises. Although heat is being continually imparted to the vessel by the stove, while the thawing of the snow lasts, no rise in temperature occurs. The heat is all consumed in transforming snow at 0° into water at 0°. Such heat is no longer either perceptible to the senses, or detectable by the thermometer. The fact

that, with a uniform addition of heat, the melting process lasts longer the greater the mass of the body to be melted, leads to the idea of quantity of heat, or *thermal quantity*. The amount of heat required to convert 1 kg. of a body into the liquid condition is called the melting heat of the body, or, since it is, in a sense, bound up within the body, or within the resulting liquid, it is also called the *bound*, or *latent, heat* (Block, 1757). To determine the latent heat of ice, let 1 kg. of dry snow at  $0^{\circ}$  be mixed with 1 kg. of water at  $80^{\circ}$  C. After the snow is melted, there will be 2 kg. of water at  $0^{\circ}$ . Consequently, all the heat given off by 1 kg. of water on cooling from  $80^{\circ}$  to  $0^{\circ}$  C., is consumed in converting 1 kg. of snow at  $0^{\circ}$ , into 1 kg. of water, also at  $0^{\circ}$ . In other words, for the mere melting of 1 kg. of ice, precisely the same quantity of heat is needed as is required to raise 1 kg. of water from  $0^{\circ}$  to  $80^{\circ}$  C. The quantity of heat required to raise 1 kg. of water by  $1^{\circ}$  C. is called the heat unit, or *calorie*. The latent heat of ice is, therefore, 80 calories, or heat-units. It is now possible to form an estimate of the enormous quantity of heat consumed in spring to melt the winter's accumulation of snow and ice, and which is wholly lost to the development of animal and plant life. On the other hand, if ice at  $0^{\circ}$  were suddenly transformed into water without the consumption of this immense thermal supply furnished gradually from the sun, the danger from the overflow of water would be far greater than is actually the case.

The latent heat of some bodies is here given—

Ice ... ..	80.0	Bismuth ... ..	12.6
Phosphorus ... ..	5.0	Tin ... ..	14.2
Sulphur ... ..	9.4	Silver ... ..	21.1
Lead ... ..	5.3	Zinc ... ..	28.1

If a vessel of water with a thermometer immersed, be exposed to a very low temperature in the open air, the mercury will sink until  $0^{\circ}$  is reached. Ice then begins to form, and the mercury remains at  $0^{\circ}$  until the bulb is entirely encased in ice. In spite of the fact that heat is continually withdrawn from the vessel by its colder environment, the thermometer nevertheless does not sink, though the freezing continues. This, of course, is explicable only on the hypothesis

that the solidification of water is accompanied by a development of heat, which replaces the heat withdrawn from the vessel and keeps the temperature constantly at  $0^{\circ}$ .

When the body freezes, therefore, the heat absorbed by it on melting is again liberated. The large quantity of heat released by the freezing of water, retards very materially the on-coming rigours of winter.

Water at  $0^{\circ}$  freezes on the withdrawal of heat, and ice at  $0^{\circ}$  melts on the application of heat. The temperature of solidification, i.e. the freezing point of water, coincides with its melting point. Under special circumstances, however, viz. by preventing agitation and excluding air, liquids may be cooled far below their melting points without solidifying them. They are said to be *under-cooled*, or *over-melted*, and the phenomenon is called *over-melting*, or *retardation of solidification*. If a vessel of water, with a thermometer immersed, be covered with a layer of oil and subjected to a brisk frost in the open air, the thermometer may be made to sink to  $-8^{\circ}$  or  $-10^{\circ}$  without freezing the liquid. A slight agitation, however, will instantly cause the freezing of a sufficient quantity of the water, and the consequent liberation of sufficient latent heat, to bring the temperature of the entire mass of water back to  $0^{\circ}$ .

If crystallized sodium sulphite be melted (melting point.  $48^{\circ}$ ) in a glass test-tube and the liquid be kept quiet, it will cool to the temperature of the room ( $18^{\circ}$ ) without freezing. It solidifies, however, on agitation, or, more certainly still, on throwing a crystal of the same substance into the liquid; the temperature at the same time rises quickly to  $48^{\circ}$ .

Most bodies expand on melting, and some quite suddenly. Phosphorus, for instance, increases in volume by 3.4 per cent. on being melted. On the contrary, some bodies, such as ice and bismuth, occupy less space in a molten than in a solid condition. From 1000 cub. cm. of ice at  $0^{\circ}$ , only 910 cub. cm. of water at  $0^{\circ}$  are obtained, and on freezing 1000 cub. cm. of water at  $0^{\circ}$ , a sudden expansion amounting to 90 cub. cm. occurs. Ice is, then, specifically lighter than water, and will float even in boiling water. The force with which this expansion takes place is so great that flasks filled with water, lead pipes, and even

thick-walled bomb-shells, are burst asunder by the freezing of their contents.

With bodies which expand on melting, the melting point is raised by an external pressure, but it is lowered with those substances which contract on liquefying. The melting point of ice, for instance, is lowered by  $0^{\circ}129$ , by an external pressure of 17 atmospheres, and with 13,000 atmospheres, water remains in a liquid condition even at  $-18^{\circ}$  (Mousson, 1858). If two pieces of ice be compressed together, from the lowering of the melting point, melting will occur at the surfaces of contact, and when the pressure is relieved, the molten water will be again solidified, the ice-blocks at the same time will be frozen firmly together. This process, which in the formation and movement of glaciers plays so important a part, is known as *regelation* (Faraday, 1850).

The freezing point of a liquid is lowered proportionally to the quantity of the dissolved substance contained in solution. Quantities of various substances which are proportional to their molecular weights, produce the same depression of the freezing point when acted upon by the same solvent. This property, which holds for dilute solutions of neutral organic substances, may be used to obtain the molecular weights of these substances (Raoult, 1882).

**110. Freezing Mixtures.**—Whenever a solid passes into a liquid condition, heat is consumed, or “bound,” or rendered latent, whether the transformation be effected by melting the solid, or by dissolving it. While the heat required to melt a body must be furnished by warming the body from without, a solid body may be dissolved without the addition of external heat. The quantity of heat required to convert the solid into a liquid (*dissolving heat, or heat of solution*) must then be taken from the constituent particles of the solution itself, and the temperature must therefore sink. If a handful of pulverized saltpetre be thrown into a glass of water, the solution will be at once cooled by a few degrees. If ammonium nitrate is dissolved rapidly in equal volumes of water, its temperature will fall by  $27^{\circ}$ ; if potassium-sulphocyanate, the temperature will drop by  $34^{\circ}$ . If, instead of water, snow or ice be used, the ice will be reduced to a liquid by reason of the presence of the salt, which strives to enter into solution with it, and from the consumption of the salt and the higher latent heat of the ice.

a very considerable reduction of temperature ensues. If a quantity of finely-crushed ice be mixed with half as large a quantity of table salt, the temperature falls to  $20^{\circ}$  below zero. This mixture is used by confectioners in the preparation of ices. Snow, with twice the quantity of calcium chloride, produces a mixture at  $-42^{\circ}$  C. Dilute sulphuric acid poured over snow melts it rapidly, and reduces the temperature to from  $40^{\circ}$  to  $50^{\circ}$  below zero.

The heat consumed in dissolving bodies is again set free, when the dissolved body is separated from the solution in a solid condition. If into an over-saturated (76) solution of sodium sulphate (Glauber's salts) a crystal of this same salt be thrown, crystallization immediately begins, and the entire mass rises considerably in temperature.

#### 111. Heat of Crystallization—Heat of Chemical Combination.

—Copper sulphate forms with water the beautiful blue crystals known as blue vitriol. On heating them they give off their water, which was held as a solid constituent within the crystals, and the salt, thus freed of its water, remains behind as a bright grey powder. If water be now added to the powder, the mass becomes blue again, a portion of the water solidifying as water of crystallization, and a considerable quantity of heat being liberated (*heat of crystallization*). The gypsum, occurring in nature, i.e. calcium sulphate with water of crystallization, loses the latter constituent on heating (known as *burning*). Pulverized burnt gypsum, ground to a paste with water, is used in the manufacture of plaster casts. Its adaptability to this purpose depends upon the fact that the gypsum, which is free from water, absorbs the water mixed with it as water of crystallization, and, consequently, the entire mass rapidly hardens. This process is also attended with the liberation of a considerable quantity of heat.

The solidification of water likewise accompanies the slaking of burnt calcium, or lime, and, as is well known, the process is accompanied by a violent liberation of heat. Burnt calcium (calcium oxide,  $\text{CaO}$ ), generated by heating native calcium (calcium carbonate,  $\text{CaCO}_3$ ) in a calcium oven, thus driving off the water, combines with water to produce calcium hydroxide ( $\text{Ca(OH)}_2$ ), or *slaked calcium*, which is a solid.

With all these processes, and especially with the latter, chemical forces also act, which compel the water to enter into the solid as one of its constituent parts. The heat developed in the formation of a chemical compound, and which is always a fixed and determinate quantity, is called the *heat of chemical combination*. The separation of the constituents entering into combination consumes the same quantity of heat as was developed by their combination. All artificial sources of heat depend upon combustion (oxidization), i.e. upon the chemical combination of the fuel with the oxygen of the air. For measuring the heat of chemical combination, the *water calorimeter* is used. This instrument contains a vessel within which the chemical action takes place. The number of heat-units produced by burning a unit of weight of the respective fuels is given in the appended list:—

Hydrogen ... ..	34·460	Stearic acid ... ..	97·20
Gasoline ... ..	11·860	Charcoal ... ..	80·80
Petroleum ... ..	11·094	Alcohol ... ..	91·90
Oil of turpentine ...	10·850	Stone coal ... ..	66·60
Wax ... ..	10·500	Fir wood ... ..	44·20

Animal heat arises in consequence of chemical processes going on within physical organisms, and especially from the combining of the carbon taken in as food with the inhaled oxygen. The heat of the body of a healthy man is 37·2°C., and is but little affected by either climate or age.

**112. Formation of Steam, or Vaporization—Evaporation.**—A vessel of water set over the fire soon *boils*. Bubbles are seen to rise in the water, which become more and more numerous as the temperature rises, and soon a violent motion of the liquid ensues, accompanied by a bubbling, or gurgling, sound. These bubbles contain not air, but water in a gaseous condition. They rise to the surface, burst, and discharge their contents into the air, with which they mix, and form a transparent, invisible gas, called vapour. The visible cloud which rises above the surface of the boiling water, is not vapour, but it consists of exceedingly fine drops of water which have been condensed from the vapour on cooling. This cloud, however, soon assumes the form of an invisible aqueous gas. By continued heating, the entire mass of water contained

in the vessel may be readily converted into vapour. (The passage of a liquid into the gaseous state is designated by the general term, *vaporisation*.) Water is transformed into a gaseous condition, not alone at the temperature of boiling water, but the transformation may occur at any lower temperature. If a quantity of water be exposed to the air in a shallow open vessel, the water slowly disappears. This formation of vapour, which takes place slowly and quietly at the surface of a liquid, is called *evaporation*. Heating accelerates the process, although it does not entirely cease with cold. Even ice and snow are observed to diminish in bulk, and gradually to disappear, even in cold, dry weather. Bodies which are easily vaporized, even at low temperatures, are called *volatile*.

In the cases just considered, the formation of steam occurred in the presence of air, with which the steam immediately mixed. The properties of vapour are studied by confining it in a Torricellian vacuum. Into the first of four Torricellian tubes, by means of a curved pipette, let a quantity of water be admitted into the vacuum; into the second, a little alcohol; into the third, a little ether; and let the fourth remain empty to be used as a barometer to determine the instantaneous atmospheric pressure. Immediately on the introduction of the liquids, the mercurial columns in the first three barometer tubes are seen to sink. In the tube containing water the depression amounts to 17 mm.: in that containing alcohol, to 44 mm.; and in that containing ether, to 435 mm.: provided that in each case the temperature of the surrounding air be kept constantly at 20°. These depressions cannot be due to the mere weight of the small quantities of liquid above the mercurial columns. To depress the mercury by 17 mm. would require a column of water 231 mm. high. They can only arise from the pressure, or tension, of a gaseous mass in what were originally perfect vacua, that is, of the vapour produced from the liquid. It is also noticed that if the temperature remains constant, the heights of the mercurial columns suffer no change, and the mass of liquid still remaining unevaporated above the mercury does not diminish. No further vapour forms within this space, and the space is therefore said to be *saturated*, or filled with *saturated vapour*.

To ascertain more precisely the behaviour of vapours under such conditions, fill a glass tube 80 or 90 cm. long, closed at one end, with mercury, leaving a small portion unfilled for the reception of the liquid to be tested—ether, for example. Let the tube which contains only the two liquids be now closed with the finger, and its end immersed under the surface of mercury contained in a deep vessel, and let the finger be then removed. No air should be allowed to enter the tube, which must be placed vertically, as shown in Fig. 108. Beneath the mercurial column, which at  $20^{\circ}$  is, say, 435 mm. lower than the barometric height at the same instant, a small quantity of liquid ether is admitted into the tube, which fills the vacuum immediately with a transparent, or with an invisible, gas. Let the space above the mercury now be increased by drawing the tube partially out of the basin of mercury below. The height of the mercurial column and, accordingly, also the pressure on the ether gas, do not change, but the quantity of the liquid ether decreases. As the space within which the vapour is formed increases, more ether vapour is formed of the same tension as before, and, so long as any liquid remains, the space will continue to be saturated. If, instead of saturated vapour of ether, a little air should be contained in the upper part of the tube, on increasing the volume the pressure would diminish and the mercury would necessarily rise. This will also be the case with the ether vapour as soon as all the liquid has evaporated; for, if the space for vapour be increased by withdrawing the tube beyond the point at which the liquid ether has disappeared, the column of mercury will be seen to rise, and it will be found that the tension of the vapour, now no longer saturated, will vary inversely as the volume. If the tube be again lowered deeper into the mercury, the tension of the unsaturated vapour will increase, according to Mariotte's law, with its density, the mercurial column will sink to its original height, and the saturated



FIG. 108. — Behaviour of Saturated Vapour.

state will be again reached. If the tube be pushed still deeper into the basin below, it will be seen that from now on the height of the mercurial column and, consequently, also the tension of the ether vapour (435 mm.), remains unchanged. At the same time liquid ether, in ever-increasing quantities, will be observed to collect above the surface of the mercury until, finally, the entire mass is reconverted into liquid. While, therefore, the unsaturated vapour follows Mariotte's law, comporting itself as a true gas in respect of changing volume and pressure, the saturated vapour does not conform to this law. Diminution of volume does not increase its tension, but merely condenses a corresponding portion of the vapour into liquid, while the remaining space continues to be filled with saturated vapour of constant tension. The tension of the vapour in a saturated condition (*tension*, or *pressure of saturation*) is accordingly the maximum tension attainable by the vapour under the prevailing temperature. The saturated vapour is consequently designated as that possessing, at the temperature considered,

*the greatest possible tension, or which is at its maximum tension.*



FIG. 109.—  
Vapour  
Barometer.

**113. Tension of Saturated Vapours.**—When a space containing saturated vapour, together with the liquid from which it was formed, is heated, a portion of the liquid vaporizes, and the space in question becomes saturated, for this higher temperature, with vapour of greater density and higher pressure. When the space is again cooled to its former temperature, the newly formed vapour is again precipitated in the condition of a liquid, the space remaining again saturated for the lower temperature with the former quantity of vapour. To every temperature a definite tension of vapour-saturation corresponds. To show this to be true for water vapour, for example, let a little of the liquid as above be brought into the Torricellian vacuum of the barometer (Fig. 109), where it will instantly evaporate partially and fill the space with saturated vapour. Let the barometric-tube be now enclosed within a larger tube containing the water, which is to be gradually

warmed from 0 to 100. With rising temperature the mercurial column sinks, until at 100 the mercury outside and inside the tube stand at the same height. The vapour tension for any temperature may be found by subtracting the height of the mercury-column in the tube containing the vapour from the simultaneously observed barometric height. The following table gives the tension of saturated water vapour to 100, expressed in terms of the height of the mercurial column (in mm.), which holds it in equilibrium.

Temperature, °C.	Tension, mm.	Temperature, °C.	Tension, mm.
-30 ... ..	0.4	40 ... ..	54.9
-25 ... ..	0.6	45 ... ..	71.4
-20 ... ..	0.9	50 ... ..	92.0
-15 ... ..	1.4	55 ... ..	117.5
-10 ... ..	2.1	60 ... ..	148.8
-5 ... ..	3.1	65 ... ..	186.9
0 ... ..	4.5	70 ... ..	233.1
5 ... ..	6.5	75 ... ..	288.5
10 ... ..	9.2	80 ... ..	354.6
15 ... ..	12.7	85 ... ..	433.0
20 ... ..	17.4	90 ... ..	525.5
25 ... ..	23.6	95 ... ..	633.8
30 ... ..	31.6	100 ... ..	760.0
35 ... ..	41.8		

As is seen from the table, water at the freezing point 0° furnishes vapour capable of depressing a mercurial column by 4.5 mm.

Water vapour is developed even from ice. To measure the tension for temperatures below the freezing point, the upper portion of the barometer should be surrounded by a suitable freezing mixture. At the boiling point of water (100°) the tension of saturated steam is the same as the pressure of the atmosphere, or it equals a pressure of *one atmosphere*, which is sufficient to sustain a column of mercury 760 mm. high. The mercury in the tube is depressed until the surface is on the same level as the surface of the mercury outside. With still higher heating the vapour would overcome atmospheric pressure and escape from the tube through the mercury. For temperatures beyond the boiling point, the process of determining the tension of vapour just outlined does not suffice. The apparatus of Fig. 110 may then be used.

A U-shaped tube, with one of its branches short and broad, and the other long and slender, is partially filled with mercury by leaving the end of the shorter tube open. The mercury rises to the



FIG. 110.—  
Vapour  
Manometer.

same height in both branches. A quantity of water is then poured in the upper end of the short branch, and boiled until all the air is expelled, whereupon the opening is quickly melted shut. At 100° the mercury stands at the same height in both branches (the longer being open at the upper end), because the saturated vapour at 100° holds in equilibrium the atmospheric pressure acting in the open branch. But if the temperature be raised higher than 100°, by immersing the lower portion of the apparatus in a hot bath of oil, for example, the mercury rises in the long branch. The latter forms an open manometer (84), and the elevated portion of the column gives the excess of the vapour tension over external atmospheric pressure.

Suppose the height of the mercurial column to be 760 mm., the tension of the vapour must then hold double the atmospheric pressure in equilibrium, or the tension amounts to two atmospheres, one of which is the normal atmospheric pressure, and the other the equal pressure due to the 760 mm. column of mercury. It is generally the custom, for the sake of clearness, to express these vapour-tensions directly in "atmospheres" (each equal to 760 mm. pressure). This is done in the following table, which gives the tension of saturated water vapour for higher temperatures :—

Temperature, °C.			Tension Atm.	Temperature, °C.			Tension, Atm.
100.0	...	...	1.0	161.5	...	...	6.5
111.7	...	...	1.5	165.3	...	...	7.0
120.6	...	...	2.0	168.2	...	...	7.5
127.8	...	...	2.5	170.8	...	...	8.0
133.9	...	...	3.0	175.8	...	...	9.0
139.2	...	...	3.5	180.8	...	...	10.0
144.0	...	...	4.0	213.0	...	...	20.0
148.3	...	...	4.5	236.2	...	...	30.0
152.2	...	...	5.0	252.5	...	...	40.0
155.9	...	...	5.5	265.0	...	...	50.0
159.2	...	...	6.0				

From this and the foregoing table it appears that the tension of saturated vapour rises at an increasing rate as the temperature rises, since not only the temperature, but, by continual vaporization, the density also increases. But to permit of the formation of vapour and of the consequent continual saturation of the space, it is necessary to keep a quantity of liquid present and in contact with the vapour. If all the liquid should become vaporized and the temperature should still rise, the vapour would expand proportionally to the increase of the temperature if the tension remain constant; or in case no expansion were allowed, the tension would increase at this same rate (Mariotte-Lav-Lussac's law). The space would not then contain all the vapour it is capable of holding at the given temperature, and would therefore not be saturated. Such an unsaturated vapour is said to be *super-heated*, because its temperature is higher than that of saturated vapour of the same tension.

When a soluble substance is added to a liquid, the tension of its saturated vapour diminishes proportionally to the quantity of the dissolved material. Materials brought into the same solvent in quantities proportional to their respective molecular weights, produce equal diminutions of pressure (Raoult, 1887).

This proposition, which, however, holds for only dilute solutions, may be used to determine molecular weights (cf. 109).

**114. Boiling, or Ebullition.**—The vaporization of a liquid at so rapid a rate as to be accompanied by the liberation of great quantities of vapour, in bubbles, throughout the heated mass is called *boiling, or ebullition*. In this process vapour is formed not alone at the surface, but the formation of vapour extends also into the interior of the liquid. Bubbles of vapour can form within the mass of a liquid only when the tension of the vapour they contain is able to hold in equilibrium the pressure upon the liquid at the place of formation. A liquid will therefore boil when it has reached the temperature at which the tension of its saturated vapour equals the external pressure. This temperature, which is also the boiling point, is accordingly dependent upon the external pressure, so that the smaller the external pressure the lower is the boiling point. The *normal boiling point* of water, which has been chosen as the

fixed point of the thermometer scale, and designated  $100^{\circ}$ , is the temperature at which saturated water vapour has a tension equal to the normal atmospheric pressure, or is the temperature at which it holds a mercurial column of 760 mm. height in equilibrium. If a thermometer should be constructed at a different atmospheric pressure, *e.g.* at a barometric height of 720 mm., since it may be found from a fuller table of tensions that water vapour reaches this tension at  $98.5^{\circ}$ , the fundamental distance thus obtained must be divided into not 100, but into 98.5 parts to obtain a correct scale. Upon high mountains, or plateaus, where the atmospheric pressure is less than at the sea-level, water boils at a lower temperature than  $100^{\circ}$ . Upon the top of Mont Blanc, for illustration, at a height of 4775 m. above the sea, where the barometric height is only 417 mm., water boils at  $84^{\circ}$ , *i.e.* at the temperature where the tension of water vapour is also 417 mm. If the boiling point of water in an open vessel be determined at an elevated station, and the corresponding tension be taken from a table, the barometric height prevailing there is known without observing the barometer, and from the atmospheric pressure so ascertained, the height above the sea of the place of observation may be computed (83). A thermometer constructed for this purpose, whose scale is graduated to very small subdivisions, and contains only a very few graduations below the boiling point, is called a *hypsometer*. Under the receiver of an air-pump, water may be made to boil at any temperature below the boiling point. In a glass flask half filled with water, let the water be brought to boiling, and allow all the air to be driven out by the escaping steam. Close the mouth now with an air-tight cork, and place the flask in a vertical position, mouth downward. Above the water, which is now cooled below the boiling point, the water vapour will exert a pressure corresponding to its temperature upon the liquid. If cold water be poured over the flask, the water within begins to boil vigorously, because the pressure of the vapour upon the liquid is suddenly diminished by cooling. Again, if by boiling, all the air be driven out of a glass tube which expands into a bulb at either end, the bulbs being partially

filled with alcohol, and the tube be then hermetically sealed, so that after cooling the enclosed liquid to the ordinary temperature it is exposed only to the low pressure of its vapour, the heat of the hand suffices to make the liquid boil (*pulse hammer*). A tube containing water and exhausted of air in this way is called a *water-hammer*, because on shaking the vessel the water strikes against the glass walls with a metallic ring.

In an open vessel a liquid can never be heated above its boiling point, which corresponds to the prevailing temperature at the instant, because as soon as the boiling has begun the heat applied from without is consumed, not in raising the temperature, but in converting the liquid into the gaseous state. In a closed vessel, on the contrary, since the vapour cannot escape, the tension of the vapour pressing against the liquid rises more and more with continued heating, and consequently, the boiling point must also rise. Under a pressure of two atmospheres, water boils at  $121^{\circ}$ ; under three atmospheres, at  $134^{\circ}$ , and so forth. Upon this principle the vapour porridge-pot (*Papin's Digester*, 1681) depends, the purpose of which is to heat the water and the food to be boiled to a higher temperature than is possible in an open vessel, thus producing quicker and more complete results. It consists of an iron pot with a close-fitting lid fastened by screws. The lid is provided with a safety valve and a tap for the reception of the articles to be cooked. The valve prevents the tension of the vapour from exceeding a certain limit. If the interior of such a vessel be connected with the receiver of an air-pump, by which the confined air may be condensed, or rarefied at will; and if the highest point reached by the mercury of a thermometer inserted into the interior be observed for each different pressure, the boiling point for this pressure is obtained, i.e. the temperature at which the tension of the vapour is equal to the pressure read on a manometer also inserted into the interior chamber of the vessel. In this way the tensions exceeding one atmosphere, given in the above table, were obtained.

When the boiling point of a liquid is given as characteristic of it, the temperature at which the tension of the saturated

vapour equals 760 mm. is meant. The boiling points of some liquids under this normal pressure are here added.

Nitrogen protoxide ...	-88° C.	Chloroform ..	61° C.
Carbonic acid ...	-78° "	Alcohol ...	78° "
Ammonium ...	-38° "	Benzol ...	81° "
Chlorine ...	-34° "	Water ...	100° "
Cyanide ...	-20° "	Oil of turpentine ...	159° "
Ether ...	35° "	Mercury ...	357° "
Carbonic disulphide ...	46° "	Sulphur ...	447° "
Sulphuric acid ...	-10° "	Zinc ...	1040° "

Substances dissolved in a liquid have higher boiling points the more concentrated their solution. Saturated solution of salt boils at 109°, calcium chloride solution at 179°. The vapour developed from the solution is, however, pure water vapour and has the temperature 100° (*cf.* 113, Diminution of vapour pressure).

While water in metallic vessels boils at 100°, it has been observed that a retardation frequently occurs with glass vessels, that is to say, the water frequently becomes heated higher than 100° before boiling, and the process then begins gradually. Such a retardation may be readily seen with water from which the absorbed air has been expelled by boiling, since in such water there are no rising air bubbles to favour the formation of vapour bubbles. Such retardation of the boiling point may be prevented by dropping a platinum wire, a few grains of sand, or other solid particles into the water. These liberate the particles of air collecting on their surfaces, and make ebullition easier. On account of the development of vapour which occurs at times very suddenly, retardation of ebullition may even lead to explosions.

**115. Leidenfrost's Phenomenon.**—If a small quantity of water be dropped upon a glowing metallic plate, a rounded droplet, which does not rest immediately upon the surface, is seen to form. A thin cushion of vapour forms beneath the droplet, which has sufficient strength to carry the droplet rapidly about over the heated surface until it gradually evaporates without boiling. When the source of heat is withdrawn and the plate cools a little, the weaker coating of vapour cannot longer support the droplet, which now comes in contact with the

heated metallic surface and evaporates suddenly and violently. This phenomenon is called, from its discoverer, "Leidenfrost's phenomenon," and the drops, "Leidenfrost's drops." All liquids are capable of exhibiting it, but the temperature must be higher, the more difficult the liquid is of vaporization, or the less easily the thin and poorly conducting layer of vapour forms which separates the droplet from the heated surface. Boiler explosions are many times caused by the water getting so low in the boiler that its walls become red-hot, and the water within, forming a huge Leidenfrost's drop, on further cooling is suddenly converted into steam (*i.e.* heated water vapour). This formation of steam many times occurs with such violence as to burst the boiler into fragments. The singular fact, that the moistened hand may be immersed with impunity in molten iron, is due likewise to the formation of a thin non-conducting layer of vapour around the hand, acting as a glove, to prevent the heated metal from coming in contact with the skin.

**116. Vaporization within an Air-filled Space.**—In a vacuum, such as the Torricellian, vaporization occurs suddenly, and the space becomes saturated almost instantly. But in a space filled with air, or other gas, the formation of vapour is a gradual process, although the same degree of saturation and hence the same tension is at length attained, as if no air or other gas were present; and this tension adds to the tension of the gas, or vapour, already present (Dalton's law, 96).

Let a little alcohol be poured into a flask into which, through a closely fitting cork, a manometer is inserted, *i.e.* a doubly bent glass tube whose lower bend contains mercury. The mercury in the open branch of the manometer will be seen to rise slowly, until at 20° it stands 44 mm. higher than the mercury within the inner branch. To the tension of the air enclosed in the flask, the pressure of the saturated vapour of alcohol at 20° is, therefore, to be added.

In still air vaporization goes on very slowly, because the atmospheric layer in direct contact with the liquid surface becomes saturated with vapour, which is transferred with extreme slowness to the adjoining layers (Diffusion, 96), and, consequently, the vaporizing process is delayed. A draught of

air, which rapidly carries away the saturated layer and replaces it with unsaturated air, very greatly accelerates the process.

117. *Heat of Vaporization.*—A closer study of the process of vaporization may be made by heating a quantity of water in a glass flask, into which, through a close-fitting cork, a thermometer and a tube are admitted, the latter terminating just below the cork and providing a means for the escape of the vapour. The thermometer will be seen to rise until boiling begins, but, as has been frequently stated, it will stand at a definite point so long as the boiling continues, *i.e.* at the boiling temperature corresponding to the instantaneous atmospheric pressure, and it is well to note that this temperature is indicated whether the bulb of the thermometer is immersed below the surface or is lifted entirely above it so as to be surrounded only by the vapour within the flask. The vapour just forming has, therefore, the same temperature as that already formed. The heat from the flame which is applied continuously to the contents of the vessel, produces then no rise in temperature. It does not act upon the thermometer. It maintains the boiling, however, by overcoming the cohesion of the molecules of the water in addition to the external pressure upon the liquid, thus transforming the water into the gaseous state. The heat required to effect this transformation is called the heat of vaporization, or, since it is lost both to perception and to thermometric detection, and seems to have become a constituent of the vapour, it is also called *bound*, or *latent heat*. To determine the amount of the heat of vaporization, the vapour is usually conducted through a worm-shaped metal tube enclosed within a cool vessel and surrounded by a carefully weighed mass of cold water at a known temperature. On passing through this coiled tube the vapour will be deposited in a liquid condition and at a temperature of 100°, at the same time surrendering to the surrounding water the heat consumed in its formation, and raising this water to a certain indicated temperature. From the weight of the water and the rise of temperature of the mass of cool water, it is easy to ascertain the amount of heat absorbed by this mass, and to find the weight of the vapour, which contains this quantity

of heat, by simply weighing the flask with its contents both before and after the experiment. It has in this way been found that 1 kg. of vapour at  $100^{\circ}$ , while condensing to 1 kg. of water at  $100^{\circ}$ , is capable of heating 10 kg. of water by  $53^{\circ}6$ , or, what is the same thing, 536 kg. of water by  $1^{\circ}$ , and that, therefore, 536 heat units, or calories, are required to convert 1 kg. of water at  $100^{\circ}$  into vapour at  $100^{\circ}$ . A familiar application of the capability of a vapour to give out so considerable a quantity of heat on condensation is made in the process of *steam-heating*. The heat of vaporization of alcohol is 208; of ether, 91; of oil of turpentine, 69 calories.

118. In **Distillation** the vapours of a boiling liquid are conducted into a cold chamber, where, after giving up their heat of vaporization, they are transformed into liquid. This process is used in the separation of a liquid from a mixture containing a more volatile substance or from other substances with which it is mixed. The separation is possible, since the volatile constituents are easily driven off, while those substances merely held mechanically in solution are either poorly, or not at all vaporized, and hence the latter remain behind, while the former may be led off into another vessel. From spring water and water of streams containing calcium carbonate, calcium chloride, etc., the water is obtained in a pure state by distillation. Freely falling rain water is distilled in Nature's laboratory, and, consequently, is pure water. Fig. 111 shows a

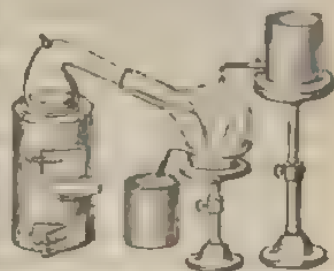


FIG. 111. — Simple Distilling Apparatus with Retort and Receiver

simple distilling apparatus. The mixture to be distilled is poured into the glass vessel with a curved neck, called the *retort*, and heated over a charcoal fire. A glass flask, into which the mouth of the retort opens, serves a double purpose, as receiver for the condensed vapour passing from the retort and as a cooling vessel. The latter purpose is subserved by causing a jet of cold water from above the receiver to play over its surface continuously. In distilling very volatile liquids the

vapour is passed through a tube surrounded by cold water, and drawn out quite long, so as to give as extended a cooling surface as possible. To economize space with large distilling apparatus, this tube is bent into the form of a spiral, and is called the *worm*. Fig. 112 illustrates the apparatus used in the manufacture of commercial alcohol. It consists of a copper boiler filled with the raw fermented liquid, the *mash*, and the tin worm coiled within the cooling vat. Many



FIG. 112.—Distilling Vessel with Worm.

solids are deposited at ordinary temperatures from their vapours in the cooling vat directly in their usual solid, crystalline condition. Substances which comport themselves thus are sal ammoniac, sulphur, iodine, etc. This process is called *sublimation*. Flowers of sulphur is

merely sublimated sulphur, which after subjection to process is rendered very pure.

**119. Heat of Evaporation.**—In the ordinary slow process of vaporization, called *evaporation*, heat is also consumed, or *bound*, or *rendered latent*, in separating the molecules of the liquid and overcoming the external pressure. To convert 1 kg. of water at  $0^{\circ}$  into 1 kg. of water vapour at  $0^{\circ}$ , 607 calories are required. If, therefore, no heat be applied from without, the necessary heat of vaporization must be extracted from the liquid itself, or from surrounding objects. These objects becoming cooled, the fact is made apparent that there exists here also a certain quantity of heat of vaporization, or, better, a *heat of evaporation*. The refreshing coolness following a thunderstorm on a hot summer day is not wholly, nor mainly, due to the fall of cool drops of water from higher altitudes. It is caused in much greater part by the rapid evaporation, and consequent *binding* of heat just after the rain. If one places himself in a draught of air while covered with perspiration, heat is rapidly withdrawn from his skin by reason of the rapid evaporation at its surface. Fanning cools one, not because the air blown into the face is cooler than that already lying against it, but because

the current of air produced accelerates evaporation. If a volatile liquid, *e.g.* ether, be poured upon the hand, a distinct sensation of cooling is felt, because the evaporation of the ether withdraws considerable heat from the surface of the hand. A thermometer whose bulb is wrapped in cotton saturated with ether, sinks in consequence of the evaporation of the ether by  $20^{\circ}$ . If a thin, flat disk containing ether be placed upon a few drops of water poured over a smooth surface, by blowing the air from the surface of the ether with a small bellows, the evaporation will take place so rapidly as to freeze the bottom of the disk to the surface. Some artificial methods of producing ice owe their efficiency to this property. In pipes surrounded by a liquid of low freezing point (*e.g.* by a solution of salt) ether, or liquid ammonium, is rapidly evaporated. Enough heat is thus withdrawn from the pipes to cool them far below the freezing point of water, and if these pipes are surrounded by other vessels containing water, the latter will of course be frozen.

By means of the apparatus of Fig. 113, water may be

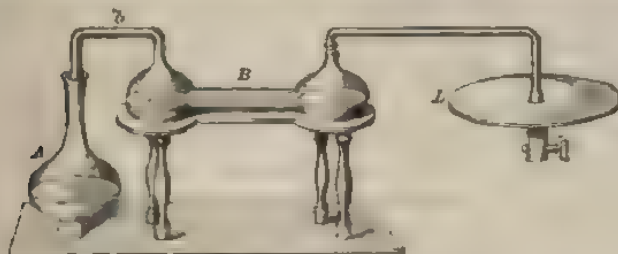


FIG. 113. Freezing Apparatus.

frozen by the cold produced in its own evaporation. From the flask, A, containing water at the ordinary temperature, a glass tube, *b*, leads by an air-tight connection into the vessel, B, consisting of two large glass globes connected by a wide glass tube. From the second globe a glass tube, *c*, leads to the air-pump, L. The vessel, B, is half filled with concentrated sulphuric acid, which absorbs the water vapour developed in A, and hence prevents the saturation of the space with vapour. If now the air-pump be put in action, in consequence of the rapid evaporation, the water cools below  $0^{\circ}$ , and finally under

a pressure of 4.5 mm. of mercury (the tension of saturated water vapour at 0°) the water at 0° begins to boil violently, and during the boiling it solidifies into ice, because the heat necessary for its vaporization at 0° is withdrawn from it. This heat manifests itself by raising the temperature of the sulphuric acid. The latter absorbs the water vapour, reducing it again to a liquid condition, which process, as we have learned, liberates a large quantity of heat (Leslie, 1813; Carré's ice-pump, 1867).

Water may also be frozen by the cold produced in its own evaporation with the aid of the *cryophorus* (Wollaston, 1813). This consists of a bent glass tube provided with a bulb at each end. The apparatus contains nothing but water and water vapour, the air having been expelled, as with the water-hammer, by boiling before closing the vessel. One bulb is then immersed in a freezing mixture of salt and ice. The water contained in it is condensed, and the tension of the vapour within the interior of the vessel so lowered that the water in the other bulb vaporizes rapidly, and is distilled over into the first. A sufficient quantity of heat is consumed in this process to convert the water into ice.

**120. Specific Gravity of a Vapour, or Vapour-Density,** is the number which expresses how many times heavier the vapour is than an equal volume of air at the same pressure and temperature. To obtain the density of a vapour, besides its weight, its pressure and temperature must be known. From these three magnitudes, by Mariotte-Gay-Lussac's law, the weight of an equal volume of air at the same pressure and temperature may be easily computed. This only needs to be divided by the weight of the vapour to obtain the specific gravity of the vapour referred to air as the unit.

To determine these magnitudes, a method first used by Gay-Lussac and improved by Hofmann (1869) may be advantageously employed. In the vacant portion of the tube of a barometer, graduated to cubic centimeters, a small flask with a ground-glass stopper is admitted. The flask contains a carefully weighed mass of the liquid to be vaporized. The barometer tube is surrounded by a wider tube, through which, from a small boiler, may be passed the vapours of a liquid (water,

or aniline) whose boiling point must be known. On heating, the liquid contained in the small flask drives the stopper out, and is transformed completely into superheated vapour, which assumes the temperature of the boiling point of the liquid. The weight of this vapour is known from the weight of the flask, its volume is read from the graduated barometer tube, its pressure is found from the difference between the barometric height and the height of the mercurial column still remaining in the graduated tube. Everything is known, therefore, which is needed for the comparison of the weight of the vapour with that of an equal volume of air at the same pressure and temperature.

For bodies not easily vaporized, Dumas (1826) used the following process. A glass globe drawn to a point, with a narrow opening, is weighed first when filled with air, and a small quantity of the material to be investigated is poured into it. The globe is now heated in a bath of water, oil, or a molten metal to a known temperature not exceeding the boiling point of the substance. The substance vaporizes, its vapour expels the air, and finally, the globe, after all the liquid is transformed into vapour, is filled with superheated vapour alone, whose pressure equals the external atmospheric pressure and can, therefore, be read from a barometer. The point is then hermetically sealed, and the globe, filled with vapour, is again weighed. When the point is broken off under water, the globe is forced full of water by the atmospheric pressure, and a third weighing gives its volume, for it will contain a number of cubic centimeters equal to the number of grams of water contained in it. The weight of the vapour is found by subtracting from the weight of the globe, filled with vapour, its weight after exhausting the air, *i.e.* the weight of the glass. The latter value is found by diminishing the weight of the globe, filled with air at the beginning, by the weight of the air contained in it, which may be readily computed. Weight, volume, pressure, and temperature of the vapour under investigation are therefore completely known.

According to the method of displacements of Victor Meyer (1879), the vessel, A, continuing upward in a tube, *a*, and closed with a stopper, *s* (Fig. 114), is brought to a temperature not

higher than the boiling point of the liquid to be investigated, by immersing A in the vapour of boiling water, or anilino, with which the enveloping vessel, V, is filled. As soon as the air

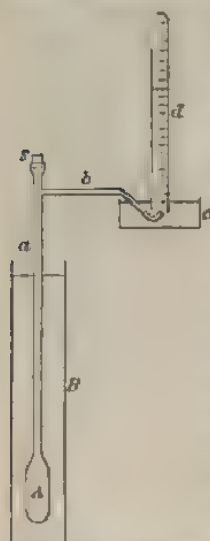


FIG. 111.—Determination of Vapour Density.

ceases to escape from the side-tube, *b*, toward the pneumatic trough, *c*, which is filled with water, the stopper, *s*, is opened, a small closed vial filled with a known weight of the liquid is thrown into the vessel, A, the stopper, *s*, is again closed, and the graduated tube, *b*, is filled with water, and placed above the mouth of the lateral tube, *b*. The vial opens of itself, and the superheated vapour of the liquid displaces a volume of air equal to its own volume, which is collected and measured in the tube, *d*. If now the temperature and pressure of the collected air are determined, all data needed to determine its weight and to compare it with that of the vapour are known.

In the following table, the specific gravities, or densities, of some gases and vapours are given :—

					Density referred to	
					Air = 1	Hydrogen = 1.
Hydrogen	...	...	...	...	0.0691	2.0
Oxygen	...	...	...	...	1.1056	32.0
Nitrogen	...	...	...	...	0.9713	28.0
Chlorine	...	...	...	...	2.4530	71.0
Hydrochloric acid	...	...	...	...	1.2470	36.5
Carbonic oxide	...	...	...	...	0.9670	28.0
Carbonic acid	...	...	...	...	1.5240	44.0
Sulphur	...	...	...	...	2.2110	64.0
Iodine	...	...	...	...	8.7100	254.0
Bromine	...	...	...	...	5.5400	160.0
Phosphorus	...	...	...	...	4.3680	124.0
Mercury	...	...	...	...	6.9760	200.0
Water	...	...	...	...	0.6220	18.0
Alcohol	...	...	...	...	1.6130	46.0
Ether	...	...	...	...	2.5650	74.0
Acetic acid	...	...	...	...	2.0800	60.0
Chloroform	...	...	...	...	4.2000	121.5
Benzol	...	...	...	...	2.7500	78.0

A comparison of these numbers shows that the densities of gases and vapours are as their molecular weights. If, therefore,

vapour-density and molecular weight are measured in terms of the same unit, putting the molecular weight of hydrogen, which was assumed equal to 2 (48), also equal to 2 km. (as is done in the third column of the foregoing table), *the vapour-density will then be equal to the molecular weight.* The determination of the vapour-density of a chemical compound leads directly to a knowledge of the molecular weight.

Since the densities, and, accordingly, also the weights of equal volumes of different gases are to each other as their molecular weights, the *law of Avogadro* (1811) holds: *Equal volumes of all gases, under the same pressure and temperature, contain the same number of molecules.*

**121. Moisture of the Atmosphere.**—Water vapour mixed with the atmosphere produces what is termed moisture of the atmosphere. In consequence of the incessant vaporization of water at the surfaces of the seas, lakes, etc., the air always contains water vapour in varying quantities. This vapour plays an important rôle in climatic change. For this reason, it is desirable to be able to form some idea of the quantity of vapour in the air at any given instant. The water vapour of the air exerts, by virtue of its tension, a pressure which must be added to the pressure of the air itself (Dalton's law). The mercurial column of a barometer, therefore, never gives the pressure of the air alone, but rather the sum of the pressures of the air and of the vapour. This vapour pressure, expressed in mm. of mercury, or the weight of water contained in a gaseous condition within a cubic meter of air (easily computed from the known density, i.e. 0.622 of water vapour), is called the *absolute humidity*. For purposes of weather prognostication, it is of less importance to know the absolute quantity of water vapour in the air, than to know whether or not the air is near its saturation point. In the first case, the air is said to be moist; in the second, dry. When the air, almost saturated with water vapour, is cooled slightly, a portion of its vapour will condense in the form of fog and clouds, or, when in contact with the skin, it produces a feeling of dampness. On the contrary, air containing less vapour than, by virtue of its temperature, it is capable of

absorbing before reaching its point of saturation, may be cooled more or less without precipitating its water in a liquid form. Comparing saturated air at  $20^{\circ}$  with saturated air at  $9^{\circ}$ , we find that the tension of the vapour contained in the former is 17.4 mm. (113), and in the latter, only 8.6 mm. Both masses are moist. If, however, the former mass of air contained (at the same temperature,  $20^{\circ}$ ) vapour of only 8.7 mm. tension, i.e. only half the quantity of vapour which it is capable of holding at this temperature, it must be considered dry, although, taken absolutely, it contains more moisture than the saturated, and, hence, moist air of  $9^{\circ}$ . The terms *relative humidity*, and *degree*, or, *ratio of saturation*, are applied to the ratio of the quantity of vapour actually present in the air to the greatest quantity which, under the prevailing temperature, the air is capable of holding before saturation. Relative humidity is ordinarily expressed in percentages. Air of  $20^{\circ}$  and 8.8 mm. vapour-tension has a relative humidity of 50 per cent., and saturated air of 100 per cent. To obtain the absolute as well as the relative humidity of the air, *hygrometers* and *psychrometers* are used.

If a flask, filled with cold water, be brought into a warm room, fine drops of water are seen to condense upon its outer surface. The air in the room contains water in a gaseous condition; but for the temperature of the room, it is not saturated. On contact with the cold wall of the vessel, the air immediately adjacent is cooled to the temperature at which the water vapour contained in it suffices to saturate it, and farther cooling, by even the slightest amount, deposits the water in a liquid form. The temperature at which water vapour begins to condense from unsaturated air is called the *dew point*. When the dew point has been determined, the amount of moisture in the air may also be obtained. Suppose that in air of  $20^{\circ}$ , deposition of moisture should begin to show itself at  $15^{\circ}$ , it is then known that at this temperature the air would be saturated by the vapour it contains. The tension of the vapour must, therefore, equal 12.7 mm. of mercury (113). But if the air were saturated at  $20^{\circ}$ , it would contain vapour of 17.4 mm. tension. The ratio of the quantity of vapour actually present, to that which the air is capable of absorbing at this temperature, i.e. its percentage

of saturation, or its relative humidity, is therefore 12.7 : 17.4, or 73 : 100. The air contains, therefore, 73 per cent. of its possible content of water vapour. To find the dew point, Daniell's hygrometer, shown in Fig. 115, may be used. The apparatus is sometimes called a condensing hygrometer. A wide glass tube is bent twice, and the ends of the vertical branches, one of which is shorter than the other, are supplied with hollow bulbs. The tube is exhausted of air, and the bulb of the longer branch is filled with the volatile liquid ether, whose vapour expands and fills the entire tube. A thermometer dips into the ether, while a second thermometer is attached to the standard of the apparatus, from which the temperature of the air may be read. The other bulb is covered with a muslin wrapper. Ether is now dropped upon the wrapper and its rapid evaporation renders latent a considerable quantity of heat, thereby cooling the bulb. The tension of the ether vapour contained in the bulb and tube is so far reduced in this way that the ether contained in the first bulb begins rapidly to evaporate, and by virtue of the consequent consumption of heat, the bulb cools. Care must now be exercised to determine at what temperature, by the inner thermometer, the vapour begins to condense on the bulb. To make the delicate, breath-like deposit distinctly perceptible, a narrow zone is gilded around the bulb. The temperature of the dew point is thus ascertained, and from it and the data of the external thermometer the moisture contained in the air may be obtained, as in the preceding example. Many bodies of the animal and vegetable kingdoms, especially such as are of a fibrous structure, such as hairs, fish-bone, catgut, bristles, etc., possess the property of absorbing the water vapour in the air, at the same time exhibiting an increase in their lengths. In dry air they lose the moisture they have absorbed, and contract again. Upon this property of such bodies, Saussure's (1783) *hair hygrometer* (Fig. 116) is based. A human hair, freed from oil, is stretched by a light



FIG. 115 — Daniell's Hygrometer

weight, so as to transmit the changes of its length due to the variations of the moisture it absorbs, to a roller provided with a pointer, which is easily movable along a graduated arc.



FIG. 116.—  
Saussure's Hair  
Hygrometer.

the instrument is placed under an inverted glass vessel, filled with dry air, the pointer takes a position corresponding to perfect dryness, and this point is designated  $0^{\circ}$ . The point over which the index stands when the instrument is placed in air saturated with water vapour, which air has a relative humidity of 100 per cent., is designated by 100. The interval between these two points is divided into 100 equal parts, called *degrees of humidity*. The data of the instrument are, however, very far from agreement with those of an apparatus showing relative humidity. Before a hair hygrometer can be used for measurements, the value of the degree must be determined experimentally for each particular instrument.

*Hygrosopes* depend upon these same principles. These instruments are widely distributed among the common people, and, as weather-prophets, are highly esteemed by them. The grotesque little figure draws on his mantle with approaching rain, and betakes himself into his hut, his motions being produced by the changing length due to moisture of a string of catgut. A pine twig, deprived of its bark, with its stiffer end fastened to a wall, shows, likewise, by its varying curvature, the varying degrees of atmospheric moisture. The spirally curved bristles of many species of geraniums, which roll up in moist weather, may also serve as hygrosopes.

By means of the *psychrometer* of August, 1829, the moisture contained in the air is determined by observing the fall of temperature due to evaporation. It consists of two thermometers fixed to a common standard with scales reading to tenths of a degree. One of the thermometers indicates the temperature of the air. The bulb of the other is covered with a bit of muslin, which is kept continually moist by means of a wick dipping into a vessel of water. By the evaporation of

the water from the muslin wrapper, heat is consumed, and the temperature of the thermometer falls. The moistened thermometer indicates, therefore, a lower degree than the dry thermometer, and the difference of their indications increases the more rapidly the evaporation takes place, *i.e.* the dryer the surrounding air. The difference of the readings of the dry and the moist thermometers is, therefore, connected with the degree of humidity of the air, so that the latter may be computed from this difference. By means of the psychrometer the absolute humidity, *i.e.* the vapour pressure expressed in mm. of mercury, as also the relative humidity, may be obtained.

**122. Liquefaction of Gases.**—Unsaturated vapours comport themselves under changes of pressure and temperature precisely as air. They follow Mariotte-Gay-Lussac's law. Their condition is, therefore, not essentially different from gases, ordinarily so-called. Gases are in fact merely unsaturated, or superheated, vapours, which are still far from their point of saturation—vapours which have risen from liquids whose boiling points are very low. Gases may, then, be condensed into saturated vapours by cooling and compression, and by still farther cooling and compression, into liquids, just as is the case with unsaturated vapours. If, for example, sulphuric acid, the biting, bad-smelling gas developed by burning sulphur, be cooled by means of a mixture of snow and salt, it will condense at first into a colourless liquid which boils at  $10^{\circ}$  below zero (114, table). With gases more readily condensed, Oersted's compressing apparatus may be used. This apparatus consists of a strong glass cylinder, *cc* (Fig. 117), into which water may be forced from a reservoir at the side by means of a condensing pump, *d*. At the bottom of the glass cylinders, a vessel filled with



FIG. 117.—Compressing Apparatus.

mercury is placed, into which dip tubes, open below and closed above, and which, under the pressure ordinarily prevailing, are filled with gas to the surface of the mercury outside. If water is pumped into the cylinder the mercury rises in the tubes and the gases are compressed in accordance with Mariotte's law. This is seen by filling a tube with ordinary air and dipping it into the vessel of mercury. The latter tube should be so arranged that it may be used as a closed manometer to measure the pressure prevailing at any moment within the cylinder. When the gas approaches its point of saturation its volume diminishes more rapidly than does that of air. The mercury will be seen to rise more rapidly in the tube containing this gas, and above its surface the liquid will appear. Thus at 0°, cyanogen and sulphuric acid are converted into liquids by a pressure of three atmospheres, chlorine by four, and ammonium by six and one-half. More difficultly condensable gases are liquefied by compressing them in a strong iron flask provided with a suitable set of valves (Natterer's *compressing apparatus*) and at the same time reducing their temperatures. Carbonic acid (carbon dioxide) is liquefied in this way by a pressure of 38, and nitrous oxide by a pressure of 50 atmospheres. Liquid carbonic acid is a thin colourless liquid of specific gravity 0.86 at 15°. On heating it expands more rapidly than air.

By evaporation of the liquids thus obtained, very low temperatures may be produced, in consequence of the large quantities of heat rendered latent in the vaporizing process.

If liquid carbonic acid be allowed to flow out of the iron flask in which it is kept, into a cloth purse, the rapid evaporation of a portion of it will produce a temperature so low that the rest of the mass will be stiffened into a crystalline condition, resembling snow. Notwithstanding the low temperature of this carbonic acid snow (its temperature is  $-79^{\circ}$ ), it may be taken into the hand without injury, because a layer of vapour is instantly formed, which prevents the substance from coming into direct contact with the skin. On pressing the frozen acid, however, a burning sensation is felt. Solid carbonic acid dissolves in ether, and the solution is a liquid of  $-90^{\circ}$  C., whose temperature under the air-pump sinks to  $-110^{\circ}$  C. Mercury freezes in

this solution. Liquid nitrogen protoxide solidifies by its own evaporation to a mass whose melting point is  $-105^{\circ}$  C.

Liquid sulphuric acid in a glowing platinum dish forms a Leidenfrost's drop. When water is added it evaporates with so great violence as to freeze in the glowing dish. By a mixture of ether and solid carbonic acid mercury may also be frozen in a glowing dish.

While most known gases were long ago reduced by pressure, and lowered temperature to a liquid condition, a few, viz. hydrogen, oxygen, nitrogen, and the air, which is merely a mixture of the latter two, also carbonic oxide and nitric oxide, have until very recently successfully resisted all attempts to liquefy them. They have, therefore, received the designation of *permanent gases* in contradistinction to the others, which are called *coercible gases*. Callodon subjected these gases at a temperature of  $-30^{\circ}$  C. to a pressure of 400 atmospheres, and Natterer to a pressure of 3000 atmospheres, without liquefying them. In 1869 Andrews showed that for every vapour there is a so-called *critical temperature*, above which the vapour remains gaseous, no matter how high the pressure, since a maximum of tension, or a condition of saturation is not attainable.

For ether vapour, the critical temperature is  $196^{\circ}$ , for carbonic acid  $31^{\circ}$ , for etheline  $9^{\circ}$ , and for the so-called permanent gases it lies far below  $0^{\circ}$  (oxygen,  $-118^{\circ}$ ; carbonic oxide,  $-140^{\circ}$ ; nitrogen,  $-145^{\circ}$ ; hydrogen,  $-174^{\circ}$ ). With both Callodon's and Natterer's experiments, the temperature was still above the *critical temperature*. In order that liquefaction may occur, it is necessary to have the most intense cold operating in conjunction with extremely high pressure. By satisfying these conditions, Cailletet in Paris, and Pictet in Geneva, succeeded at almost the same time, near the end of 1877, in liquefying the hitherto so-called permanent gases. Cailletet compressed the gas in a narrow, thick-walled glass tube by means of an hydraulic press. Oxygen, cooled by the aid of sulphuric acid to  $-29^{\circ}$  C., remained gaseous under a pressure of 300 atmospheres. A tap was at this juncture suddenly opened, through which a portion of the gas escaped into the air. For the work which was performed by the suddenly expanding gas, so much heat was consumed, as to

depress the temperature somewhat more than  $200^{\circ}$  lower still. Simultaneously with this sudden expansion, a foggy appearance was observed within the tube, which consisted of fine droplets, or bubbles, of liquid oxygen. Similar phenomena were exhibited by nitrogen, carbonic oxide, air, and even by hydrogen. But while Cailliet, by compressing these gases, and allowing them suddenly to expand, thereby lowering their temperatures, merely succeeded in producing the hazy clouds of liquid, Pictet succeeded by high pressures, and extremely low temperatures, in obtaining large quantities of liquid oxygen and hydrogen. His process may be best explained by a sketch (Fig. 118). The oxygen is developed from potassium chlorate heated

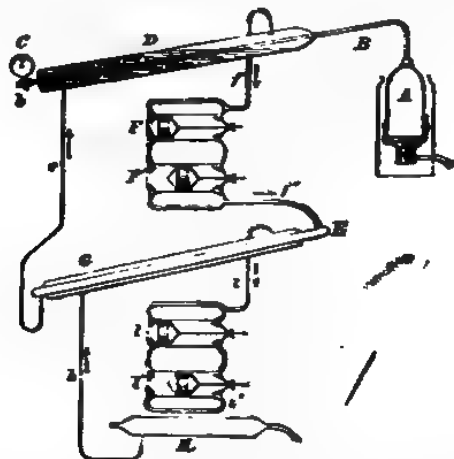


FIG. 118.—Pictet's Apparatus for Liquefying the Permanent Gases.

in a strong iron vessel, A. To the iron vessel a thick-walled copper tube, B, 3.7 m. long, is screwed. The tube carries at C a manometer, for reading the tension in the tube, and is closed at *b* by a screw tap. In this tube the gas is compressed by its own pressure, rising with the continued liberation of gas from the potassium chlorate. The tube, B, is surrounded by a wider tube, D, containing liquid carbonic acid (or nitrogen protoxide), which by means of the communicating pumps, F and F', is kept in continual circulation in the direction of the arrows between the tube-shaped receptacle, E, and the tube, D, by way of the

narrow tube, *eff*. By aid of the pumps, so rapid an evaporation of the carbonic acid is effected, that the temperature, in consequence of the enormous heat-consumption, sinks to  $-130^{\circ}\text{C}$ . To obtain a quantity (2 kg.) of carbonic acid in the form of a liquid, the receptacle, K, is surrounded by a tube, G, in which liquid sulphuric acid, arriving from the vessel, H, through the tube, *h*, kept in continual circulation by the pumps, *l* and *p*, is rapidly evaporated and lowers the temperature to  $-60^{\circ}$ . The vessel, H, constructed like a tubular boiler, is kept cool by a current of cold water. After the apparatus is in operation, the pressure of the oxygen rises in the tube, cooled to  $-130^{\circ}\text{C}$ ., to 525 atmospheres, sinks then again and remains at 470 atmospheres. This depression and the final constancy of the pressure indicate that a part of the gas is liquefied. If the tap be now opened a stream of liquid escapes with great violence, of which electrical illumination renders two parts distinguishable, an inner, transparent one, and an outer one, blindingly white. The latter is composed of dust of frozen oxygen, since a portion of the liquid is solidified by the cold of evaporation produced by the extremely vigorous vaporization. Pictet succeeded in determining the specific gravity of the liquid oxygen. It was found to equal 0.9787. Hydrogen was liquefied at a pressure of 650 atmospheres and a temperature of  $-140^{\circ}$ , which is produced by using liquid nitrogen protoxide instead of carbonic acid. On opening the tap an opaque stream of liquid of steel-blue colour escaped, at the same time the solidified hydrogen upon the floor produced a rattling sound as of falling shot.

The critical temperature at which a liquid under any pressure passes into a gaseous condition is also called, in accordance with Mendelejeff's suggestion, the *absolute boiling point*. The ideas of "vapour" and "gas" may now be more closely discriminated by considering the critical point, and by calling any gaseous body below the critical point "vapour," and above it "gas." According to this definition, a vapour may be converted into a liquid by pressure alone, while a gas must, at the same time, be reduced in temperature.

123. Graphical Representation of the Behaviour of Gases and Vapours. — The law of Mariotte-Gay-Lussac, followed by gases, may be graphically

illustrated as follows. Conceive the gas in a horizontal cylindrical tube, hermetically sealed at O (Fig. 119), and confined at the other end by a movable piston, K. If now the piston is pushed toward O to any desired point, as V, the distance, or abscissa, OV =  $v$ , represents the volume  $v$  of the gas. At the definite absolute temperature, T, the corresponding pressure,  $p$ , of the gas is found from the equation  $pv = RT$  (105). Erect now upon OV at V the perpendicular ordinate VP, and make it equal to  $p$ , viz. equal to the height of the corresponding mercurial column. Proceed in like manner for all other positions of the piston, keeping the value of T the same. Then, for all possible values of the volume  $v$ , the tops of the ordinates will form a continuous curve, NPM (an equilateral hyperbola), which, rising from the right toward the left, will represent the increase of pressure with the decrease of volume. Repeating this construction for other temperatures, a number of such curves are obtained, covering the entire plane of the drawing, and, because each corresponds to the same temperature, they are called "*isothermals*."

Imagine the tube to be filled now with unsaturated vapour, and that, on

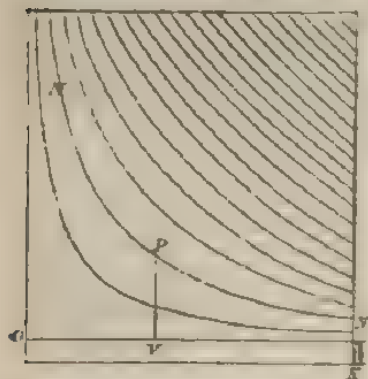


FIG. 119.—Behaviour of Gases.

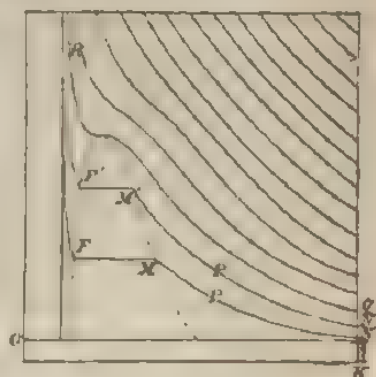


FIG. 120.—Behaviour of Vapours.

forcing the piston inward, the pressure rises first along the curve, NPM (Fig. 120), until, at M, the maximum tension, or the condition of saturation, is reached. From here on the diminution of volume is not accompanied by a rise in pressure, but a liquefaction of a portion of the vapour occurs. The curve, therefore, from M to F, is a horizontal straight line, until, at F, all vapour is transformed into liquid. From here the volume does not appreciably change, notwithstanding a very strong rise in pressure, and the line of pressure rises very steeply. With a higher temperature, saturation is reached at a higher pressure and a smaller volume, as is illustrated by the curve, NPM F'. At the critical temperature, the pressure is represented by the curve, QR, in which the horizontal portion is no longer apparent, whence it follows that a maximum of pressure, or the saturation of the vapour, does not again occur. With still higher temperatures, the trend of the isothermal is the same as with perfect gases. The curve, QR, corresponding to the critical temperature in the plane of the drawing, forms the limit, or boundary, of the vaporous region below and the gaseous region above.

**124. Specific Heat** is the name applied to the quantity of heat required to raise the temperature of 1 kg. of the body by  $1^{\circ}\text{C}$ . The amount of heat needed to raise a body's temperature by  $1^{\circ}\text{C}$ . equals, therefore, the product of its weight into its specific heat, and is called its *thermal capacity*. Experience teaches that equal masses of different materials require very different amounts of heat to produce equal rises of temperature. For example, to raise 1 kg. of water and 1 kg. of mercury from  $0^{\circ}$  to  $100^{\circ}$ , it is found that, with the same rate of applying heat to the two liquids, mercury reaches the desired temperature more quickly than water. If 1 liter of each liquid is used and, therefore, 13.6 times as much mercury as water by weight, the desired temperature will be reached by mercury with the flame of one Bunsen burner more quickly than by the water with two such flames. When a body cools to its original temperature, it gives up to surrounding objects the heat which it consumed while its temperature was rising. By observing the rise in temperature of these objects it is then possible to determine the total quantity of heat expended in raising the temperature. All methods of determining the specific heat of bodies depend upon the amount of heat given up by the body in cooling. If three equally heavy balls, one of copper, one of tin, and a third of lead, be heated in boiling water to  $100^{\circ}$ , and laid upon a disk of wax, the copper ball falls quickly through the hole melted by it; the tin ball sinks deeply within the wax, while the lead ball sinks into it but slightly. From this it appears that copper has emitted the most heat, and, consequently, has the greatest specific heat, while lead has the lowest. By this experiment it is, however, impossible to obtain an accurate estimate of the ratio of the specific heats of these substances. To do this, it is necessary to measure the quantities of heat given off, and to express them in heat units. The thermal unit, the *calorie*, has already been selected and defined. As was stated in (109), it is the amount of heat required to raise 1 kg. of water  $1^{\circ}\text{C}$ ., or, what is the same thing, the specific heat of water is taken as the unit. Instruments for measuring specific heat are called *calorimeters*. To determine the specific heat of a body by the

melting process, the ice calorimeter (Fig. 121) of Lavoisier and Laplace may be used. It consists of three metallic vessels,

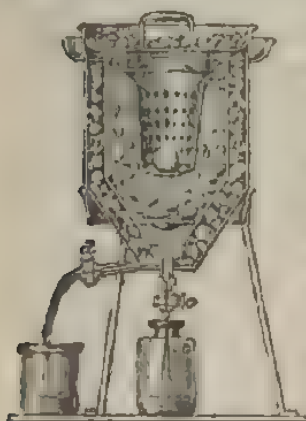


FIG. 121.—Ice Calorimeter of Lavoisier and Laplace

each enveloped by the next larger, the innermost of which, *c*, is perforated like a sieve, or it may be replaced by a wire basket. The intervening space, *aa*, between the middle and outermost vessels, as also the hollow lid of the former, are filled with lumps of ice, which serve to keep the temperature of the space, *bb*, between the innermost and middle vessel, which is likewise filled with ice, at a uniform temperature. The water produced within the space, *aa*, by the external heat, flows off through the tap, *d*. If, now, a body of known weight

and temperature, *e.g.* an iron sphere heated to  $100^{\circ}$  in the vapour of boiling water, be placed in the innermost vessel, while it cools from this temperature to  $0^{\circ}$ , the body will melt a

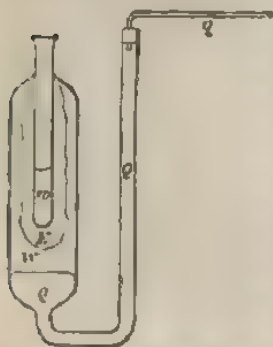


FIG. 122.—Ice Calorimeter of Bunsen.

definite quantity of ice which may be determined by weighing the water passing out through the tap, *a*. Since it is known that to melt 1 kg. of ice, 80 thermal units are required, it is easy to compute the quantity of heat given off by the body while cooling. From this the quantity of heat per kg. which the body contains at  $1^{\circ} \text{C.}$ , is readily found, and this is its specific heat. The method of Black (1772) is much simpler. In this process, the heated body is placed in a cavity hollowed out of a plane block of ice, the hollow being then covered with

a plate of ice. After the body has cooled to  $0^{\circ}$ , the water from the melting ice is soaked up by means of a sponge and weighed. The more precise ice calorimeter of Bunsen (Fig. 122) depends

on the fact that the melting of ice is accompanied by a contraction, *i.e.* the water arising from the melting ice occupies a smaller space than does the ice itself. The test-tube, *w*, is fused into the wide glass vessel, *W*, which is continued downward into the U-tube, *QQ*. The vessel, *W*, is filled with water free from air, which is confined by the mercury, *QQ*, in the lower part of *W* and in the tube. Allowing very cold alcohol to flow through the test-tube, *w*, it will become coated with an envelope of ice. If, now, a body, heated to a known temperature, is thrown into the test-tube containing a little water at  $0^{\circ}$ , ice will be melted, and, in consequence of the resulting contraction, mercury will rise in the vessel, *W*, and within the narrow tube, which is inserted by means of a cork into *Q*, the mercury withdraws toward *Q*. The amount of its displacement gives the quantity of water produced by melting, and, accordingly, also the quantity of heat given up by the body to the ice.

If 1 kg. of water at  $10^{\circ}$  is mixed with 1 kg. of water at  $50^{\circ}$  and no heat is lost, the mean temperature of the mixture will be  $30^{\circ}$ . The kg. of water which cooled from  $50^{\circ}$  to  $30^{\circ}$ , gave off the 20 heat units required to heat the other kg. from  $10^{\circ}$  to  $30^{\circ}$ . If, on the other hand, 1 kg. of water at  $10^{\circ}$  is mixed with 1 kg. of oil of turpentine at  $60^{\circ}$ , the mixture will show only  $24^{\circ}$ . To furnish the 14 heat units required to heat 1 kg. of water from  $10^{\circ}$  to  $24^{\circ}$ , the temperature of the kg. of oil of turpentine must therefore fall  $36^{\circ}$ . Conversely, these 14 thermal units will also suffice to heat 1 kg. of oil of turpentine by  $36^{\circ}$ . To heat 1 kg. of oil of turpentine by  $1^{\circ}$ , therefore,  $\frac{1}{36}$  or 0.4 thermal units are required, or the specific heat of oil of turpentine is 0.4. To study this mixing process with more definiteness, Regnault (1840) used the apparatus represented in Fig. 123. The upper part is formed of three enveloping metallic cylinders, the innermost, *A*, being closed above by a cork through which a thermometer is inserted, and below by a metallic lid, which may be easily removed. In the middle of *A* a hollow cylindrical wire basket is suspended by a thread passing through the cork. This little basket is for the reception of the body to be investigated, the latter being either broken

into fragments, or hermetically sealed in thin-walled glass tubes. Within the hollow of this vessel, the thermometer is

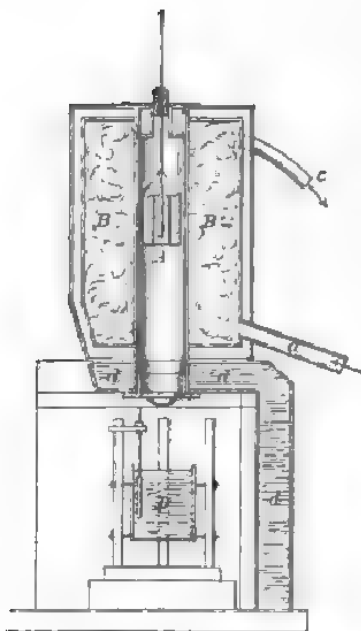


FIG. 123.—Water Calorimeter of Regnault.

enclosed. Steam is now admitted into the space, B, from a boiler at the side. The vapour heats the body to  $100^{\circ}$ , and passes outward through *c*. When this temperature has been reached, the wire basket is let downward after the removal of the lower lid, into the water-calorimeter, D, which is filled with a known quantity of water. The temperature of the mixture is now observed, and from it the amount of heat transferred from the body to the water, and accordingly also the specific heat of the body may be readily derived. By means of a double metallic wall, *d*, filled with water, the calorimeter, D, is protected against the heat from the boiler and from the vapour-chamber, BB.

If *m* denotes the weight of water in the calorimeter, *t* its temperature, *M* the weight of the body, *T* the body's temperature, *c* its specific heat, and *s* the resulting temperature of the mixture, the body gives off a quantity of heat represented by  $Mc(T - s)$ , and the water absorbs a quantity of heat represented by  $m(s - t)$ . Both quantities must be equal, therefore  $M(T - s) = m(s - t)$ , whence there results—

$$c = \frac{m(s - t)}{M(T - s)}.$$

In accurate determinations it must be borne in mind that the calorimeter itself, and also the immersed thermometer absorb heat, and that during the experiment heat is radiated to surrounding objects. These circumstances may be readily considered in the computation. If water is mixed with water,  $c = 1$  in the foregoing equation, and we have  $M(T - s) = m(s - t)$ , i.e. variations of temperature are inversely as the corresponding weights. It results from this, that when two masses of water, or of any homogeneous substances are mixed, the temperature of the mixture—

$$s = \frac{MT + mt}{M + m}$$

This equation is called *Richmann's rule*.

A third method of determining specific heat is that known as the *method of cooling*, used by Dulong and Petit. It is based upon the principle that a heated body in a vacuum where it can cool only by radiation, other circumstances being the same, cools the more slowly the greater the quantity of heat it contains. With equal fall of temperature the quantities of heat given off by different bodies will accordingly bear the same ratio to each other as their times of cooling. The specific heat of a body increases with rising temperature, and continually approaches a fixed limit. Between 0° and 100°, however, the change is so small that within this interval the specific heat may be regarded as invariable.

The specific heats of some solid substances are—

Aluminium ... ..	0.214	Tin ... ..	0.056
Sulphur ... ..	0.203	Iodine ... ..	0.054
Iron ... ..	0.114	Antimony ... ..	0.051
Copper ... ..	0.095	Mercury ... ..	0.033
Zinc ... ..	0.095	Platinum ... ..	0.032
Silver ... ..	0.057	Lead ... ..	0.031

and those of some liquids—

Alcohol ... ..	0.566	Benzine ... ..	0.392
Glycerine ... ..	0.555	Chloroform ... ..	0.233

Of all bodies, therefore, water has the greatest specific heat, viz. 1. The specific heat of ice is only 0.505.

Dulong and Petit found (1819), by comparing the above numbers, the important law that the specific heats of solid chemical elements are related to each other inversely as their atomic weights, so that the product of the atomic weight and the specific heat for all these bodies is the same, and equals almost 6.4. This Dulong-Petit law may also be expressed as follows: *the quantities of the solid elements expressed by their atomic weights require equal amounts of heat for equal elevations of temperature, or, the atomic heats of the solid elements are equal.* Neumann (1831) proved also that the specific heats of chemical compounds of similar composition are in the inverse ratio of

their atomic weights, and Kopp proved the proposition that the molecular heat of a compound equals the sum of the atomic heats of its constituent elements.

To determine the specific heat of gases, Regnault caused the gas to flow in a uniform current, first through a coiled tube surrounded by heated oil, within which the temperature was raised to  $t$ , and then through the coiled tube of a vessel filled with cooler water, where the temperature fell to  $t'$ . In this experiment the quantity of heat,  $mc(t - t')$ , was transferred to the cool water every minute, where  $m$  denotes the weight of the mass of gas passing every minute, and  $c$ , its specific heat. If the cool water, whose mass is  $M$ , attains a constant excess of temperature above its surroundings, the number of degrees ( $T$ ) by which the water cools per minute is observed after the current of gas has stopped flowing. Obviously, as before, while the stream of gas was flowing, the quantity of heat,  $MT$ , escaping, is replaced by the quantity of heat  $mc(t - t')$  given off in the same time by the gas. Consequently,  $mc(t - t')$  equals  $MT$ , from which equation the specific heat of the gas results. In this way the following values were found:—

Hydrogen	...	...	...	...	...	3.4090
Air	...	...	...	...	...	0.2375
Carbonic oxide	...	...	...	...	...	0.2425
Oxygen	...	...	...	...	...	0.2175
Chlorine	...	...	...	...	...	0.1214
Nitrogen	...	...	...	...	...	0.2438

If these numbers are multiplied by the specific gravities (the weights of equal volumes) of the respective gases, almost the same product is obtained for all, i.e. equal volumes of different gases require equal quantities of heat for equal elevations of temperature. Since the specific gravities of gaseous bodies are to each other as their molecular weights (Avogadro's law, 120) it is also true that the molecular heats of all (perfect) gases are equal.

In the mode of experimentation just described, the heated gas is in equilibrium with the external pressure of the air, since it is compelled to expand against this pressure. To perform the required work, a portion of the applied external heat is

consumed, as was the case with vaporization. To heat the mass of gas in a rigid vessel of fixed volume to the same temperature, less heat would be required, since no expansion could then occur and, therefore, no work is to be performed. With gases, two different specific heats are distinguished, viz. a greater specific heat with constant pressure and a smaller specific heat with constant volume. With solids and liquids, this distinction need not be made, because their expansion being small, the work of overcoming the external pressure is inconsiderable. But with gases this consumption of heat becomes perceptible, since, for example, a gas which expands while overcoming a pressure, cools when it receives no heat from without, by reason of the fact that it must furnish from its own thermal supply heat enough to perform the work of expansion. If the air under the receiver of an air-pump is saturated with water vapour, at the first stroke of the piston a fog is produced by the cooling, and a thermometer connected with the receiver (*e.g.* Breguet's metal thermometer, Fig. 103) sinks. Conversely, heat must be developed on compressing a gas. The so-called pneumatic tinder-box depends upon this principle. If a piston be pushed with sufficient rapidity into a cylinder containing air, the compressed air becomes so highly heated as to kindle a bit of tinder attached to the piston.

The direct determination of the specific heat with constant volume is not possible, since the weight of the gas contained in the rigid shell is altogether too small, relatively to the weight of the shell itself. The ratio of the two specific heats  $c$  and  $c'$  may, however, be found by ascertaining the small rise of temperature,  $\delta$ , of a mass of air, when it is suddenly compressed by as much as it would have expanded under constant pressure, if the temperature had risen  $1^\circ$ . The quantity of heat,  $c$ , required for the latter purpose must then suffice to heat this mass of air, if its volume is constant, by  $1 + \delta$ , i.e. if  $c'$  denote the specific heat with constant volume, we must have,  $c = c'(1 + \delta)$ , or  $\frac{c}{c'} = 1 + \delta$ . By a very ingenious method, Clement and Desormes (1819) executed some experiments such as that described. For air, the best experiments gave the ratio  $\frac{c}{c'} = 1.41$ ; for other gases the value was almost the same. From this the specific heat of air with constant volume is—

$$c' = \frac{c}{1.41} = \frac{0.2375}{1.41} = 0.1684.$$

**125. Conduction of Heat.**—If a metallic wire is held in the flame of a candle, the heat passing along through the wire from its heated end soon becomes manifest at the other end, so that it cannot long be held with the fingers. A wooden rod of the same length, however, may be set on fire at one end and burnt almost to the fingers without perceptible heating of the part held. A silver spoon dipped into hot soup quickly becomes hot at the handle, while a wooden one, under the same circumstances, heats very slowly and very slightly. This transmission of heat in bodies from warmer to cooler portions within the mass of the body is called *conduction*. From the examples already adduced, it is readily seen that the conductivity of different substances is very different. Of all bodies, the metals are the best conductors. Wood, ashes, straw, silk, feathers, hair, wool, etc., and loose, porous substances of the animal and vegetable kingdoms are, in general, the poorest conductors. Stone, glass, and porcelain conduct heat somewhat better than do the latter substances. The conductivity of different metals is also very different, as may be easily shown by the following experiment. Let a copper bar, together with an iron bar of the same cross-section, be placed in horizontal positions with ends touching. Upon their lower sides, at equal distances from the place of contact of the bars, let a number of small wooden balls be stuck with wax. If the ends in contact are now warmed the heat will be transmitted through the copper bar the more rapidly, and with equal heating more balls will fall from it than from the iron bar.

If a metal rod (Fig. 124) is heated at one end and thermometers are placed upon it at equal distances apart, being held in place by slight depressions in the rod, it will be observed that after a little time each thermometer will attain a fixed height, and, consequently, that a state of equilibrium of temperature along the rod will be reached. By this condition of equilibrium, it is merely meant that just as much heat is received from the source by any cross-section of the rod, as this section transmits to the other side. This emission from any given cross-section, however, is not confined to the flow of heat within the rod toward the cooler end, but it also includes

the flow through the surface of the bar to the colder surroundings, in consequence of which, heat is continually lost to the bar. It is, therefore, desirable to distinguish an internal and an external heat-conductivity. By *internal conductivity* is meant the quantity of heat passing through a cube of the substance of 1 cm. edge during one minute, when two opposite faces have a difference of temperature of  $1^{\circ}\text{C}.$ , and the remaining surfaces are regarded as impermeable to heat. By *external conductivity*, however, is meant the quantity of heat given off by a body one degree higher in temperature than that of its environ-

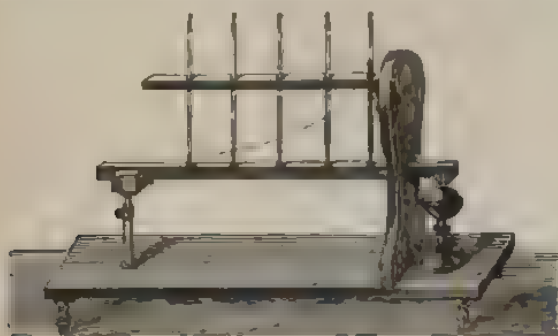


FIG. 124.—Conduction of Heat.

ment through 1 sq. cm. of its surface during one minute. The quantities of heat are here supposed to be measured in terms of the smaller heat-unit, or gram-calorie, viz. in terms of the quantity of heat required to warm 1 g. of water by  $1^{\circ}\text{C}.$  If the metal bar has attained the condition of thermal equilibrium, or the "statical condition," it appears that the successive differences of height of the thermometers follow the law that each successive one is the same fractional part of the preceding difference, or that when the distances from the source of heat increase in arithmetical series, the thermometer heights decrease in a geometrical series. This law is represented by the dotted curve, which connects the top of the mercurial columns (Fig. 124). This law was discovered by Despretz in 1822.

The fall of the curve from the warmer toward the cooler end of the rod represents the fall of temperature at each point

of the rod. By the phrase "fall of temperature" is meant the ratio of the difference of the temperatures of two very near points to the distance between the points. The quantity of heat passing through any separate cross-section of the rod by inner conduction during one minute may be obtained by multiplying the product of cross-section and fall of temperature by the internal conductivity of the substance. In the stationary condition, this quantity of heat must be equal to the entire amount of heat lost by the end of the rod beyond the cross-section through external conduction during one minute. The latter quantity of heat may be computed from the surface of this portion of the bar and the indication of the thermometers, when the external conductivity is known. The external conductivity is found by observing the loss of heat per minute of a portion of the same substance, whose weight, surface, and specific heat are determined when the difference of temperature above the surrounding objects of the portion considered is known. It is thus possible to obtain both heat-conductivities, in accordance with the above definitions, in *absolute measures*.

The relative (internal) conductivity of different bodies can be found by observing rods of the same form, whose external conductivities are equalized as nearly as possible by varnishing the surface, or plating it with silver. Under these circumstances, the internal conductivities are as the squares of the distances from the source of heat, at which equal differences of temperature occur after the stationary condition has been reached. Wiedemann and Franz (1853) determined, with the aid of a thermo-electric element (see below), the temperatures of the substances tabulated below, and obtained the following numbers for the relative (internal) conductivities of metals (first column of the following table), the conductivity of silver being put equal to 100.

						Internal Conductivity	
						Relative	Absolute.
Silver	...	..	...	.	...	100	83
Copper	...	..	...	..	...	74	61
Gold	...	...	...	..	...	53	44
Brass	...	...	..	..	...	23	20
Zinc	...	.	...	...	...	19	16
Tin	...	...	..	...	...	15	12

						Internal Conductivity.	
						Relative.	Absolute.
Iron	...	...	...	...	...	12	10
Lead	...	...	...	...	...	9	7
Platinum	...	...	...	...	...	9	7
German Silver	...	...	...	...	...	6	5
Bismuth	...	...	...	...	...	2	1.5

The absolute conductivities (second column) were computed from the relative by using as a standard the value 10 for iron, determined directly by Neumann. Through a cube, whose edge is 1 cm., sixty-one gram-calories pass during one minute, when the difference of temperature of two opposite faces is 1° C., and during one second, therefore, very nearly one gram-calorie passes.

In daily life, we make manifold applications of the good and poor conductivities of substances. To avoid burning the fingers, tea-pots, stove-doors, and poker are provided with wooden handles. Trees and shrubs are wrapped during the winter with straw to protect them against freezing. Clothing, which is made from poor conducting materials, prevents the rapid radiation of the heat of the body to surrounding objects. On the other hand, straw and other poor conductors of heat are used to prevent the entrance of external heat into refrigerators and ice-houses, and ice which is to be shipped is packed in sawdust. Fire-proof money-safes contain ashes between their double walls, to delay the admission of heat. In a cold room, metallic latches and door-bells feel colder than the table-cloth, though all are really at the same temperature, because the metal conducts heat from the hand more rapidly than does the cloth. In a room heated to a temperature higher than that of the human body, the metal feels warmer than the cloth, because the metal, in this case, conducts more heat to the hand than does the cloth. If a cylinder, composed half of copper and half of wood, is surrounded by a close-fitting envelope of paper and held over a flame, the paper chars as far as it covers the wood, but over the copper part of the cylinder it remains uninjured, because the metal, by rapidly conducting the heat away, does not allow the paper to reach the burning temperature. In a similar way a very remarkable property of

wire gauze, when immersed in flame, is accounted for. If a piece of fine wire gauze is held in a gas-flame, the flame appears to be cut in two. The metal threads conduct away the heat so rapidly that the gas-flame is cooled below kindling temperature. If a gas, without being lighted, is allowed to flow from a burner, and a wire screen is held in the current, the latter may be lighted above the screen without kindling the portion of the current below it. *Davy's safety-lamp* depends upon this property. The flame of an oil lamp is surrounded by a cylindrical wire gauze closed at the top. With such a lamp, when a miner enters a subterranean chamber in which carbonic acid gas is mixed with the air, forming the so-called *fire-damp*, a gaseous mixture which explodes in an open flame, the combustible gas passes slowly through the meshes of the gauze to the flame. It is then consumed within the cylinder in a succession of weak explosions, which are not strong enough to ignite the outside gas.

Liquids are poor conductors of heat. In these the heat is distributed mainly by means of currents (*convection*), arising from the expansion of the interior layers, which, becoming less dense, rise in the liquid and are replaced by the colder and denser layers from above. By means of this circulation within the liquid, upon which the heating of water depends, the rise in temperature of the liquid is very greatly promoted. When, on the contrary, a liquid is heated from above, the heat is distributed downward very slowly by reason of its poor conductivity. In a test-tube, held obliquely, water may be boiled, while a piece of ice, held at the bottom of the tube by a sinker, is not appreciably melted. The absolute conductivity of water is only 0.009.

Gases conduct heat also very poorly. Layers of air enclosed between double windows, for example, are well adapted to the prevention of the outward radiation of heat. The above animal and vegetable substances (straw, wool, etc.), called poor conductors, owe their "heat-holding" capability mainly to the poorly conducting air held within their interstices. The heat-conductivities of gases are, moreover, very unequal. Hydrogen conducts heat better than any other gas. The absolute con-

ductivity of air is 0.0033; that of hydrogen is about seven times as great (Stefan, 1872).

**126. Radiation of Heat.**—When the face is turned toward a hot stove, the sensation of heat is experienced. If a screen is interposed between the face and the stove, the sensation immediately disappears. The sensation is not, therefore, due to the temperature of the air of the room. An effect of some sort must proceed from the stove, which is destroyed by an obstruction placed between the face and the stove. We describe this effect by saying the stove “radiates heat.” This radiant heat is propagated through the air in straight lines without warming the air directly, although an indirect effect through the mediation of bodies against which the heat impinges, is noticeable, provided the body does not transmit all the heat incident upon it, but absorbs a portion. The fantastic figures produced by hoar-frost upon window-panes disappear under the influence of the radiation from the stove, even though the temperature of the air in the room is below the freezing point. These invisible rays, emanating from the heated body, are reflected by a mirror, are refracted by prisms and lenses, are dispersed by rough surfaces, and are absorbed according to the same laws as are light rays. Moreover, light rays are at the same time heat rays, for they raise the temperature of a body which absorbs them. We shall, therefore, treat the phenomena of radiant heat later in connection with luminous phenomena, limiting ourselves here to the mere mention of a few facts of everyday experience.

A body exposed to heat rays will rise in temperature higher and higher the more completely it absorbs the rays falling upon it, or the fewer of these rays it turns aside by diffuse reflection. With the same exposure, therefore, *dark* bodies are more highly heated than *light* ones. For this reason we wear darker clothes in winter and lighter in summer. Darkly coloured soil is more strongly heated under the influence of solar radiation than white, chalky soil. The soot of resinous pine-wood, which absorbs all kinds of radiation almost perfectly, and for this reason looks black, is more highly heated than any other known body on exposure to thermal radiations. If soot is

sprinkled over the surface of freshly fallen snow, it will be observed that the snow melts more rapidly under the soot than elsewhere, and that, following the trace of the soot, a deep trench is melted into the snow. Those bodies which absorb heat rays best also radiate their heat most easily. Radiating power increases in the same ratio as absorbing power. Hot water cools more rapidly in a pot covered with soot than in one whose surface is not so covered. A thermometer, with its bulb covered with soot, or lampblack, rises in the sunshine more rapidly and to a higher point than a precisely similar thermometer without a coated bulb. It also cools much more rapidly in the shade than does the latter. Different thermometers, on exposure to the sun, indicate different temperatures according to their absorptive powers. For this reason the data of a thermometer exposed to sunshine are worthless as indications of atmospheric temperature.

It is evident that only those rays which *penetrate* into a body are absorbed by it and warm it. A smoothly polished body, which reflects a part of the rays from its surface, with a given exposure, is less strongly heated than is the same body with a rough surface. The latter surface permits a part of the radiations to penetrate to a certain depth within the body before dispersing them. On the other hand, a warm body radiates its heat more freely through a dull, than through a polished surface, because, at the polished surface, a part of the heat radiated from the interior of the body is reflected again into the mass. In a brightly polished metallic coffee-pot, the liquid retains its heat longer than when the surface of the pot is less highly polished. In this connection it may be noted that the best absorbents are, at the same time, the best *radiators*.

That the radiating power of a body is equal to its absorbing power follows, moreover, from the so-called *principle of dynamical equilibrium of temperature* (Prevost, 1809). Every body emits rays of heat, and, at the same time, receives other rays. When the body has reached the same temperature as that of surrounding bodies, experience teaches that it does not change its temperature longer, although radiation both toward and from it continues. This is, however, possible only if, during a given

interval of time, the same amount of heat is absorbed as is radiated.

**127. Mechanical Theory of Heat.**—*First proposition.* To explain thermal phenomena, it was formerly assumed that heat is caused by a subtle, imponderable, thermal material, which, by penetrating into a body in greater or less quantities, was thought to produce its different temperatures, its expansion, its melting, its vaporization, etc. This so-called *theory of emission* satisfactorily accounted for neither the phenomena of heat radiation, nor for the fact that heat may be produced by friction or any other sort of mechanical work. The *mechanical theory of heat* generally accepted at present assumes, on the contrary, that heat is a species of energy, that it is the energy of motion (kinetic energy) of the molecules, whose movements are so small as to be imperceptible to the eye. They are, however, capable of producing an impression upon the sensibility, which we call "heat." To explain how the production of heat by mechanical work is possible according to this view, we may use the illustration of a smith hammering a piece of iron. While he lifts the hammer he performs a certain quantity of work, by virtue of which the hammer, during its fall, attains a certain quantity of energy, which latter is expended in belabouring the iron. After striking the iron lying upon the anvil, the falling hammer comes to rest; its progressive motion is suddenly stopped; the energy, however, which resides in it is in nowise lost. It is merely transferred to the piece of iron, giving rise to vibratory motions within the mass. The energy of the hammer reappears without diminution in this undulatory motion of the molecules. The anvil is set into rapid vibration, similar to the vibration produced by striking a bell: in both cases the vibrations are perceived by the ear as loud, ringing sounds. In hammered iron, however, vibrations of the molecules are also produced, which are perceptible as heat. The iron is heated, and, by continued hammering, it may even be made to glow. The work performed by the smith at each stroke is greater the heavier his hammer and the higher he raises it. If the hammer weighs 1 kg., and he raises it 1 m. high, the work expended is

1 kilogram-meter. The energy with which the anvil is struck is measured by the same magnitude. This energy corresponds precisely to the quantity of heat produced in hammering the iron. The energy is expended, partly in the iron, partly in the hammer, partly in the anvil, and a fourth portion is expended in the propagation of sound through the air.

In the retardation of motion by friction heat is also developed. A metallic button rubbed on wood, or leather, becomes hot. The savages obtained fire by rubbing two sticks together, and we ourselves accomplish the same result by heating the phosphorus of a match to its temperature of ignition by means of friction. When a train of cars is stopped by means of brakes, both wheel and brake become heated.

From experiments on the friction of cast-iron upon water, in which, on the one hand, the work expended, and, on the other, the amount of heat developed, were accurately determined, Joule (1850) found that a work of 424 kgm. is consumed in heating 1 kg. of water through  $1^{\circ}$  C. The number 424 kgm. is called the *mechanical equivalent of the heat unit*. It expresses the constant relation between work and heat, by the aid of which the one may be transformed into the other. That not only work may be transformed into heat, but that the converse is also possible, is illustrated by the steam-engine. The energy of motion with which a train of cars moves arises obviously from the heat of the fire under the boiler of the locomotive, and, as was shown by Hirn from experiments with steam-engines, for every 424 kgm. of work performed in moving the train, one heat unit disappears by being converted from the form of invisible molecular motion into the energy of moving masses. This proposition, first recognized by Robert Mayer in 1842, is designated the *first proposition* of the mechanical theory of heat.

**128. Structure of Matter.**—The view that heat is not a material substance, but rather that it is work, or kinetic energy, together with the assumption that bodies are composed of separate molecules, furnishes a consistent, clear, though without an hypothetical explanation of all the phenomena of heat with which the facts of experience have thus far familiarized us.

A solid is to be regarded as an aggregation of molecules, bound together without immediately touching one another into a connected whole by the internal forces of cohesion. By reason of the combined effect of the forces exerted by neighbouring molecules, every molecule has a definite position of equilibrium. From this position it can be removed and brought into a new position of equilibrium only by the action of external forces. So soon as these external forces cease to act, the molecular forces will drive this molecule back into its original position of equilibrium. The elasticity of solids also finds a satisfactory explanation in this theory. The molecules are not in a state of rest, however, in their momentary positions of equilibrium, but oscillate rapidly about this position. The energy with which the vibrating molecules strike the finger is perceived by us as heat. The degree of heat, or the temperature of a body, is accordingly proportional to the energy of motion of its molecules. To warm a body means merely to set its molecules into more energetic vibration, or to increase their amplitude of swing. Since the vibrating molecules pass farther from their positions of equilibrium than before, they require a greater space within which to perform their motions, and, consequently, strive to assume new positions of equilibrium more widely separated than in the cooler state. The volume of the body is, therefore, increased on heating, or the body expands. The molecular forces, however, oppose the separation of the molecules. To overcome these forces, a certain quantity of external heat, or work, is consumed in the performance of internal work. If an external force acting against the expansion exists, as, for example, the pressure of the gas surrounding the body, this must also be overcome. The energy (heat or work) required to expand the body performs, therefore, also *external work*. If the body is again brought into its original condition, it will give out the entire amount of heat which was consumed in both the internal and the external work, in the performance of which heat originally disappeared.

On continued heating of a solid, the aggregation of its molecules grows less and less compact. The molecules separate and their distances reach, finally, the narrow limits within

which molecular forces are operative. Cohesion is no longer strong enough to bring the molecules back into their original positions of equilibrium. They abandon, then, their former places and take on a translatory motion, moving upon and displacing each other, without wholly separating, their mutual attractions having been greatly weakened. The body then passes into a liquid condition, or it melts. After the melting point has been reached, no further heat applied from without is consumed in raising the temperature, but it is used in performing the internal work of overcoming the forces which formerly held the molecules in equilibrium. This heat performed upon internal work and, therefore, lost to perception, is the *latent heat* of melting. All this internal work, when the body solidifies, must reappear in the form of heat, or, in the language of the emission theory, the heat "bound" in melting is again liberated in freezing.

On the free surface of the liquid, those molecules, lying beyond the limits of the spheres of action of their neighbours, are no longer drawn back by them. They fly off rectilinearly into the space above the liquid with the velocities they possess at the instant they transgressed those limits. These molecules, freed from the bonds of cohesion and flying hither and thither, are now in a gaseous condition and form the vapour developed from the liquid. This vaporization, viz. the releasing and flying off of separate molecules from the surface of the liquid, occurs at all temperatures, but of course in greater quantities the higher the temperature of the liquid, i.e. the more vigorous the motion of the molecules. The pressure of a vapour is produced by the impact of the flying molecules against the walls of the containing vessel, whence they are reflected after the manner of elastic balls. On returning to the surface of the liquid the molecules are either reflected from it, or held down and incorporated again with the liquid, according as the kinetic energy at the place of impact is greater or less than the cohesive force of the molecules. In an enclosed space, therefore, the number of molecules of vapour will increase until, within a given time, as many molecules return to the surface of the liquid as leave it. The stationary condition of

saturation, or the maximum of tension, is then reached. Any other gas which may perchance be present cannot, of course, interfere with this process (Dalton's law).

Since during evaporation those molecules which happen to possess the greatest velocity are the ones which fly off, the average kinetic energy of the remaining molecules must be less, i.e. the vaporizing liquid cools (*cold of evaporation*), if the loss of energy is not replaced by heat from without. On the interior of the liquid vapour can arise only after the motion of the molecules has become so vigorous that their tendency to fly off exceeds the pressure of the liquid plus the pressure of the air resting upon it. If the temperature required for this, i.e. the boiling point, is reached, the liquid is rapidly and violently converted into vapour. It boils, since all external heat is consumed in internal work, viz. in tearing asunder the last bonds of cohesion, as heat of vaporization, or, as it was the custom to say formerly, the heat is all *bound*. From this it follows that the boiling point of the liquid must be lower the smaller the pressure upon it.

**129. Kinetic Theory of Gases.**—We are thus led to that view of the molecular constitution of gases, which is known as the *mechanical*, or *kinetic* theory of gases (Daniel Bernoulli, 1738; Kromig, 1856; Clausius, 1857). According to this view, all the molecules of a gas are in rapid rectilinear motion. They fly about in all possible directions through space, and traverse spirally-curved, zigzag paths, continually rebounding from each other and from obstructions to their movements, like elastic balls. All known properties of gases may be explained on the assumption just made of the motion of their molecules. The pressure exerted against the walls by a gas contained in a closed vessel is produced by the incessant impacts of the rapidly moving molecules of the gas. Since these impacts during short intervals of time occur in all directions, from their combined effect there will result a perpendicular pressure whose magnitude is proportional to the energy of the colliding molecules, and, accordingly, increases in the ratio of this energy, i.e. proportionally to the increase of temperature (Gay-Lussac's law). If the confined gas is compressed without

change of temperature to  $\frac{1}{2}$ ,  $\frac{1}{3}$ , etc., of its initial volume during the same time, and upon equal superficial areas of the wall, twice, thrice, etc., as many molecules will impinge with the same energy as before. The pressure, or tension, will, therefore, have double, threefold, etc., its initial value. We arrive thus at Mariotte's law: *the pressure of a gas varies inversely as its volume.*

Let us now consider equal volumes of different gases under the same temperature and pressure. That the temperatures are equal signifies merely that the same energy resides in their molecules, or that each molecule of the one gas strikes the vessel wall with the same violence as each molecule of the other. The pressure of all gases being the same, it is evident that, with each gas, the same number of molecules strike against the unit of surface during the unit of time. We arrive thus at Avogadro's law, *that with the same pressure and temperature, the same number of molecules is contained in the same volume of different gases.* The molecular weights of gases are consequently as the weights of equal volumes of them, or as their specific gravities.

At the point where the wall of the containing vessel interposes itself before the molecules of the gas, these molecules, by virtue of the energy with which they strike, exert a pressure upon the wall. Where these particles find an orifice, however, they pass out. The velocity of efflux, or the effusion through narrow orifices, is merely the velocity of the molecules flying outward. The energy of the molecular motions which gives rise to the pressure upon the walls of the vessel is, however, proportional to the product of the mass of the molecule, or of the molecular weight, into the square of the velocity. If, therefore, two gases exert equal pressures, the products of their molecular weights, or what, according to Avogadro's law, is the same thing, of their specific gravities, by the squares of their velocities, must be equal. If, therefore, different gases flow outward under the same pressure, the squares of their velocities of efflux will be inversely as their specific gravities (59).

When a gas is heated without change of volume, *i.e.* while it remains in a closed vessel of constant volume, the external heat has to perform neither external nor internal work, because

neither the external pressure nor the resisting molecular forces have to be overcome. In this case, therefore, all external heat is applied in raising the temperature, *i.e.* in increasing the molecular energy of the body. But when the heated gas is permitted to expand, thus keeping itself in equilibrium with the fixed external pressure, as in the former case, no internal work has to be performed. On the contrary, a portion of the external heat is consumed in external work, *i.e.* in overcoming the external pressure. The amount of heat required to warm 1 kg. of gas under these circumstances, or the specific heat with constant pressure must accordingly be greater than that with constant volume, because in it is contained a quantity of heat consumed in external work which equals the difference of the two specific heats. Since now the work required to heat the expanding gas by 1° C. is known, the work performed by one thermal unit, or the mechanical equivalent of heat, is easily derived. In this way Robert Mayer, the founder of the mechanical theory of heat, determined first the mechanical equivalent. If the gas which has been expanded by heat is again compressed to its original volume, the quantity of heat consumed in its expansion and transformed into work is again liberated in the form of sensible heat.

The following is Robert Mayer's computation. One kilogram of air at 0° and 760 mm. pressure, occupies a space of 0.773 cubic meters. In a cylinder of 1 square meter cross-section and closed by a movable piston, this mass of air expanded when heated 1° by  $\frac{1}{273}$  of its volume, and pushed the piston against which an external atmospheric pressure of 1.033 kg. per sq. cm. rested—and therefore upon the entire piston a pressure of 10,330 kg.—by  $\frac{0.773}{273}$  m. backwards. For the work here performed amounting to  $\left(\frac{10330 \cdot 0.773}{273}\right)$  kgm., a quantity of heat was consumed equal to the difference of the two specific heats (124), therefore equal to 0.069 thermal units. The work of 1 thermal unit then equals  $(10330 \cdot 0.773) : (273 \cdot 0.069)$  equals 424 kgm.

To heat different gases by equal amounts, *e.g.* by 1° C., the

energy of motion of their molecules must be increased by equal amounts, i.e. the molecules of all gases require equal quantities of heat to raise their temperature by equal amounts. (The quantities of heat required to warm the molecular weights through equal intervals are equal.) Since, according to Avogadro's law, all gases contain in equal spaces an equal number of molecules, and consequently the molecular weights are to each other as the weights of equal volumes (or as the specific gravities), it may also be said that equal volumes of different gases require equal quantities of heat for equal elevations of temperature. The specific heats of gases, i.e. the quantities of heat required to warm 1 kg. of each by  $1^{\circ}\text{C}.$ , are to each other inversely as their molecular weights, or as their specific gravities. This law was derived on the assumption that the external heat has to perform no internal work. It is, therefore, rigorously correct only for perfect, or ideal, gases. It is very nearly true for air, oxygen, nitrogen, hydrogen, carbonic oxide, and nitric oxide (cf. 124).

**130. Second Proposition of the Mechanical Theory of Heat—Entropy.**—Heat can be transformed into mechanical work (e.g. by means of the steam-engine) only when an exchange of heat from a body of higher temperature (from vapour) to one of lower temperature (cold water) occurs. Sadi Carnot (1824) perceived the analogy between the mechanical performance of heat and that of running water, which likewise performs work only when it sinks from a higher to a lower level. He assumed that both heat and water sink without diminution of energy from one level to another. Clausius (1850) showed, on the contrary, that, according to the proposition of the equivalency of heat and work, a portion of the heat disappears as heat in the performance of a quantity of work equivalent to it, while the other part of the external applied heat passes as such into the cooler body. He then propounded the second proposition of the mechanical theory of heat, viz. that heat can never of itself pass (i.e. without a corresponding consumption of some other form of energy) from a colder into a warmer body. While mechanical work can be easily and completely converted into heat by friction, impact, etc., it is impossible to retransform the entire quantity of heat

again into work. In this latter process heat is being continually transmitted to cooler surrounding bodies. The result of this is that the entire quantity of mechanical energy contained in the whole universe passes from day to day more and more into heat, which is distributed in all directions, and tends gradually to equalize the present differences of temperature. William Thompson (1851) called this process *dissipation*, or *degradation* of energy. The sum-total of the energy at present in the universe may then be conveniently divided into two parts, one of which, as heat of higher temperature and as mechanical, chemical, electrical, etc., is still partially convertible into heat; but the other, already transformed into heat, and collected in colder bodies, is irrecoverably lost for purposes of work. This latter portion, which Clausius called "Entropy," increases incessantly at the expense of the former, or, as Clausius expresses it, "*The entropy of the universe strives ever toward a maximum.*" If after an inconceivably long interval of time this maximum should be attained, still the energy originally present in the universe would not be lost, but would be distributed uniformly throughout the universe in the form of heat. Differences of temperature would then no longer exist, and the fundamental condition for the transformation of heat into other forms of energy would then be wanting. All mechanical motions would then necessarily cease, and cosmic processes would have reached an end.

**131. Steam-Engine.**—The steam-engine offers the most important example of the transference of heat into work, or into the energy of motion. Its most essential part is the cylinder (Fig. 125), in which a steam-tight piston moves back and forth, according as the steam (heated water vapour is called *steam*) is admitted in front of or behind the piston. A suitable admission of the steam to the one side, or the other, of the piston is accomplished automatically by means of the valve motion. At the side of the cylinder is a chamber (to the right in the figure), into

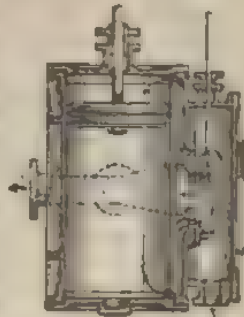


FIG. 125. Steam Engine

which steam enters from the boiler, in the direction of the lower arrow. Through one wall of the chamber three canals open, the two outer of which lead toward the ends of the cylinder, while the third is so arranged that the steam, after having performed its work, passes out of the cylinder through it into the tube visible at the left. Along this wall of the chamber a hollow cup-shaped slide moves, which, when in the position shown in the cut, allows the steam to pass from the chamber (steam-chest) through the upper canal into the cylinder behind the piston, while the steam from the previous stroke of the piston passes through the lower canal into the hollow of the slide, and thence, by way of the middle canal, into the exhaust pipe. After the piston has reached the other end of the cylinder, the slide has been carried upward by the valve rod in such way that the lower canal communicates with the steam-chest, and the upper canal with the hollow of the slide. The proper working of the slide is effected by the engine through the aid of an eccentric disk fixed rigidly to the shaft of the drive-wheel. If the slide is so arranged as to cut off admission of the steam into the cylinder, before the piston has reached the end of its travel, the expansion of the confined steam still performs work against the piston, and at the same time cools in consequence of its expansion. Engines of this sort are called *expansion engines*. The steam escaping outward, through the hollow of the slide, has a lower tension than that entering from the steam-chest, and the piston is driven by virtue of the difference of tension. With engines which work at a low steam-pressure but little above the tension of one atmosphere (low-pressure engines), it is the custom to pass the escaping steam into a chamber surrounded with cold water, called the *condenser*, in which the steam condenses in greater part on cooling, and, accordingly, its tension is reduced to that corresponding to the temperature of the condenser. If, on the contrary, as with high-pressure engines, the steam is allowed to escape directly into the air, so that upon one side of the piston the pressure of one atmosphere always exists, the working steam must then have a pressure of five to eight atmospheres. Since high-pressure engines require neither condensers nor pumps to maintain a supply of cool water, and are for this reason much

more simply constructed than low-pressure engines, they are used where economy of space is of especial importance, or where cool water is not available, as with locomotives. The motion of the piston to and fro is converted into a rotatory motion by attaching to a crank a so-called *connecting-rod*, which communicates with the piston-rod by means of a joint. This mechanism turns a shaft which is rigidly connected with the drive-wheel. When the engine is in operation a part of the heat applied to the boiler is absorbed by the cold water and by surrounding objects. Another part is transformed into work, and disappears as heat. The quantity of heat lost, as Hirn has proved by an extended series of experiments with steam-engines (cf. 127), is equivalent to the work performed; but if the water is vaporized in an open vessel without performing work, the entire quantity of heat applied is transmitted to the cooler environment.

## VI. MAGNETISM.

**132. Magnetism.**—Many pieces of iron ore found in nature, i.e. magnetic iron ore, or magnetite, called also *magnetic oxide of iron*,  $\text{Fe}_3\text{O}_4$ , possess the property of attracting and holding pieces of iron. This property is called *magnetism*, and a piece of iron ore which possesses it is called a *natural magnet*. By touching, or stroking, a piece of iron with a magnet, magnetism may be transferred temporarily to the iron; if steel is used, the magnetism produced in it is permanent. Artificial magnets are made in this way, and the process is called *magnetization*. If a steel bar, after having been magnetized, is besprinkled with iron filings, they remain sticking in bushes, or tufts, quite thickly at the ends, and gradually becoming less numerous toward the middle, while just at the middle no filings at all adhere to the bar. The two points near the ends toward which the forces of attraction are directed are called the *poles*. The middle, where no attraction occurs, is called the *equator of the magnet*, also the *neutral*, or the *indifferent region*. The line connecting the poles is called the *magnetic axis*.

If a magnetized iron bar is suspended by a silk thread attached at the middle so as to turn in a horizontal plane, its axis assumes a direction deviating inconsiderably from a north and south line (the *geographical meridian*). This behaviour of the magnet is due to the effect exerted upon it by the earth. That pole which always turns toward the north is called the *north pole*, and the other the *south pole*. The end of the magnetic needle pointed to the north is also sometimes called the *marked end*, or the *red pole*, of the needle, and, in contradistinction, the other is called the *unmarked end*, or the *blue pole* of the needle. If a second magnet be brought near the suspended bar, the north and south

poles manifest opposite tendencies of such sort that *like poles repel and unlike poles attract each other.*

**133. Molecular Magnets.**—If a magnet is broken in two in the middle (*e.g.* a magnetized knitting-needle), each piece will be found to form a perfect magnet with two opposite poles, two poles arising at the section of fracture, each of which is the opposite of that of the corresponding portion of the needle. However far this subdivision may be continued, each piece of the magnet, no matter how small, is always found to be a perfect magnet with two equally strong poles. This behaviour justifies the assumption that every molecule of a magnet is itself a magnet with two opposite poles, called a “molecular magnet.” This assumption in no wise contravenes the fact that the magnetic property is exhibited only at the ends of a magnetic bar; on the contrary, it accounts for this fact satisfactorily. If, for simplicity, we imagine a slender magnetic bar to consist of a single row of molecular magnets, with like poles all turned in the same direction, we shall have everywhere, throughout the length of the bar, two opposite poles of the adjacent magnets lying together, whose attraction and repulsion mutually destroy each other. Hence it is only at the ends of the bar, where the free poles of the extreme molecules are operative, that any effect will be apparent.

**134. Magnetic Influence, or Induction—Coercive Force—Saturation.**—If the north pole of a magnet be brought near a piece of soft iron, the latter immediately becomes a magnet, the nearer end becoming a south, and the other a north, pole. The latter is now also capable of attracting a second, this a third, etc., piece of iron. The iron is attracted by the magnet, because under the magnet's influence it is converted into a magnet, which turns its opposite pole to the pole of the magnet to which it is approached. This influence is called *induction*. The magnetism of soft iron quickly disappears, and the particles of iron borne by it fall off almost as soon as the inducing magnetic pole is removed, or, in general, as soon as the magnetizing force ceases to act. Steel acts differently. It is not so easily magnetized as soft iron; but when, after continued application of the magnet, it has once acquired the magnetic property,

it will retain this property, even after the substance has been taken from under the influence of the magnetizing force. The force with which steel resists magnetization is known as *coercive force*. It is greatest with the hardest and brittlest steel. Annealing reduces it, and on heating to redness and gradually cooling it becomes as weak as with soft iron.

To explain the phenomena of magnetic induction, it is assumed that every unmagnetized particle of iron, or steel, consists of ready-made molecular magnets, which are so irregularly disposed, that toward any given direction as many north as south poles are turned, and therefore that the attracting and repelling effects mutually destroy each other. On bringing a mass of unmagnetized substance near a magnetic pole, these molecular magnets rotate about their centres of gravity in such way as to turn their unlike poles toward the inducing magnetic pole. By means of this orderly arrangement of the majority, if not of all, of the molecular magnets, a piece of iron, or steel, is converted into an externally active magnet. While in steel the molecules oppose to the rotation, a considerable resistance (the *coercive force*) analogous to friction; on the other hand, they maintain their orderly arrangement with equal persistency. Molecules of soft iron, however, return



FIG. 126. — Magnetic Needle.

readily to their former disorderedly arrangement after having been drawn out of this condition by inductive action. A piece of iron, or steel, can be magnetized only to a certain degree, i.e. to *saturation*, which condition is attained when all the molecular magnets are directed in the same sense.

### 135 Forms of Magnets—

**Keeper — Lifting Power or**

**Portative Force.**—The most useful forms are: the right parallelepiped, or cylindrical magnetic bar; the magnetic needle (Fig. 126), a flat strip of magnetized steel, which ordinarily has the

form of a slender rhombus and in the middle is provided with a small agate socket. The latter renders the needle easily movable about a steel point which fits into it. The third form is the horse-shoe magnet, whose poles lie beside each other in such way as to enable them to act simultaneously and concurrently. A piece of soft iron is usually kept against the poles. By the inducing action of the magnet, this iron is in turn converted into a magnet with its poles lying upon the opposite poles of the horse-shoe magnet. This bar of soft iron is called the *keeper*, or *armature* (Fig. 127, mm). Since, in forming the south pole of the armature, both poles of the magnet contribute, its magnetization is stronger than it would have been if it had been touched by but a single pole. As each pole of the armature strives to keep the molecules of the magnet directed toward it, the armature assists the magnet in holding its strength.



FIG. 127.—Horse-shoe Magnet with Armature

Larger magnets, for more thorough magnetization, are made of a number of single horse-shoe steel magnets, with like poles placed beside each other and screwed together. Such a compound magnet (Fig. 127) is called a *magnetic magazine* (*plate magnet*).

The *lifting power*, or *portative force* of a magnet, tested by loading the armature with plates, increases much more gradually than with the mass of the magnet. According to Haecker (1842) this portative force is proportional to the two-thirds power of the weight of the magnet. While a magnet weighing 60 g. carries 25 times its weight, one of 50 kg. cannot carry 3 times its weight, and one of 1000 kg. is scarcely able to carry its own weight.

**136. Methods of Magnetization.**—On account of the great coercive force of steel, mere contact with a magnet is not sufficient to produce strong magnetization. Various methods of *stroking* are therefore in vogue, *e.g.* beginning in the middle of the magnet, one end of the bar, or of the horse-shoe magnet to be magnetized, is moved ten or twenty times over the north pole, while the other end is moved the same number of times

over the south pole of a powerful magnet. Of course the end which is stroked by the north pole of the magnet becomes a south pole, and conversely. The various artificial methods of *touch* which were devised to magnetize steel to saturation have practically lost their value, for, since the discovery of electro-magnetism, greater inequalities of magnetizing force are now available than was the case formerly.

**137. Terrestrial Magnetism.**—On hanging a bar magnet some distance above a magnetic needle, in the position it assumes under the influence of the earth, the bar will assume a position parallel to the needle, and both bar and needle will turn their north poles to the north. If the needle is drawn aside from its position and then allowed to swing freely, it returns quickly to its former position. But if the metallic bar is gradually lowered, at a certain height of the bar above the needle it will be noticed that the needle has lost its tendency to resume its original position, for if now drawn aside it does not strive to resume its former position. Lowering the magnetic bar still farther, the needle will be seen to reverse its position and to direct its north pole toward the south. It appears, therefore, that the effect of the earth upon the magnetic needle may be destroyed by a magnet situated at a suitable distance. When this condition has been attained, let a magnetic bar with its south pole directed toward the north be brought near the magnetic needle, whereupon its tendency to direct its north pole toward the north returns, and at a certain distance of this second bar this tendency attains the same magnitude, as under the effect of the earth alone. Hence the earth acts as a magnet, whose north pole is turned southward, and, accordingly, it may itself be regarded as a huge magnet (Gilbert, 1600), whose poles are so remote from the magnets with which we experiment, that the forces exerted by one of them upon the poles of a magnetic needle, are equal and opposite (couple), and are, therefore, able to draw the needle into any definite direction, but are, of course, incapable of producing a motion of translation of the magnetic needle toward the magnetic pole of the earth (terrestrial magnetism).

**138. Astatic System.**—A magnetic needle, which has been

freed from the effect of terrestrial magnetism, by placing a magnet with like directed poles near it, so that the system is free to follow any impulse imparted from without, is called *astatic* (without definite direction). The same effect is also obtained, by so fastening two magnetic needles of approximately equal strengths (Fig. 128) one above the other, in such way that opposite poles are like-directed, and this *astatic pair* will then swing with perfect freedom.

**139. Magnetic Meridian—Declination.**—The vertical plane through the axis, *ab* (Fig. 129), of a magnetic needle, which is movable in a horizontal plane, after the needle has assumed the position of equilibrium under the influence of terrestrial mag-

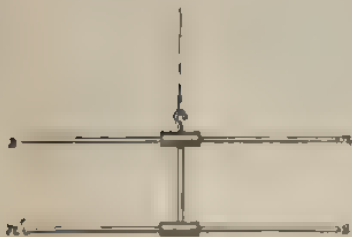


FIG. 128.—Astatic Pair.



FIG. 129.—Declination.

netism, is called the *magnetic meridian*. It forms, with the north and south line, or with the geographical meridian, an angle which is called the *magnetic declination*, or *deviation* (Columbus, 1492). At different places on the surface of the earth, the declination has different values, and is designated east, or west, according as the north end of the needle deviates eastward, or westward, from the north and south line. In the region about Munich, at the beginning of the year 1896, the declination was west, while it was equal in Berlin to  $10^{\circ}1$ , in Munich to  $10^{\circ}9$ , in Paris to  $15^{\circ}1$ , with an annual decrease of  $0^{\circ}1$ .

The declination chart (Fig. 130) furnishes an idea of the nature of the magnetic deviation upon the surface of the earth.

On this chart all places of equal declination are connected by curves, called *isogonals* (Halley, 1700). All isogonals run together in two points, one of which lies in the Arctic ocean north of North America, in the neighbourhood of Melville peninsula, and the other in the Antarctic ocean near New Holland. These points are regarded as the magnetic poles of the earth. A line of declination  $0^\circ$ , that is, a line upon which the magnetic needle points directly toward the north, cuts off the eastern part of Brazil, runs eastward of the West Indies, crossing North America in the neighbourhood of Philadelphia and traversing



FIG. 130. — Declination Chart for 1800.

Hudson bay. Thence it proceeds through the north magnetic pole of the earth, across the White and Caspian seas, crosses the Indian ocean west of Hindoostan and the East Indies, and turns then toward New Holland, returning finally into itself at the south magnetic pole of the earth. Upon the Atlantic ocean and in Europe and Africa, the declination is everywhere toward the west. Upon the other hemisphere, terminated by the line just traced, it is toward the east, excepting within a limited region in Eastern Asia, where a second closed curve of  $0^\circ$  declination has been traced, within which the declination is again westward.

A *declinometer*, or *declination compass*, is an instrument for

the measurement of declination. A simple form of this apparatus is shown in Fig. 131. Within a circle graduated to degrees, a magnetic needle swings freely about a point in a horizontal plane. A telescope is attached to the side of the box, which may be rotated about a vertical axis. The axis of the telescope lies parallel to the diameter  $0^{\circ}$ - $180^{\circ}$  of the graduated circle. When the apparatus is so placed that the needle points to  $0^{\circ}$ , the axis of

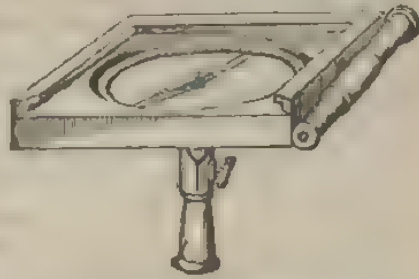


FIG. 131.—Declination Compass.

the telescope lies in the magnetic meridian. On the contrary, when the telescope is placed exactly in the north and south line, the point of the needle indicates, upon the circle, the magnetic deviation. This instrument is used in surveying as a surveyor's compass for the determination of angles. The magnetic needle indicates the angle which the line of sight makes with the magnetic meridian.

The compass (Chinese, 2300 B.C.) used by mariners to orient themselves at sea, consists of a magnetic needle turning upon a point. The needle carries a circular paper disk fixed to it, whose circumference is divided into thirty-two parts, beginning from the north pole of the needle. The paper disk is called a compass card, and the divisions are called compass marks. The whole is enclosed within a box which is suspended upon gimbals.



FIG. 132.—Inclination.

**140. Inclination.**—If a magnetic needle, movable about a horizontal axis passing through its centre of gravity (Fig. 132), is so placed that its plane of rotation coincides with the magnetic meridian, it inclines in the northern hemisphere of the earth with its north pole downwards, and in the southern

with its south pole downwards. The angle at which it is inclined to the plane of the horizon is called the *inclination* (Hartmann, 1544). At the beginning of 1896, this inclination was at Berlin  $66^{\circ}7'$ , at Munich  $63^{\circ}7'$ , at Paris  $65^{\circ}0'$ , with an annual diminution of  $0^{\circ}03'$ . The inclination increases toward the north. Above the north magnetic pole of the earth, which Captain Ross actually reached, at  $70^{\circ}5'$  north latitude, and  $96^{\circ}42'$  longitude west from Greenwich, the needle coincides with a vertical line, and for this reason the mariner's compass is of no value in these high latitudes. The inclination diminishes toward the south and near the equator, the needle

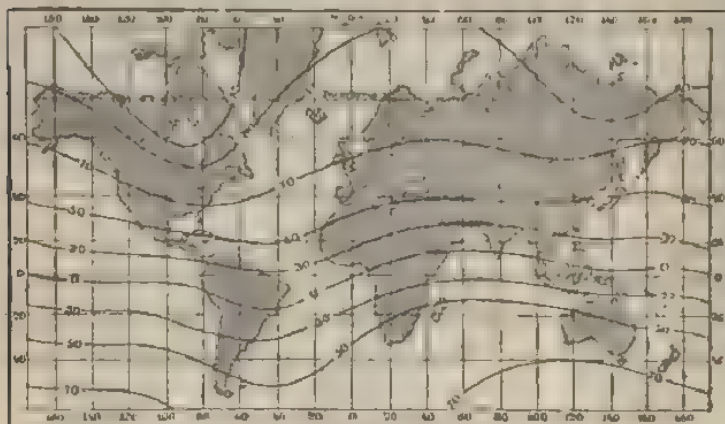


FIG. 133.—Inclination Chart for 1860

assumes a horizontal position, and still farther toward the south, the south pole of the needle becomes depressed, the amount of depression increasing more and more toward the southern magnetic pole. The variation of inclination is graphically represented by the *declination chart* (Fig. 133; Haugsteen, 1826), upon which points of equal inclination are connected by a curved line. Such lines are called *isoclinics*. The zero isocline, on which the needle stands horizontal, passes around the torrid zone, first on one side and then on the other of the geographical equator. It is called the *magnetic equator* of the earth. It is also sometimes referred to as an *acclinic*

line. The instrument for measuring the inclination is called an *inclination compass*, or *inclinometer*, or *dip needle* (Fig. 134; Normann, 1576). The arrangement of parts of this instrument is at once apparent from the figure.

**141. Intensity of Terrestrial Magnetism.**—The position of the dip needle gives the direction toward which, at each place of observation, the earth's magnetic force acts, precisely as a pendulum hanging at rest indicates the direction of gravity.

If a magnetic needle, whether an inclination or a declination needle, is drawn aside from its position of equilibrium, it will return again to this position after having performed a series of vibrations. These vibrations follow the same laws as do those of a pendulum. If the same magnetic needle is allowed to vibrate at different places on the surface of the earth, the number of vibrations per second will furnish the ratios of intensity of the terrestrial magnetic forces at these places. The forces are found to bear the same ratio to each other as the squares

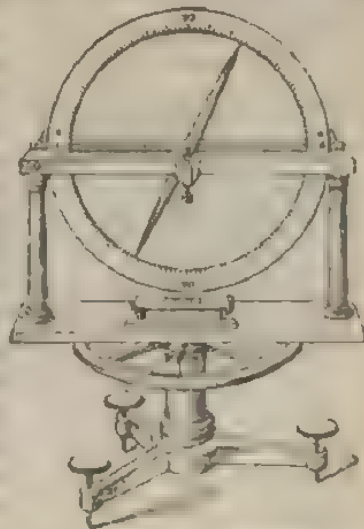


FIG. 134. —Inclinometer.

of the observed numbers of vibrations during a given period of time (40). From the vibrations of a dip needle, it is possible in this way to determine the total intensity of the earth's magnetic force, called the *total intensity*, while by means of the declination needle, it is possible to determine only the horizontal component of this entire force, or the *horizontal intensity*. But since a declination needle admits of more accurate observation than does an inclination needle, it is preferable to determine the horizontal intensity ( $H$ ) immediately, and from it to compute the total intensity ( $T$ ) with the aid of the known inclination ( $i$ ), from the equation  $H = T \cos i$ .

The distribution of the total magnetic force over the surface of the earth is represented graphically by lines of equal intensity, or by *isodynamic lines*. The little chart (Fig. 135) shows that the magnetic force in a general way increases from the equator toward the poles; that the greatest value is, however, not at the magnetic poles, but that in the northern hemisphere there are two points of maximum magnetic force, one in North America a little west of Hudson bay, and the other in northern Asia. An arbitrary unit is used in expressing the numbers described above for representing magnetic intensities. In terms of a unit which will be explained later, at the beginning of 1896

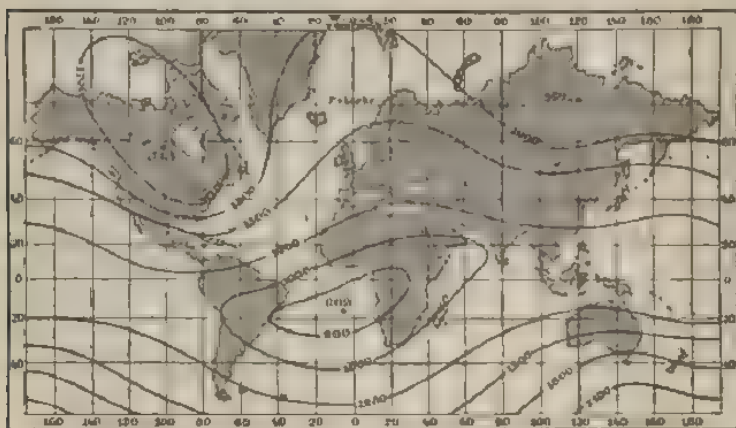


FIG. 135. Isodynamic Lines for 1895.

the horizontal intensity was at Berlin 0.186, at Munich 0.203, and at Paris 0.196, with a yearly increase of about 0.00015.

**142. Variations.**—The three magnitudes, declination, inclination and intensity, are called the *elements* of terrestrial magnetism, because by their means both direction and magnitude of the magnetic force are completely determined. All these elements undergo continuous changes even at the same place, the fluctuations being partly sudden and irregular, and partly regular, occurring periodically during the day, or during the course of many years. The former are called *disturbances*, and the latter *variations*. Daily variations

are connected with the daily motion of the sun, and amount to only a few minutes. Secular variations, on the contrary, continuing in the same sense throughout the course of years, gradually accumulate to considerable magnitudes. In France, for instance, in 1580 the declination was  $11^{\circ}5'$  east, in 1663 it was  $0^{\circ}$ , it then became west, and increased until in 1814 it was  $22^{\circ}5'$  west. Since then the western declination has diminished somewhat. The inclination also exhibits both daily and secular changes. In Paris during 1671 it was  $75^{\circ}$ , since then it has diminished, and in 1894 it was only  $65^{\circ}1'$ . The intensity is likewise subject to both daily and secular variations.

These disturbances have been found to be connected in some unknown way with earthquakes, volcanic outbreaks, and particularly with the phenomena of the northern lights.

**143. Magnetometer.**—To determine the declination and its variations with precision, the *magnetometer* (Gauss, 1833) is used. A bar magnet, *m* (Fig. 136), is suspended by an untwisted silk thread so as to admit of free motion in a horizontal plane. Above its centre, perpendicular to the magnetic axis of the bar, a small mirror, *o*, is attached to it. In the mirror may be seen by means of a view-telescope turning about the centre of a horizontal circle, i.e. by a *theodolite*, the image of a scale, *ss*, fixed horizontally beneath the telescope and graduated to millimeters. In the centre of the field (on the cross-hairs) will be seen the middle division, *a*, of the scale, provided the magnetic axis of the bar magnet coincide with the line of sight (axis of the telescope). But if the magnet deviate by even a very little from this position, some other division, *c*, will be seen on the cross-hairs. From the distance, *ac*, read in the telescope and the distance, *am*, the small angle, *amd*, by which the magnetic axis deviates from the line, *am*, may be determined with great accuracy. By means of the horizontal graduated circle, the angle between the magnetic axis and the known



FIG. 136  
Magnetometer.

direction NS of the astronomical meridian, i.e. the declination is then readily obtained.

**144. Coulomb's Law.**—When one pole of a very long bar magnet is brought near a pole of a very light magnetic needle, suspended by a silk fibre, the needle being freed from the influence of the earth's magnetism (being astatic), while the other pole is removed to so great a distance that its effect upon the needle may be neglected, the magnetic needle will vibrate according to the laws of the pendulum, until it finally attains its position of equilibrium. From the number of vibrations at different distances from the pole an accurate estimate may be formed of the ratio of the forces which the pole of the magnet exerts upon the pole of the needle at these various distances. The forces are found to be related to each other as the squares of the numbers of their vibrations (40). Hence the force with which two magnetic poles mutually attract, or repel, is *inversely proportional to the square of their distance* (Coulomb, 1785). Coulomb also proved this law by means of the torsion balance (52).

The force between two magnetic poles is moreover dependent upon the pole strengths. A magnetic pole which exerts double the force of another pole upon the same magnetic needle at the same distance, is said to have twice the intensity of the other pole, i.e. it is assumed that *force and pole-strength are proportional*. The reciprocal action of two magnetic poles is accordingly proportional to the product of the intensities of the poles.

*As an absolute unit of magnetic polar intensity it has been decided to use the pole which exerts upon an equally strong magnetic pole, a force of 1 (1 dyne) at a distance 1 (1 cm.).*

We have, therefore, for the force  $F$ , acting between two magnetic poles of intensities  $m$  and  $m'$  at a distance  $r$  (cm.) expressed in dynes,

$$F = \frac{mm'}{r^2},$$

where the intensity is regarded as positive, or negative, according as the pole is north, or south. Instead of pole-intensity it is the custom to use also the phrase, "*magnetic mass*."

**145. Magnetic Field—Lines of Force—Equipotential Surfaces.**

—The space about a magnet over which its magnetic effect extends is called its magnetic field. This field is really of infinite extent, but for convenience it may be regarded as limited by those points at which, on account of too great distance, the magnetic force becomes inappreciable. For every point of the field, by means of Coulomb's law the force may be determined which acts upon a unit magnetic pole situated at that point. This force is called the strength of the field at this point.

A small magnetic needle movable in all directions about its centre at every point of the field assumes the direction of the force acting at that point. If the needle is moved through its field always in the direction toward which its north pole points, its centre describes a line (usually curved) called a *line of force*. This line proceeds from the north pole of the magnet and terminates at the south pole. In each of its points the lines of force in the field of a magnet may be very beautifully represented by sprinkling iron-filings over the surface of a sheet of paper lying upon the poles of the magnet.



FIG. 137.—Lines of Force.

The filings are converted under the influence of the magnet into small magnets, which arrange themselves end on, along the lines of force (Fig. 137).

Surfaces which everywhere intersect the lines of force of a magnetic field perpendicularly are called equipotential surfaces. No magnetic force whatever is directed along the surfaces, and a magnetic pole may therefore be displaced along them without the performance of any work. On the contrary, work must be done to move a magnetic pole against the magnetic force from one equipotential surface to another. By means of the lines of force, together with the equipotential surfaces, the constitution of a magnetic field may be completely represented.

Every element of an equipotential surface is pierced by an infinite number of lines of force. To represent by these lines not only the direction but also the magnitude of the force at every point of the field, only such a number of lines of force are conceived to be drawn as that every superficial unit of each element of the surface shall be pierced by a number equal numerically to the force acting at the point in question. The strength of the field is then expressed by the number of lines of force per unit of surface, or by the "density" of the lines of force.

A magnetic field in which the magnetic force upon the unit magnetic mass (i.e. the strength of field) is everywhere equal and like-directed, in which, therefore, all lines of force are parallel, is said to be *homogeneous*. The magnetic field of the earth can, everywhere within moderate limits, be regarded as homogeneous. Its lines of force are parallel to the direction of the dip needle, and its strength equals the local total intensity.

The horizontal lines of force would be found by advancing from any given point upon the surface of the earth continuously in the direction of the declination needle. The lines so obtained, called magnetic meridians, run from one magnetic pole of the earth to the other, and furnish, as do the isogonals, a clear representation of the law of magnetic deviation over the surface of the earth.

**146. Magnetic Moment.**—In a homogeneous magnetic field the poles of a bar magnet are acted upon by oppositely equal parallel forces constituting a couple tending to rotate the bar magnet about its support, but producing no motion of translation. If  $m$  denote the magnetic mass,  $H$  the strength of the field,  $Hm$  is the force acting at each pole. If  $l$  denotes the distance between the poles (approximately the length of the magnetic bar), and  $\alpha$  the angle between the magnetic axis and the direction of the force,  $l \sin \alpha$  is then the lever-arm of the couple, and consequently  $Hml \sin \alpha$ , or when the magnetic bar is perpendicular to the lines of force ( $\alpha = 90^\circ$ ),  $Hml$  will be the moment of rotation. The product  $ml = M$ , i.e. the moment of rotation of a magnet standing perpendicularly to the lines of force in a homogeneous field of unit intensity is called its *magnetic moment*.

**147. Reciprocal Action of two Magnets.**—We shall limit our attention to the following simple case. Upon a magnetic needle ( $+ \mu, - \mu$ ), movable in a

horizontal plane, the needle being placed in the magnetic meridian, let a bar magnet lying in the same horizontal plane act, the latter magnet having its axis perpendicular to the magnetic meridian through the centre of the needle (138). Let the length,  $l$ , of the bar magnet, and also that of the needle, be small in comparison with the distance,  $r$ , of their centres. Let  $m$  and  $\mu$  be the magnetic masses, or polar intensities, of bar and needle, we shall have for the force  $K$ , with which the two poles  $+m$  and  $-m$  act upon the pole  $+\mu$ .

$$K = \frac{m\mu}{(r - \frac{1}{2}l)^2} - \frac{m\mu}{(r + \frac{1}{2}l)^2} = \frac{2m\mu l}{r^3 - \frac{1}{4}l^3}$$

and if  $l$  is so small that  $\frac{1}{4}l^3$  may be neglected in comparison with  $r^3$ ,

$$K = \frac{2m\mu l}{r^3} = \frac{2\mu ml}{r^3},$$

or since  $ml = M$  is the magnetic moment of the bar,

$$K = \frac{2\mu M}{r^3},$$

i.e. the action of a bar magnet (having two oppositely equal poles rigidly connected) upon a distant pole is approximately inversely proportional to the third power of the distance.

#### 143. Determination of Horizontal Intensity and Magnetic Moment (Gauss).

— Upon the pole  $+\mu$  of the magnetic needle the horizontal force,  $H\mu$ , acts in a direction parallel to the meridian, where  $H$  denotes the horizontal component of the earth's magnetism. The needle is drawn out of the meridian by the force  $K$  through the angle  $\phi$ , until its direction coincides with the resultant of  $K$  and  $H\mu$ . This occurs when



FIG. 138. — Action of two Magnets.

$$\frac{K}{H\mu} = \tan \phi, \text{ or } \frac{2\mu M}{r^3} = H\mu \tan \phi,$$

whence

$$\frac{M}{H} = \frac{1}{2} r^3 \tan \phi.$$

The ratio  $\frac{M}{H}$  is found, therefore, by observing the deviation  $\phi$  and measuring the distance  $r$ .

If now the bar magnet  $(+m, -m)$  be suspended from its centre, it will vibrate like a pendulum under the influence of the horizontal component of the earth's magnetism. Its time of vibration,  $t$ , must satisfy the equation (40, and 53),

$$t = \pi \sqrt{\frac{k}{MH}},$$

if  $k$  denote the moment of inertia of the bar, while  $MH$  represents its moment of rotation. If, therefore, the moment of inertia,  $k$ , and the time of vibration,  $t$ , are determined, we have

$$MH = \frac{\pi^2 k}{t^2}.$$

By means of the deviation formerly observed and this time of vibration,

we have thus found the ratio  $\frac{M}{H} = A$  and the product  $MH = B$ . From this we have in absolute measure the horizontal intensity,

$$H = \sqrt{\frac{B}{A}},$$

and the magnetic moment of the bar used,

$$M = \sqrt{AB}.$$

The foregoing values of the horizontal intensity were obtained in this way.

**149. Influence, or Induction, or Inductive Action, of a Magnetic Field.** A bar of iron held in the direction of the dip needle becomes magnetic through the action of terrestrial magnetism, the lower pole becoming a north pole, and the upper a south pole. If the bar be reversed the poles will exchange places. If held in any other direction, only the component of the total intensity in this direction tends to magnetize the bar, and this component, of course, becomes smaller, the greater the angle between its direction and the direction of the dip needle. It vanishes entirely when the bar is held perpendicular to the direction of the dip needle. The magnetizing effect of the earth upon rods and bars placed vertically is very considerable, since the direction of the dip needle differs but little in our regions from the vertical. Steel bars held in the direction of the dip needle, or even vertically, become permanently magnetic, especially when continually subjected to blows in this position. Shocks, indeed, seem to promote the rotation of the little molecular magnets. This explains why the tools in a locksmith's workshop usually exhibit magnetic properties.

The polar intensity (*magnetic mass*) developed upon each end-surface of a bar per superficial unit, is obviously proportional to the intensity,  $T$ , of the magnetic field, and therefore equals  $\kappa T$ , where  $\kappa$  is a numerical factor peculiar to the substance composing the bar, and is called the *coefficient of induced magnetization*. If  $q$  is the cross-section,  $l$  the length,  $v = lq$  the volume of the bar under the influence of the magnetic field,  $T$ , it attains a magnetic intensity, or a magnetic mass,  $m = \kappa T l$ , and the moment is  $M = \kappa l q T = \kappa v T$ . The number,  $\kappa$ , expresses the ratio of the induced magnetic moment at any point of the magnetic field per volumetric unit to the intensity of the field at that point, and is therefore a measure of the magnetic susceptibility of the magnetized substance. For soft iron  $\kappa = 32$ , but for micaceous iron ore,  $\kappa = 0.15$ .

## VII. ELECTRICITY.

**150. Electrification.**—When a glass rod, or a bar of sealing-wax, is rubbed with a piece of cloth, it acquires the property of attracting light bodies, such as fragments of paper, little pieces of pith, etc. As this property was first observed by the ancients (Thales, 600 B.C.) in amber, which was called by the Greeks “electron,” the condition into which the body is put by rubbing it is described as *electrical*, and the cause of this condition is termed electrical excitation, or *electrification* (Gilbert, 1600).

**151. Conductors and Non-conductors.**—Besides those mentioned, other bodies, *e.g.* sulphur, precious stones, mica, silk, resins (shellac, sealing-wax, amber), caoutchouc (hard rubber, ebonite), guttapercha, paraffine, etc., exhibit this property. On the other hand, it is impossible to electrify by friction a metallic bar held in the hand. If, however, a metal bar is provided with a handle of glass, or hard rubber, and the latter be held in the hand, the metal bar will likewise become electrified on being rubbed, though it loses this property instantly when touched with the finger. We infer from this that, when held directly in the hand the effect which we call electricity is really produced by the rubbing, but that it immediately escapes through the metal and the hand, whereas, when grasped by the glass, or ebony, handle, the escape of the electricity is prevented. While, therefore, a metal propagates, or conducts, electricity, glass and rubber do not. The former is, therefore, called a *conductor*, and the latter are *non-conductors* (Gray, 1729). Electricity imparted to a conductor at a single point is distributed at once over the entire body and escapes into the earth, which is also a conductor, when the body is connected with the earth by a third conductor. With a non-conductor, on the contrary,

the electricity remains upon the point where it was produced, and, when touched by a conductor, only the electricity of the point touched will escape.

The metals are the best conductors. Less perfect conductors are the human body, carbon, graphite, water, acids, saline solutions, leather, many varieties of stone, and the earth. On the other hand, the non-conductors, or, speaking more correctly, the very poor conductors, are the bodies already enumerated above, which, by reason of this property, hold the electricity developed by friction on their surfaces. Some liquids are also poor conductors, such as oil, petroleum, alcohol, carbon disulphide, as also the air and all gases.

**152. Insulation.**—In order that a conductor may preserve the electrical condition into which it has been brought by any means whatsoever, it must be surrounded by non-conductors, that is to say, it must be completely separated, or *isolated*, or *insulated*, from all other conductors, and especially from the earth. From this application of non-conductors, they are also called *insulators*. A metallic body held by a glass handle, or standing upon glass legs, is insulated, for the air which surrounds the body and still touching it is a non-conductor, if dry. The water vapour contained in moist air is also a poor conductor, but it deposits upon the surface of the solid insulator a thin layer of water, which makes it a conductor.

**153. Two Kinds of Electricity.**—To observe conveniently the attraction of an electrified body, a small ball of cork, or pith, may be suspended by a metallic, or linen, thread from a conducting standard. This simple apparatus is called the *electric pendulum*.

If the pith-ball is suspended by means of a silk thread to a glass support, it is insulated. If a glass rod is rubbed and then brought near the electrified ball, the ball is attracted to it and remains in contact with it for a short time, after which it is permanently repelled by the rod. The ball will be more strongly attracted by a bar of sealing-wax after rubbing, when in the condition under which it was repelled by the glass and before having touched the rod. If the ball has been put into its unelectrified state by touching it with the hand, and then

brought near the electrified bar of sealing-wax, it will be first attracted by the bar and then permanently repelled, after which it will be more strongly attracted by the glass rod than originally.

Bars of glass and of sealing-wax are therefore in different electrical states, since they exert upon the pith-ball diametrically opposite effects. If other non-conductors be similarly tested with the insulated electrical pendulum, it will be found that they comport themselves either as glass, or as sealing-wax (*resin*).

There are, therefore, two different electrical conditions for whose explanation we assume two different electricities, called *vitreous electricity* and *resinous electricity* (Dufay, 1733). If a glass bar after excitation is suspended in a horizontal position, it will be repelled by a second glass rod after excitation and attracted by an excited bar of sealing-wax. A bar of sealing-wax suspended likewise is attracted by the bar of glass and repelled by a second bar of sealing-wax. Since this motion of heavy masses is merely a consequence of their electrical conditions, we are justified in ascribing the observed effect to the electricities. It follows, therefore, that *like electricities repel and unlike attract each other*. The attraction, or repulsion, of two electrified bodies, which are small in comparison with their distance, is always in the direction of the straight line connecting them.

**154. Electrical Transmission, or Conduction.**—It is now clear that the insulated ball of the electrical pendulum mentioned above became charged with vitreous electricity on touching the glass rod and with resinous electricity on touching the rod of wax. We observe, also, that light bodies, which have been attracted by an electrified body and at first cling to it, but after a time are repelled by it, have been charged with the same kind of electricity. The electricity of a body may also be transferred without qualitative change to an insulated conductor by simple contact, or, as we say, it may be charged with electricity *by conduction*. Consequently, any body may be charged with either vitreous or resinous electricity by friction or by transmission. Transmission of the sort just mentioned is ordinarily called *conduction*.

**155. Quantity of Electricity—Electrical Mass.**—If an electrified spherical conductor is brought into contact with an exactly similar, though unelectrified, conductor, after they are separated, both will be equally strongly electrified and both will be weaker than was the first conductor before the contact. This may be proved by means of the electrical pendulum. The original charge of the first conductor has therefore distributed itself uniformly over the surfaces of both. An electrical charge may, therefore, be divided, and consequently also multiplied. Hence the need of the expression quantity of electricity, or electrical mass, whose magnitudes are expressible in a properly chosen unit of electrical mass. It is assumed that the electrical mass, or the quantity of electricity, of a body is proportional to the attracting, or repelling, force which the body is capable of exerting upon a second body under conditions unmodified by everything save the influence of the first body, *e.g.* the ball of an electrical pendulum.

**156. Positive and Negative Electricity.**—Let one of two similar insulated metallic balls be charged with vitreous and the other equally strongly charged with resinous electricity. As to whether they are equally strongly electrified, may be tested by noting whether the electrified ball of the pendulum is drawn aside from the vertical position of equilibrium by equal amounts in both cases. If the balls are brought into contact with each other they lose their electricities completely. The two unlike electricities combine with each other in equal quantities mutually destroying, or neutralizing, each other. Two magnitudes comporting themselves thus are designated as opposites, the one being called positive and the other negative. When, for example, a hole is excavated in the ground, the earth thrown out is a positive magnitude, the hollow is the corresponding negative magnitude; when both are united, that is, when the earth is thrown into the hollow, they neutralize each other, so to speak, and the original smooth surface is restored. The behaviour of the two opposite electricities with respect to each other is appropriately represented by calling the one positive (+), and the other negative (-). As to which should be considered positive, the phenomena themselves give us no

hint. It has, therefore, been conventionally established that vitreous electricity shall be called positive, and resinous negative (Lichtenburg, 1777).

**157. Simultaneous Production of both Electricities.**—Just as it is impossible to dig the hole without throwing up the heap of earth, so is it also impossible to produce the one electricity without the simultaneous production of an equal quantity of the other. If a bar of glass is stroked with a scrap of rubber and brought near the negatively electrified ball of the pendulum, the rod attracts it, while the rubber repels it. Consequently, the latter must be negatively and the former positively electrified by the rubbing. If the glass rod and the piece of rubber act together upon the pendulum, no effect whatever is perceptible, hence the opposite electricities were produced in equal quantities.

This justifies the view that the two electricities are not created by friction, but that they are present in any unelectrified body, and are combined with each other in equal masses. The friction merely separates them, allowing the one electricity to collect upon the body rubbed, and the other upon the material used in rubbing. Unelectrified bodies are for this reason said to be in a *neutral state*.

By rubbing dissimilar bodies against each other in pairs, and testing them with a pendulum, whichever sort of electricity each body may acquire, it will be possible to arrange all bodies in a series, each of which, when rubbed by one of the following, becomes positive, and, by one of the preceding, it becomes negative (Canton, 1754). The most important bodies of this friction series are : hair (cat's fur, tail of a fox), polished glass, wool, paper, silk, roughish glass, caoutchouc, resin (sealing-wax), amber, sulphur, metals, collodium (gun-cotton). The farther apart two substances are in this series the better their effect. Resin is, therefore, used with felt, glass with metal, or amalgamated leather.

**158. Location of the Electric Charge.**—Electricity can be in equilibrium only on the surface of a conductor, never on its interior. For, since the parts of the same kind of electricity mutually repel within a conductor, they must separate from

each other, inasmuch as the conductor offers no resistance to this separation, until they are brought to a stand by a non-conductor, which exists only at the surface of the conductor.

If, for example, a metallic ball, insulated on glass, is electrified and covered by two hollow metallic hemispheres, fitted with glass handles, it is found, on removing the hemispheres, that the ball is entirely unelectrified. Its electricity has been transferred to the hemispheres which for the moment formed its surface (Coulomb).

Let a metallic column be placed upon an insulated metallic plate from which a pith-ball is suspended by a slender wire. If the metallic plate be charged with electricity, the pendulum will be strongly repelled by the column. If, now, a bell-shaped vessel, made of wire gauze and held by a glass handle, is inverted over the column and the pendulum, the latter will hang perfectly slack. It has now become a part of the interior of the entire conductor, on the surface of which all the electricity is now collected, which is seen by means of strips of gold-leaf stuck to the outer surface of the gauze cover. The latter hang no longer in a passive state, but, on the contrary, are violently repelled. If the entire conductor, consisting of the plate, the pendulum, and the cover, is charged with electricity, after having been first brought into an unelectrified state, the strips of gold-leaf will be repelled at the surface, while the electrical pendulum within remains quietly at rest.

On the interior of a conductor over which electricity is spread and equilibrium is established, the neutral state always exists. No electrical force acts on the interior, or, rather, the internal electrical forces hold each other in equilibrium. Pieces of metal used in experiment on electrical equilibrium need not, therefore, be solid. Hollow ones will suffice equally well.

**159. Electrical Density.**—Upon an insulated sphere wholly freed from external effects, electricity distributes itself uniformly, i.e. it has everywhere the same density. By this is meant that upon equal surfaces the same quantity of electricity is present, and the quantity present upon unequal surfaces is proportional to the area of the surfaces.

Upon conductors of other than spherical form, electricity is not uniformly distributed. It is, however, always distributed in such manner that its effects upon every point of the interior neutralize each other. The density at any point of the surface is given by the ratio of the quantity of electricity present upon a limited element of area surrounding this point to the magnitude of the area.

To compare the densities at different points of the surface of a body, the surface is touched with a metallic disk provided with an insulating handle (proof-plane), or with a metallic ball (proof sphere), which bears away upon its surface from the point touched a portion of the charge proportional to the area of its surface, without appreciably diminishing the entire charge. A comparison of the quantities of electricity removed by the proof-plane, when touched at different points, furnishes the ratio of the densities at these points.

Upon an ellipsoid, for example, the density at any point of the surface is proportional to the distance of the centre of the ellipsoid from the element of surface at the point. This distance is the length of a perpendicular dropped from the centre of the ellipsoid upon the tangent plane at the point in question. The electricity consequently collects most thickly at the ends of the maximum axis. If this axis is very long in comparison with the other axis, the density will increase rapidly toward the ends, and attain here a greater value the more pointed the ends. If the axis of rotation of an ellipsoid of revolution becomes continually smaller and smaller, the ellipsoid is gradually converted into a circular disk upon which the density increases outward at first very slowly, then more rapidly, and, finally, most rapidly just at the edge. In general, electricity collects with greatest density at those places where the radius of curvature is least, and especially at edges, corners, and points.

**160. Electrostatic Pressure.**—The repulsion of the particles of electricity, distributed over the surface of a conductor, would drive the electricity ever farther from the surface, were it not for the fact that the resistance of the surrounding non-conductor sets a limit to it. This force, after equilibrium prevails,

is at every point of the surface perpendicular to the corresponding surface element; for if it were oblique, a motion of translation must occur along the surface, and equilibrium could not obtain. At every point of the surface, this force directed outward, in case it acts upon the unit of electrical mass, is proportional to the density at the point, and exerts accordingly upon the electrical mass covering an element of area, which mass is in turn proportional to the density, an electrostatic pressure proportional to the square of the density.

If the conductor is surrounded by air, this pressure acts against atmospheric pressure and diminishes it. It may, in fact, be proved (van Marum), that a hollow glass globe filled with hydrogen is apparently lighter, and its buoyant force greater, after electrification than before.

**161. Action of Points.**—When the density becomes sufficiently great, the electrostatic pressure increases until, in spite of the resistance of the surrounding insulator, it is driven from the surface, and a loss of electricity results. If the electrified body be surrounded with air, the atmospheric particles in contact with the body, as also the particles of dust floating in the air, become similarly electrified and are repelled with greater intensity the greater the electrical density. The electrified air escapes, from points especially, with such force that a current of electrified air is distinctly perceptible to the hand, and this current is strong enough to blow aside the flame of a candle (Franklin, 1747). Electricity is therefore said to *flow* from points.

When a light metallic wheel consisting of five or six wires terminating in points all bent in the same direction is placed upon an insulated pivot, it will rotate in the direction opposite to that in which the curved wires point. This apparatus is called an *electric whirl*, or *vane*. The reaction of the electrified air escaping from the points in the direction opposite to that in which the currents move (cf. 69), is of sufficient force to set the wheel in rapid motion.

A conductor terminating in a point can be only feebly electrified, because the electrical current escaping from the point rapidly relieves the charge. For this reason, when a

conductor is required to hold its electrical charge, it must be given a form as nearly round as possible, by the removal of all sharp edges and corners. If, on the contrary, it is desirable that it shall lose its electricity with readiness, it is provided with points. Flames and currents of smoke rising from smouldering masses act similarly to points.

**162. Coulomb's Law.**—Coulomb (1788) determined the force with which two small electrified bodies mutually repel and attract, by means of a torsion balance, constructed by himself (cf. 52). He suspended, by a delicate silver wire (glass or quartz fibre), a horizontal bar of shellac (Fig. 139), which carried at its end a light gilt ball of pith. The bar was suspended within a glass cylinder, from the cover of which a vertical glass tube rose, through which the suspending wire was passed. The position of the bar was read from a graduated circumference about the middle zone of the cylinder. The glass tube was provided above with a metallic cap carrying a circle whose circumference was graduated into degrees. Upon this cap was fitted a movable metallic plate, provided with an index at its edge. To this plate the suspending fibre was attached. Through a hole in the glass lid, by means of a shellac handle, a second equal ball was brought close beside the first. When the fixed ball was electrified and brought in contact with the movable ball, the latter, becoming similarly electrified, was repelled by the former. The horizontal bar turned and twisted the thread firmly fastened at its upper end until the force, which, by virtue of torsional elasticity (cf. 52), resisted the twisting, held the force of repulsion in equilibrium. To bring the movable ball nearer the fixed ball, the upper metal plate had to be rotated, thereby twisting the wire



FIG. 139. -Torsion Balance.

still more, and the angle of rotation was read from the graduated head above. With the balls at various distances apart, the electrical force of repulsion was measured by the elastic force of the twist which held the former force in equilibrium. The elastic force was known to be proportional to the angle through which the suspending fibre was twisted, therefore proportional to the sum of the angle formed by the displaced bar with its position of equilibrium, and the angle through which the torsion head was turned. When the initial distance from the position of equilibrium was 1, and this had afterward been reduced successively to  $\frac{1}{2}$  and  $\frac{1}{3}$  its value, the corresponding twists, and accordingly, also, the forces of repulsion, were as  $1 : 4 : 16$ , i.e. inversely as the squares of the distances. The law of attraction for *unlike* charges of electricity was found to be precisely the same.

If, in this same way, the forces with which the fixed ball repels the movable ball at a fixed distance are determined after having diminished the charge of the latter by contact with an equally large unelectrified ball to  $\frac{1}{2}$  and then to  $\frac{1}{3}$  of the original charge, it is found that the repelling forces are as  $1 : \frac{1}{4} : \frac{1}{9}$ , therefore as the electrical masses operating. This is, moreover, obvious on recalling the definition of electrical mass as given above (155).

We have then the law of Coulomb: *The force with which two electrical particles act upon each other is directly proportional to their electrical masses and inversely proportional to the squares of their distances.*

Coulomb confirmed this law by a wholly different method. He suspended by a silk fibre, near a large insulated metallic ball charged with electricity, a horizontal bar of shellac at the height of the centre of the large ball. To one end of the horizontal bar he attached a small conducting ball charged with electricity, opposite in kind to that on the large ball. The bar moved in the direction of the centre of the sphere. It can be shown that if the law of inverse squares of the distance holds, the entire electrical mass must act just as though it were concentrated at the centre of the sphere. If, after reaching its position of equilibrium, the small ball be drawn slightly out of this position, it will vibrate about it in accordance with the

laws of the pendulum, but more slowly the farther it is removed from the large ball. By the aid of a chronometer Coulomb counted the number of vibrations in equal times, and measured in each case the distance from the bar to the centre of the sphere. According to the law of the pendulum the forces are as the squares of the number of vibrations (40). The ratios of the forces acting at the various distances were thus determinable. It was found that they were to one another in the inverse ratio of the squares of corresponding distances.

Both with the torsion balance and with the method of vibrations, the loss of electricity during the experiment acted as a disturbance. Coulomb was, however, able with the torsion balance to determine and allow for this loss, and thereby to remove every objection to the validity of this important fundamental law of the action of electrical forces.

Coulomb's law is, however, much more easily and accurately shown by the principle now to be proved, that the electrical charge is wholly upon the surface of a conductor, and that at every point on the interior the electrical forces mutually destroy each other.

Upon the surface of a sphere electricity is distributed with uniform density. Conceive now that through any point,  $P$ , of the interior, a slender double cone is passed with its vertex at  $P$ . It will intercept upon the surface of the sphere two surfaces,  $\sigma$  and  $\sigma'$ , whose areas are to each other as the squares of their distances,  $r$  and  $r'$ , from the point  $P$ . The electrical masses with which they are charged are in the same ratio, and act proportionally to their magnitudes, and in opposite directions, upon an electrical particle situated at  $P$ . Since equilibrium maintains at  $P$ , the two opposite forces must equal each other. This can only be possible when the greater electrical mass (at  $\sigma'$ ) is weakened in consequence of its greater distance ( $r'$ ) in the inverse ratio of  $r' : r$ . The action of electrical masses must, therefore, be in the inverse ratio of the squares of their distances.

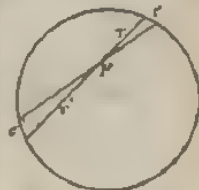


FIG. 140.—Coulomb's Law.

If  $\sigma$  and  $\sigma'$  denote the electrical masses of two small particles separated by a distance  $r$ , and  $f$  signify a positive constant depending upon the choice of the unit of electrical mass, the force,  $F$ , will be expressed by  $F = f \cdot \frac{\sigma \sigma'}{r^2}$ . If both electrical masses are of the same kind,  $F$  is positive, and denotes a repellent force tending to separate the particles. If the electricities are opposite,  $F$  is negative, and denotes an attractive force tending to diminish the distance between them.

If for the unit of electrical mass we select that mass which,

acting upon an equal mass at a distance 1 (1 cm.), exerts a force of 1 (1 dyne),  $f = 1$ , and Coulomb's law assumes the following simple form:—

$$F = \frac{ee'}{r^2}.$$

**163. Action of an Electrified Sphere.**—It follows from Coulomb's law that a sphere, over whose surface electricity is uniformly distributed, acts upon an electrified point as though the entire electrical mass were condensed at the centre of the sphere (cf. 45).

Let the external point, P, be situated at a distance,  $OP = r$ , from the centre, O (Fig. 141) of the sphere. Let a cone be described having its vertex at P and tangent to the sphere (i.e. the cone PQQ'), its base, QQ', will intersect the straight line OP in P'.



FIG. 141.—Action of a Sphere.

The radius  $OQ = R$  is then a mean proportional between  $OP' = r'$  and  $OP = r$ , i.e. we have  $r' : R = R : r$ . If now M represent any point upon the surface of the sphere, and its distance from P is  $PM = \rho$ , and from P' is  $P'M = \rho'$ , then, since  $OM = OQ = R$ , the triangles OPM and OMP' must be similar. The angle OMP' will then equal the angle OPM =  $\phi$ .

Imagine a cone of very small opening to be drawn with its vertex at P. Designate the surface element of a sphere described about P' as a centre with a radius 1 by  $\omega$ . The area of the portion of the surface of a sphere included within this cone, having its centre at P' and its radius  $\rho'$ , will then be  $\rho'^2\omega$ . Let the surface element intercepted on the given sphere at M by the same cone be denoted by  $\sigma$ . Since the straight lines PM and OM, drawn perpendicular to the surface elements  $\rho'^2\omega$  and  $\sigma$ , form with each other the angle  $\phi$ , we have  $\sigma = \frac{\rho'^2\omega}{\cos \phi}$ . If  $\delta$  is the electrical density upon the sphere, and if a unit of electrical mass is situated at the point P, the force exerted by the surface element at M upon the point, P, in the direction, MP, is, according to Coulomb's law—

$$\frac{\sigma\delta}{\rho^2} = \frac{\rho'^2\delta\omega}{\rho^2 \cos \phi}.$$

Decomposing this force into two components, one perpendicular to, and the other along OP, the former will be destroyed by the opposite component due to the point M', situated symmetrically to M, and the component along OP will be found, by multiplying the foregoing expression by  $\cos \phi$ , to be equal to  $\frac{\rho'^2\delta\omega}{\rho^2}$ . From the similarity of the triangles OMP' and OPM, we have  $\rho' : \rho =$

$R : r$ , and this component equals also,  $\frac{R^2\delta\omega}{r^2}$ .

To obtain the total force acting upon P along OP, it is only necessary to add the corresponding components for all points of the surface of the sphere.

We obtain then, instead of the latter expression, the sum of all the elements of surface of a sphere described about  $P$  with radius unity, i.e. the complete surface of this sphere, which equals  $4\pi$ .

The force exerted by the sphere upon the point  $P$  is, therefore,

$$F = \frac{4\pi R^2 \delta}{r^2},$$

or, since  $4\pi R^2$  is the surface of the given sphere, and consequently  $4\pi R^2 \delta$  expresses the electrical mass,  $E$ , distributed over the entire sphere,  $F = \frac{E}{r^2}$ , in which equation the above proposition is contained.

At the surface of the sphere  $r = R$ , and the force acting upon the unit of electrical mass is  $F = 4\pi\delta$ . If the electricity be conceived as a layer of extremely low density upon the surface,  $F = 4\pi\delta$  is then the force acting upon a point of the exterior surface of the layer and charged with a unit of electrical mass upon its internal surface, i.e. upon the conductor itself the force is zero, as also throughout the interior. Within the layer, however, the force increases continuously from the value 0, at the surface of the conductor, to the value  $4\pi\delta$ , upon the outer limit of the layer. Upon the electrical mass  $\delta$ , distributed over the superficial unit of the layer, a force of intermediate value between 0 and  $4\pi\delta$  will act. If we assume that this intermediate value is the arithmetical mean  $2\pi\delta$ , the electrostatic pressure exerted upon the electrical charge per superficial unit of area equals  $2\pi\delta^2$ . These values of force and pressure at the surface hold not only for the sphere, but with complete generality, for any form of conductor.

**164. Electrical Field—Tension (Potential)—Equipotential Surfaces—Lines of Force.**—The region within which the influence of the electrified body is exerted is called the electrical field, and the force acting at one of its points upon unit electrical mass is called the intensity, or strength, of the field at that point. Strictly speaking, the field is of infinite extent, but it may be regarded as limited by those points at such distances, that, according to Coulomb's law, the action of the electrical forces becomes imperceptibly small.

Imagine a positively electrified body, and at any point of its field a particle charged with a unit-mass of positive electricity, then the electrical charge of the sphere, which is regarded as constant, while repelling the electrical particle to the outermost limit of the field, performs a definite quantity of work. The work required to carry the electrified particle against the repellent force from the extreme limit of the field (therefore, speaking strictly, from an infinite distance), or, generally, from a point of the field where the action is imperceptible, to its original position is precisely equal to the former

work. This work is the measure of the electrical capacity of the potential energy existing in this part of the field. It is called the electrical tension, or the electrical potential at this point. For all points which are at the same distance from the centre of the sphere, or at the same *level* with respect to the surface of the sphere, the electrical potential has obviously the same value. If a series of spherical surfaces be described about the electrified sphere, each with a diameter greater than the preceding, they will represent all the surfaces of equal potential, or all *equipotential surfaces*. Upon each one of these the tension has everywhere the same value, but it increases in passing from one to another toward the electrified sphere.

To move an electrified particle *along* an equipotential surface requires no expenditure of force, for the only repellent force which could resist displacement (from the symmetry of the body) acts directly from its centre, and, consequently, perpendicularly to the spherical equipotential surfaces. On the other hand, to carry a particle *from* one equipotential surface to another a certain quantity of work must be performed, or a certain quantity of work is consumed. This work is equal to the difference of the corresponding values of the potential, regardless of the path along which the particle passes from one surface to the other.

But this is not alone true of the simple case of the sphere, which has been the only case thus far considered. However the electrified bodies may be constituted, or situated, the distribution of tension, or potential, in their fields may always be represented by a series of such equipotential surfaces. These surfaces are, however, in general not spherical, but curved surfaces of various forms. If lines be conceived as drawn so as to pierce the successive surfaces everywhere at right angles, each of these lines will represent the direction of the force at its point of intersection with the surface. For this reason these lines are called lines of force. In the case of the sphere the lines of force are straight lines radiating from the centre. In general, however, they are curved lines.

If the direction and magnitude of the force everywhere throughout the field are the same, the lines of force are parallel

straight lines, and the potential surfaces are planes perpendicular to them. The field is then said to be *homogeneous*.

**165. Fall of Potential.**—The fall of potential at any point of the field is the ratio of the small difference of the values of the potential at the ends of a short line, to the length of this line. It expresses the magnitude of the electrical force acting at the end of the line in the direction of the line. It is greatest in the direction of the lines of force, for, in this direction, the full strength of the force operates, while, in any other direction, only a component of the full force is effective. The *fall of potential* and, consequently, also the force, in the direction perpendicular to the direction of the lines of force, i.e. along the equipotential surfaces, is 0.

In consequence of the fall of potential, freely moving particles of positive electricity pass continually from points of higher to points of lower potential, just as water always flows from a higher to a lower level when under the influence of gravity alone.

**166. Equilibrium in Conductors.**—When a conductor has attained a condition of electrical equilibrium, no further motion of the electrical particles occurs; the electrical forces and, accordingly, also the electrical potential, are everywhere 0. This merely says that in a position of equilibrium, every point in and upon a conductor has the same potential, or upon the conductor the potential is everywhere constant. The electrical charge always arranges itself upon the surface in such way as to make the potential the same throughout the entire conductor. The surface of the conductor is, consequently, in case of equilibrium, an equipotential surface.

**167. Dielectrics.**—Since, on the interior of conductors in equilibrium no fall of potential and, accordingly, no force exists, lines of force do not extend into the interior of conductors. They are distributed only within surrounding non-conductors, and always proceed from, or end at the surface of the conductors to which they are perpendicular. So soon as equilibrium is established, therefore, the electrical field comprises, not the space occupied by the conductors, but only

that space which is composed entirely of regions containing insulating substances. For this reason Faraday has called non-conductors *dielectrics*, to indicate that electrical forces exist within them, and are transmitted from particle to particle through their masses.

**189. Electrical Capacity.**—If an electrical connection is made between two conductors of different electrical potential, positive electricity will flow from the body of higher potential to that of lower, until the potential upon both bodies, now acting as a single conductor, is everywhere the same. This is precisely analogous to the case in which two vessels, filled with water, are connected by a tube in which the water soon assumes the same level in both.

But, just as a vessel of greater capacity must contain a greater quantity of water, if filled to a given level, so, also, for example, a sphere of greater radius requires a greater quantity of electricity to charge it to a definite potential than is required by a smaller sphere, *i.e.* the larger sphere has a greater electrical capacity. By the electrical capacity of a conductor, we mean the quantity of electricity which is required to raise its potential by one unit. The quantity of electricity,  $E$ , required to charge a conductor to a definite potential,  $V$ , is then equal to the product of its capacity,  $C$ , into this potential, or  $E = CV$ . It may also be said that the capacity of a body is the ratio of the quantity of electricity upon it to its potential, or  $C = \frac{E}{V}$ .

The earth acts as a reservoir of such enormous capacity that all quantities of electricity artificially produced, when distributed over its surface, cannot appreciably raise its potential. Its capacity is, so to speak, infinitely great.

The idea of electrical capacity is allied to that of thermal capacity, *i.e.* the quantity of heat required to raise the temperature of a body by  $1^\circ \text{C}$ . While, however, thermal capacity is dependent only upon the weight and material of the body, electrical capacity is independent of the conductor. It depends rather upon its magnitude and form, and, as we shall soon see, it is further influenced by the presence of other conductors within the field.

**189. Values of Potential and Capacity.**—The potential at any point of a field cannot be determined absolutely. The difference of its potential from that of the earth is what we always seek to ascertain, the electrical tension of the earth being assumed as 0, just as the altitude of a station is referred to the level of the sea, or temperature, to the melting-point of ice. A positively electrified body has then a positive potential (*i.e.* above 0), and a negatively electrified body, a negative potential (below 0). If, for example, a negatively electrified body is in electrical connection with the earth, positive electricity flows over upon it from the earth, whose potential is higher (zero) than that of the body, until the electrified body has become neutral, having also assumed the potential zero. Every conductor which is connected with the earth (*put to ground*) has a zero potential.

The value of the potential of the electrified mass,  $e$ , considered as condensed into a point, upon a second point at which the unit of electricity is situated, and at the distance  $r$  from the first, is  $\frac{e}{r}$ .

The electrical mass  $e$  acts upon the electrical unit at the distance  $r$  according to Coulomb's law, with a force  $\frac{e}{r^2}$ , and performs, while displacing the unit by the very small distance  $r_1 - r$ , to the distance  $r_1$ , the work  $\frac{e(r_1 - r)}{r^2}$ . If, however, as was assumed,  $r_1$  is but little greater than  $r$ , in place of  $r^2$ , the product,  $rr_1$ , may be used, with an error which becomes smaller, the smaller the distance  $r_1 - r$  is taken. This work may be thus expressed—

$$\frac{e}{rr_1} (r_1 - r) = e \left( \frac{1}{r} - \frac{1}{r_1} \right).$$

If, now, the particle be moved by successive steps from  $r_1$  to  $r_2$ ,  $r_2$  to  $r_3$ , ... finally from  $r_{n-1}$  to  $r_n$ , the total work performed equals the sum—

$$e \left( \frac{1}{r} - \frac{1}{r_1} + \frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{r_2} - \frac{1}{r_3} + \dots + \frac{1}{r_{n-1}} - \frac{1}{r_n} \right), \text{ or } e \left( \frac{1}{r} - \frac{1}{r_n} \right).$$

When the distance  $r_n$  becomes infinitely great, then  $\frac{1}{r_n} = 0$ , and the work which the electrical mass  $e$  performs in repelling the electrical unit to an infinite distance (to the limit of the field), and which, on the other hand, must be expended to carry the electrical unit from an infinite distance to  $r$ , or the potential  $V = \frac{e}{r}$ .

If any number of electrical masses,  $e, e', e'', \dots$  act over distances  $r, r', r'', \dots$  upon a point charged with the mass-unit of electricity, the potential at this point is—

$$V = \frac{e}{r} + \frac{e'}{r'} + \frac{e''}{r''} + \dots = \Sigma \frac{e}{r}$$

Since a sphere acts upon an external point as though its entire charge,  $E$ , were condensed at its centre, its potential upon a point at distance  $r$  from the centre is  $V = \frac{E}{r}$ , provided  $r$  is greater than the radius of the sphere. At the surface, where  $r = R$ , and also everywhere upon the interior, the potential has the constant value given by  $V = \frac{E}{R}$ .

The charge on the sphere is, consequently,  $E = RV$ , whence it follows that the capacity of the sphere equals its radius (168).

**170. Energy of the Electric Charge.**—While an unelectrified insulated conductor is being charged, a continually increasing amount of work is required for every later addition to its electrical mass, since the added electricity suffers repulsion from that already present. That work is required, is evident from the consideration that the potential of the body must increase from its initial value 0, to its final value  $V$ . Since the potential increases directly as the charge, the work performed per unit of electrical mass will be ultimately the same as if the body maintained, during the entire process of electrification, a constant potential, equal to the arithmetical mean between the initial value 0 and the final value  $V$ , viz.  $\frac{1}{2}V$ . For the electrical unit this work is, accordingly,  $\frac{1}{2}V$ , and, for the electrical mass,  $E$ ,  $W = \frac{1}{2}VE$ , or, also, since  $E = CV$  ( $C$  being the capacity of the conductor),  $W = \frac{1}{2}CV^2 = \frac{E^2}{2C}$ .

This work, in a sense stored up in the conductor, and given out by it again (*e.g.* as heat), as soon as the conductor returns to its unelectrified condition, is called the *energy* of the electrical charge, or the potential of the conductor upon itself.

**171. Electrical Induction, or Influence.**—If an unelectrified conductor, such as the insulated cylinder of Fig. 142, is brought near (into the field) of an electrified body, *e.g.* a

positively charged metallic sphere, the latter body becomes also electrified under the influence of the former. This mode of electrification is called *induction*, or *influence*. That the second body becomes electrified, is readily seen by attaching the ends of strips of gold foil to the upper surface of a horizontal cylinder. When the cylinder is in a neutral condition, the strips of foil lie motionless upon its surface. When an electrified sphere, however, is brought near the cylinder, the strips are repelled by it, and assume an upright position upon its surface. The repulsion is strongest near the ends of the cylinder, and is wholly wanting about a zone lying between the nearer end and the middle of the cylinder. This is called

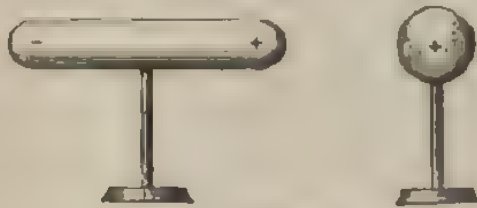


FIG. 142.—Induction.

the zone of indifference, or the *neutral zone*. If an excited rod of glass is lowered toward the strips of foil, those lying on the side of the *neutral zone* toward the electrified sphere will be attracted by the rod, and those beyond this zone will be repelled by it. The nearer end of the cylinder is, therefore, charged with electricity opposite to that of the inducing charge, and the remote end with electricity of the same kind as the inducing charge, or, the remote end is positively, and the near end negatively electrified.

The electricity attracted to the nearer end is called *induced electricity of the first kind*, and that repelled to the remote end, *induced electricity of the second kind*.

The density of the induced electricities is greatest at the ends of the cylinder, diminishes continuously thence toward the neutral zone, where it is zero, as can be readily shown by means of a proof plane.

The second conductor also reacts upon the first, and so

alters the distribution of its electricity that the greatest density occurs at the point nearest to the second conductor.

To insure equilibrium, the electricity must be so distributed upon both conductors that the effect of the body's own electricity upon each of its points is oppositely equal to the effect of the electricity of the other body upon the same point. It follows then that the electrical force at every point of the body is zero, or, that the potential upon each of the conductors is constant. Upon this depends the true nature of induction.

If the inducing body is removed from the vicinity of the cylinder, or if its charge is led off by touching it with the finger, the cylinder returns again to its original neutral condition, as is readily apparent from the falling of the strips of foil. The two opposite induced electricities are, therefore, produced in equal quantities, and, after the cessation of the inducing action, they completely neutralize each other.

**172. Electrification by Induction.**—If, while a conducting body is under the influence of an electrified conductor, it is connected with the earth, by touching it with the finger or other conductor, the electricity of the second kind escapes to earth and the gold foil at the further end sinks quietly down upon the surface of the body. The induced electricity of the first kind remains and is seen to collect about the nearer end of the cylinder to still greater density, for the strips of foil rise here into more nearly vertical positions. It is wholly immaterial where the conductor is touched. The induced electricity of the first kind will not escape even if the nearer end, where the charge is densest, is touched.\* It cannot escape inasmuch as it is needed to maintain equilibrium upon the surface of the cylinder.

If, after connection with the earth has been broken, the inducing conductor is removed, the induced electricity of the first kind, i.e. the negative electricity, distributes itself over the entire conductor in accordance with a law of density depending upon the form of the conductor. The strips of gold foil now rise at both ends with equal vigour. The strips at both ends are charged

\* For this reason, it was formerly the custom to say that the electricity of the first kind is *bound* by the attraction of the inducing charge.

with negative electricity, for an excited bar of hard rubber repels and depresses them.

By inductive action, therefore, an insulated conductor may be charged without bringing it into contact with an electrified body, the *induced* charge being opposite to that of the *inducing* charge.

**173. Suction of Points.**—If the remote end of the conductor on which the charge is induced, terminates in a point, the induced electricity of the second kind flows off from it and the conductor remains charged with induced electricity of the first kind, just as though the repelled charge had been led off by electrical connection with the earth.

If the point is at the nearer end, the opposite electricity flows off toward the inducing body and partially neutralizes its charge. The conductor now remains charged with electricity of the second kind. Since appearances are as if the point attracted, or sucked, electricity from the first body into the second, this phenomenon is sometimes designated *suction of points*. All these phenomena are clearly indicated by the strips of gold leaf.

**174. Action of Screens.**—If a metallic plate in connection with the earth be inserted between the inducing body and the insulated conductor, the strips of gold foil collapse. The conductor returns to its neutral condition. Induction now extends immediately to the metallic plate, the front side of which becomes covered with electricity of the first kind, and, since the electricity of the second kind escapes into the earth, the back side remains unelectrified. The electrified body and the metallic plate now form a compound body upon which the electricity is in equilibrium, and can, therefore, not act through the metallic plate upon a conductor situated behind it. The lines of force proceeding from the electrified body terminate upon the front side of the metallic plate which forms a screen protecting the body behind it from the influence of the electrified body. A glass screen interposed between the two bodies does not have this effect. It permits the passage of the lines of force, or is *dielectric*.

If an insulated electrified sphere is covered with a woven

wire screen in such way as to be enclosed within a conducting envelope, no electrical influence whatever will be apparent outside of the screen, when the latter is in communication with the earth. The induced electricity of the first kind upon the interior of the envelope holds the electricity of the sphere in equilibrium. The sphere within the screen is likewise protected from the electrical influence of bodies situated without it. If the conducting envelope surrounding the electrified body remains insulated, its external surface becomes electrified under the influence of the electrified body with like electricity to that upon the body, while the inner surface of the screen becomes charged equally strongly with electricity of the opposite kind. The electricity of the inner surface holds that of the enclosed body in equilibrium, no matter where it may be situated on the interior of the screen. If, now, the body be brought in contact with the envelope, it loses its charge completely. Consequently, the quantity of electricity produced upon the interior wall must have been oppositely equal to that of the inducing body, since it was just sufficient to neutralize the charge. It is, therefore, also true that the electricity upon the exterior wall of the envelope equals that of the inducing body. The action of the body on points outside of the envelope is then, no matter where the body is situated on the interior, always the same as if its quantity of electricity were distributed over the outer surface of the envelope. Experience shows also, in general, that the effect of any number of electrical masses on the outside of an enclosing surface is the same as if the entire mass were spread over this surface (Faraday).

**175. Explanation of Electrical Phenomena by Induction.** - The phenomena of attraction, mentioned at the beginning, find their complete explanation in induction. When an excited rod of glass is brought near an insulated pith ball, the latter becomes negatively electrified on its front surface and positively electrified on its back surface. Since the negative side is the nearer to the rod, the attraction overcomes the repulsion, the ball comes in contact with the rod, and its negative electricity, awakened by induction, is then neutralized by an equally great quantity of the positive electricity of the glass rod, and the ball,

now containing only positive electricity, is repelled by the rod. Contrary to appearance, no actual communication of like electricities has occurred on contact, but there has occurred merely a neutralization of induced electricity of the first kind by an equal quantity of the electricity of the charged body. An electrified ball will even be attracted by a body, electrified by a strong charge of the same kind of electricity, if the attraction of the nearer electricity of the first kind is stronger than the repulsion of like-named electricity already present increased by the contribution due to the induced charge. If the ball is suspended upon a conducting fibre, it is more vigorously attracted than if insulated, because the electricity of the second kind immediately escapes, and, consequently, does not act against the attraction. Subsequent repulsion, obviously, cannot occur in this case.

**176. Electroscope.**—The use of various forms of *electroscopes* (e.g. straw electroscope, gold-leaf, or aluminium-leaf electroscope) to determine the electrical condition of bodies depends upon the principle of induction.

The gold-leaf electroscope (Fig. 143) consists of a brass bar cemented into a glass tube, the bar terminating above in a ball, or plate, and carrying at its lower end a pair of strips of gold foil. To protect the pendulum from currents of air and, at the same time, to insulate the metallic body, the tube is set into the neck of a glass vessel by means of a cork, or a metallic holder. If an electrified body, such, for example, as an excited glass rod, be held a little distance above the plate, the strips of foil spread apart by reason of the positive electrical charge covering both strips. The positively electrified glass rod exercises its inducing action upon the insulated metallic body of the electroscope, repelling the positive electricity into the strips of foil and attracting the negative into the plate. When the glass rod is removed, the strips of foil collapse, the two separated electricities exactly neutralizing each other. If



FIG. 143.—Gold-leaf Electroscope.

the metallic plate be touched with the finger, while the glass rod is near it, the induced electricity of the second kind escapes and the gold-leaves collapse, while the electricity of the second kind remains condensed upon the plate. If, after removing the finger, the glass rod be also removed, the negative electricity spreads over the entire metallic body and the gold strips permanently diverge. The electroscope is now charged by the inducting action of the positive glass rod with negative electricity. By means of an electrified bar of hard rubber the apparatus may be positively charged in the same manner. If the glass rod be brought to the electroscope when negatively charged, the strips of foil will contract, because the glass rod, by reason of its inductive action, repels positive electricity into the strips and withdraws negative electricity from them, thus diminishing the negative charge. If, on the other hand, the negatively electrified bar of rubber be brought near to the electroscope, it will repel an additional quantity of negative electricity into the leaves, and they will spread still farther apart. The charged electroscope indicates, therefore, not only the presence of electricity on a body, but it shows also whether this electricity is positive, or negative, since in the former case the leaves diverged with a positive, and, in the latter, with a negative charge upon the electroscope. On the contrary, the contraction of the leaves does not necessarily indicate that the body to be tested is electrified; for the leaves contract also when the hand, or any other unelectrified conductor, is brought near the electroscope. The electricity upon the metallic part of the electroscope acts inductively upon the hand, whose unlike electricity of the first kind attracts a portion of the electricity of the apparatus into the plate, whereby the mutual repulsion of the gold-leaves is weakened.

**177. Electrical Spark.**—If a conductor be brought near a highly charged body, opposite electricities will be collected with increasing density upon the nearest portions of the two bodies, since the unlike electricity produced in the conductor by induction and attracted to its extreme point, attracts the opposite electricity of the charged body also toward the nearest

point of its surface. If the density of the two electricities, and accordingly also their tension, becomes great enough, they will break through the non-conductor (e.g. the air) lying between the two bodies and unite with a hissing sound, or with a loud report accompanied by an *electrical spark*. The latter are merely glowing particles of the bodies between which the spark passes, which have been torn loose by the escaping electricity, and together with the air along their paths, are heated to glowing and sometimes even to brilliant luminosity.

If the conductor is connected with the earth, and the electrified body is also a conductor, the discharge takes place from the latter toward the former.

If the electrified body be discharged through a series of conductors separated from each other by non-conducting spaces, as is shown in Fig. 144, where a succession of rhomboidal plates of tinfoil are glued upon a glass plate, or along a spiral line around a glass tube, a little spark passes across each intervening space at every discharge, producing a beautiful luminous phenomenon.

**178. Brush Discharge, or Electrical Glow.**—When issuing from a point, positive electricity forms a luminous brush and the



FIG. 144.—Mimic Lightning



FIG. 145.—Electrical Egg

negative, a luminous point, which, on account of their low brightness, can be seen only in the dark. The discharge in such a case is accompanied by a low hissing noise.

When electrified clouds collect about the tops of towers, the points of weather-vanes, of lightning-rods, of the masts of ships, and even about the hair and garments of men, such luminous brushes are frequently observable. They are produced by the passage of the electricity induced in the objects on the surface of the earth from the points mentioned above toward the clouds. This phenomenon is called *St. Elmo's fire*.

In the so-called electrical egg (Fig. 145), consisting of an

egg-shaped glass vessel exhausted of air, within which two metallic rods (*b* and *b'*) terminate in balls, and provided with metallic terminals and a tap, electricity passes across a considerable distance, by reason of the feeble resistance to discharge through the rarefied air within. The accompanying luminous phenomenon consists of a violet-red sheaf of light rays proceeding from the positive almost entirely to the negative ball. The latter, on the other hand, is surrounded by a blue envelope of light, called the *negative glow*, which is separated from the positive sheaf by a dark intervening space.

**179. Electrophorus.**—To obtain still stronger electrical charges than would be possible by the means already mentioned, the *electrophorus* (Wilke, 1762) may be used. A disk of resin, or of rubber (*g*, Fig. 146) is melted in a metallic mould, *mn*, or laid

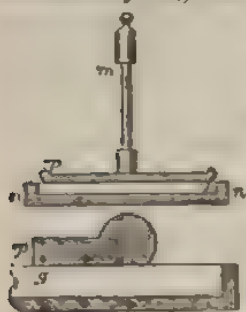


FIG. 146.—Electrophorus.

upon a metallic plate. The cake of resin is electrified by stroking it with cat's fur, or with the tail of a fox. Its negative electricity acts by induction upon the metallic base, the repelled, or negative, electricity escapes into the earth, while the positive is attracted to the lower surface of the cake, a portion being drawn over upon this surface. This positive electricity upon the lower surface of the cake holds the negative of its

upper surface bound, thus preventing it from spreading, or passing off upon a conducting body brought in contact with it. For if the metallic cover, *p*, provided with the insulating handle, *m*, be placed upon the cake and then lifted upward without being touched, it will be found when tested with an electroscope to be unelectrified. If, however, it is touched with the finger while lying upon the plate, it will be found to be strongly charged with positive electricity when lifted from the plate. The charge will be strong enough to produce a spark when the cover is brought near a conductor. The negative electricity of the surface of the cake draws positive electricity by its inductive action to the lower side of the cover and drives negative electricity to its upper side. When

the cover is raised without being touched, these two electricities at once recombine, and the cover passes into an unelectricified, or neutral, condition. When, however, it has been touched with the finger before being raised, negative electricity escapes into the earth, the positive induced electricity, however, remains upon the lower surface of the cover. If now, after touching, the finger is removed and the cover raised, this positive electricity, withdrawn from the influence of the cake, spreads over the entire surface of the cover. Since by this mode of procedure no electricity is withdrawn from the cake, it may be repeated indefinitely, each time with the same result, and an inexhaustible supply of electricity may be obtained. In the process, however, no electricity has been created outright, but by raising the positively electrified cover, the attraction between

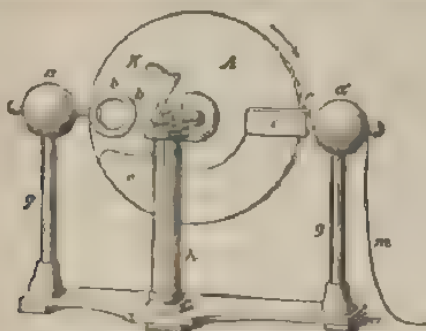


FIG. 147.—Electrical Machine.

it and the negatively electrified cake is overcome, and the work thus performed is stored up as electrical energy in the cover.

**180. Electrical Machines** (Otto von Guericke, 1663) are used to produce electricity of great tension (of high potential) by the aid of friction. A glass disk (Fig. 147, A), fastened upon a horizontal glass axis, *i*, borne by a support, *h*, is turned by means of the crank, *K*, in the direction of the arrow, between the two leather cushions, *α*, pressing lightly against its faces. To heighten their power of excitation, the cushions are coated with Kienmayer's amalgam, a mixture consisting of one part tin, one part zinc, and two parts mercury. The friction

electrifies the glass disk positively and the cushion negatively. The negative electricity of the cushion passes off through a wire, or chain, to the earth. This connection with the earth is made, because if the negative electricity were allowed to remain upon the cushion, farther production of positive electricity would be retarded. The positive electricity adhering upon the glass disk and prevented from escaping into the air by means of the strip, *a*, composed of a non-conducting material, such as oilskin, or silk, passes at length between the two wooden rings, *bb*, communicating with the conductor, *a*. The latter is composed of a hollow brass sphere insulated upon the glass standard, *g*. Metallic points extend from the wooden ring toward the faces of the disk, and are connected by means of a groove filled with tinfoil. The positive electricity of the disk acts inductively upon the insulated conductor, *ab*, consisting of the metal ball and the wooden ring, repels the positive electricity to the sphere, and attracts the negative to the points. The negative electricity, however, flows from these points toward the disk, and combining with the positive electricity upon its surfaces, neutralizes the electricity upon the disk, and is itself at the same time neutralized. The conductor remains charged with a quantity of positive electricity of the second kind equal to the negative electricity of the second kind, which was lost on the disk by flowing from the point. Since the result is the same as if the points had soaked up the positive electricity and transferred it to the conductor, the wooden rings are sometimes called suction apparatus. To render the negative electricity of the rubbing cushion available for use, it is also connected with a hollow brass sphere, *a'*, borne upon a glass post, *g*, and acting as a negative conductor. The negative electricity collects upon this sphere if it be insulated and at the same time the positive conductor be put to ground.

By means of this electrical machine, numerous experiments exemplifying the laws of electrical action may be performed. Among these many assume the form of interesting toys. If the knuckle of a finger, or any other conductor connected with the earth, and having a rounded end, be brought near the

terminal of an electrical machine in action, sparks from 5 to 25 cm. in length will pass across. The longer sparks are not rectilinear, but resemble the zigzag lines characterizing streaks of lightning.

The mutual repulsion of similarly electrified bodies may be shown by the aid of the *paper brush* (Fig. 148). This apparatus consists of a conducting rod, which may be inserted in a hole provided for it in the upper surface of the conductor of an electrical machine. To the upper end of this rod is attached a conducting disk, from whose periphery narrow strips of thin paper hang. When the apparatus is put in place upon a



FIG. 148.—Paper Brush.



FIG. 149 — Pith-ball Dance.

machine and the disk turned, the strips spread apart like the ribs of an umbrella.

The *pith-ball dance* illustrates the attraction and electrification of unelectrified by electrified bodies. In a glass cylinder (Fig. 149), closed above and below by a metallic cover, little balls of cork, or pith, are placed, while a chain, hanging from the conductor, leads electricity to the upper cover. The latter then attracts the unelectrified balls, but as soon as they come in contact with the cover, being thereby similarly electrified, they are repelled. The balls, having discharged their electricity to the lower cover, which is in communication with the earth, are again attracted, and thus they continue to dance between bottom and cover, at the

same time transmitting the electricity from the conductor to the earth.

Fig. 150 represents an apparatus for producing *electric chimes*. By means of a wire, *abc*, connected with the conductor,

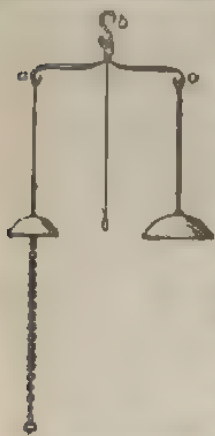


FIG. 150.—Electric Chimes.

two metallic bells are suspended, the one at *c* by a metallic fibre, the other at *a* by a silk thread. The latter communicates with the ground by means of the chain shown in the cut. Between the bells, and equidistant from them, a metallic ball is suspended by a silk fibre. When the first bell is electrified by the conductor it attracts the ball to itself, and after contact and consequent participation in its electrical charge, the first bell repels the ball to the second, where it gives up its electrical charge, and is again drawn back by the first bell. By a repetition of this process of striking first against the one bell, and then the other, a rapid succession of chimes is produced.

Easily inflammable liquids, such as ether, carbon disulphide, are ignited by the sparks of an electrical machine, and certain gases may be exploded by means of them. The *electric pistol*



FIG. 151.—Electric Pistol.

(Fig. 151) may be used to illustrate the latter fact. This device consists of a thin vessel, closed by a cork, into which extends a metallic wire cemented in a glass tube, *tt'*, and terminating in the small balls, *b* and *b'*. If the vessel is filled with a mixture of air and hydrogen, or with illuminating gas, and the outer knob, *b*, is brought against the conductor of the machine, a spark will pass between the inner knob and the glass wall, the gas explodes, and the cork is violently expelled with a loud report.

One may electrify his own body (Dufay, 1734) by means of the *insulating stool*, which is merely a thin block of wood borne upon glass feet, or placed upon a rubber plate. By merely

touching the conductor of an electrical machine while standing upon the stool, or upon a rubber plate, or with rubber overshoes upon the feet, the body will be immediately electrified. The hair of the head stands on end, and in a dark room brushes of light are seen to stand with their bases upon the tips of hairs; but so soon as a spark passes between the conductor and the body, these electrical manifestations disappear. While the body is in this condition a vessel containing ether, and connected with the ground, may be ignited by the spark which passes between it and the finger.

The steam-engines and hydro-electrical machines of Armstrong (1830) are based upon the principle that when steam issues through an orifice from a boiler it becomes electrified, (usually with positive electricity), while the boiler, if insulated, becomes at the same time oppositely electrified. The electricity thus developed arises from the friction of the particles of water on the walls of the orifice (Faraday, 1846), as this water is rapidly borne along by the steam. The production of electricity is most rapid when the walls of the orifice are of wood. In the same manner liquid carbonic acid is electrified on issuing from the iron flasks used for preserving it.

**181. Condensing Apparatus—Condensers.**—If an insulated conductor, for example, a metallic plate supported upon a vertical glass standard, is brought into electrical connection with the conductor of an electrical machine in action, the plate will take up a quantity of electricity corresponding to its capacity, or, what is the same thing, it will continue to absorb electricity until its tension, or potential, is the same as that of the conductor. This tension, however, sets a limit to the quantity of electricity which may collect upon the plate, as may be recognized from the fact that an electrical pendulum suspended from the back side of the plate can be made to diverge no further after this tension has been reached. If after the connection with the conductor has been broken, a plate similar to the former is brought near to its front surface, the pendulum contracts more and more, the nearer the second plate is brought, while an electrical pendulum, hanging upon the back side of this latter plate, diverges less than that of the

former plate. The plate first mentioned acts inductively upon the other, attracts the negative electricity of the first kind to the front surface, and drives the positive electricity of the second kind to the back surface. While the positive electricity of the first plate, drawn by the induced negative electricity of the second, accumulates with increasing density upon its front surface, the density upon the rear surface continually diminishes, and with it the repulsive force exerted upon the pendulum also diminishes. If now, the first plate is again connected with the conductor, electricity again passes over upon it, increasing the charge already present, and the pendulum rises again to its former height, thereby indicating that the potential of the conductor has been again attained. Hence it follows that the potential of the first plate has been reduced by the reaction of the second. But the first plate contains now a greater quantity of electricity than it was formerly capable of taking up. Its capability to take up electricity, or its *capacity*, has therefore been increased by the presence of the second plate.

If now the second plate be touched, the positive electricity escapes, its pendulum collapses, indicating that its potential has become zero. The negative electricity, however, which no longer reacts against the positive, condenses upon the front surface, attracts more of the positive charge of the first plate toward its front surface and the pendulum upon its rear surface contracts. Electricity may again be transferred from the conductor to the plate until the tension of the conductor is reached. The capacity of the first plate is, therefore, augmented by the presence of the second plate, if the latter is connected with the earth.

Two metallic plates separated by a nonconducting layer of air, glass, shellac, etc., the one of which is connected with a source of electricity, and the other with the earth, form thus a *condensing apparatus*, or a *condenser*, by means of which electricity may be collected and condensed in greater quantities than would be possible upon a single plate. The former is called the *collecting*, and the latter the *condensing plate*. The term *condenser* is applied more particularly to those apparatus which are used to collect in measurable quantities electricity of so low potential as to be inappreciable by means of the

electroscope directly. With the condenser of Volta (1782), the horizontal collecting plate is screwed directly to the stem of a gold-leaf electroscope (Fig. 152), while the condensing plate may be placed upon it by means of an insulating handle. The surfaces of the plates turned toward each other are varnished, and accordingly are always separated by a thin layer of resin. If the lower plate is connected with a weakly electrified body, and the upper face is *touched with the finger*, the two opposite electricities upon the adjacent faces of the plates condense on both sides of the coat of varnish until the collecting plate has reached the potential of the electrified body. If the upper plate is now removed the electricity collected upon the lower plate next to the layer of resin spreads over the entire metallic part of the electroscope and charges it to a far higher tension than characterizes the body to be tested. That this is true is seen from the diverging of the gold-leaves, and that the final tension is higher is clear when we recall that the capacity of the collecting plate sinks to its original small magnitude after the removal of the condensing plate.



FIG 152.—Gold-Leaf Electroscope with Condenser

The *air-condenser* of R. Kohlrausch (1838) consists of two vertical plates separated by an insulating layer of air.

The capacity of a condenser is directly proportional to the surfaces of its plates, and inversely proportional to the distance separating them.

If  $V$  denote the potential of the collecting plate,  $V'$  that of the condensing plate, and  $d$  the thickness of the layer of air between the plates,  $\frac{V - V'}{d}$  is the *fall of potential* from the first to the second plate, and it is also the force tending to drive the electricity from the first toward the second plate. This force is also expressed by  $4\pi s$ , if  $s$  denote the density of the electricity in question. We must then have—

$$4\pi s = \frac{V - V'}{d}.$$

Since  $d$  and  $V - V'$  are constant,  $\delta$  must also be constant, and  $E = S\delta$  is the quantity of electricity collected upon the surface  $S$  of the plate. Multiplying both sides of the foregoing equation by  $S$ , and dividing by  $4\pi$ , we obtain—

$$E = \frac{S}{4\pi d}(V - V'),$$

or, since the condensing plate is connected with the earth ( $V' = 0$ )—

$$E = \frac{S}{4\pi d} V,$$

the capacity of the condenser is accordingly  $\frac{S}{4\pi d}$ .

**182. Leyden Jars—Franklin's Plate.**—Leyden jars are used to collect the electricity of strong sources, such as electrical machines, in large quantities. They were first used by Kleist (1745) and later by Cunnæus (1746), and are sometimes called Kleist's jars. The Leyden jar consists of a glass vessel coated within and without to within a few inches of the top with tin, or zinc, foil. They are, therefore, merely condensers with glass as the insulating substance. The portion of the vessel not coated with foil is varnished to insure better insulation. The vessel is also provided with a varnished wooden cover through which passes a metallic rod terminating upward in a ball and communicating below with the inner coating of the jar. The flask is charged by connecting one of the coatings, usually the inner, with a source of electricity, *e.g.* with the conductor of an electrical machine, or the cover of an electrophorus, while the other coating communicates with the ground. Such communication may be readily effected by placing the jars upon a conducting support. The two electricities, that imparted to the inner coating (*e.g.* positive electricity), and the negative induced electricity (of the first kind) upon the outer coating, are spread over the surfaces of the coatings next the glass, the former with the tension of the conductor, and the latter with no tension, or with the potential zero. Both strive to unite with each other. This tendency many times becomes so strong that the glass wall is incapable of resisting the electrostatic pressure exerted upon it, whereupon a perforation of the glass occurs, rendering the flask useless. The larger the surface of the coating the greater is the capacity of the flask, or the greater is the quantity of electricity which may collect upon it. But instead

of employing a single large flask, which would be very unwieldy, the *electric battery* (Fig. 153) is used. This consists of several Leyden jars connected in such way that all the external coatings, on the one hand, and all the internal coatings, on the other, are in electrical communication.

The process of discharge may be studied by the aid of Henry's *quadrant electroscope* (1774), by placing it either upon the conductor, or upon the battery itself. It consists of a vertical metallic column, to which is suspended from the middle of a graduated arc a pith ball stuck on a stiff wire. This pendulum rises and stands in equilibrium as soon as the

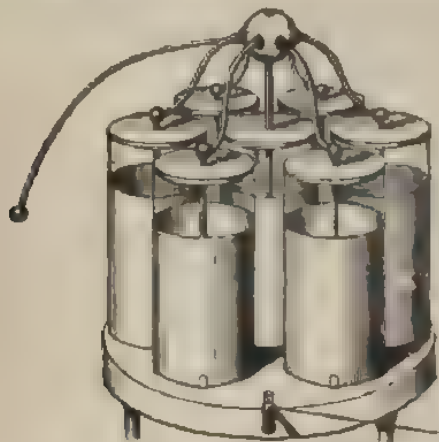


FIG. 153.—Electric Battery.

potential of the conductor, or the maximum limit of charge is reached.

A flask, or battery discharges, i.e. the two opposite electricities collected upon the coatings combine, when the external coating and the knob leading to the internal surfaces are brought into electrical connection, or when both coatings are connected with the earth. If the outer coating be grasped with one hand and the knob with the other, a strong tendency to contraction of the muscles of the arm is felt, and in case of a high charge, a violent pain in the breast is experienced. This so-called electric shock may be transmitted through a

number of persons grasping one another's hands so as to form a continuous circuit.

To prevent discharge through the human body during experiments with the Leyden jar, an insulated *discharging rod* is used. This is composed of two slender rods usually of brass, connected by a joint and provided with knobs at the ends, a glass or rubber-coated handle being connected also with the rod at the joint. A simpler form consists of a flexible wire coated with rubber and terminating in knobs (Fig. 154). One knob of the discharging rod is brought into contact with the outer coating, and the other is brought near the knob of the flask. While the knob of the discharger is still at a considerable



FIG. 154.—Simple Discharging Rod.

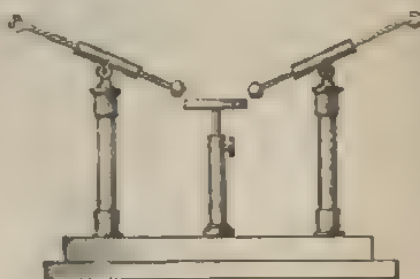


FIG. 155.—General Discharger.

distance, sometimes called the *striking distance*, from the knob of the jar, a bright spark passes between the knobs, accompanied by a loud report. The striking distance was measured by Riess in 1837 by means of the *sparking micrometer*. It consists of a pair of metallic balls supported upon glass footings, whose distance apart may be measurably altered. The striking distance is proportional to the density of the accumulated electricity.

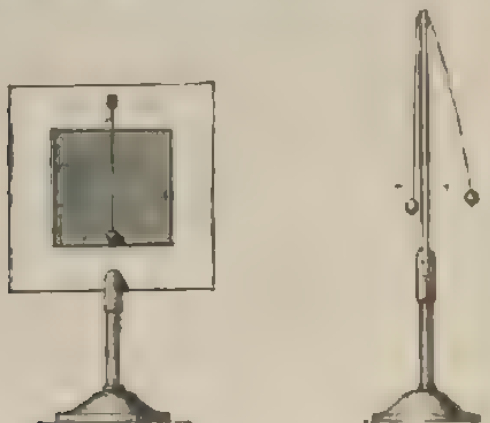
To be able with convenience to expose objects to the discharging stroke, Henry's (1779) *general discharger* (Fig. 155) was devised. Supported upon glass footings and movable by means of joints, two short tubes, through which pass metal rods, provided with knobs at their inner ends and hooks at their outer ends, are placed as shown in the figure. The rods may be

pushed inward and outward through the tubes. One of the knobs is connected with the outer coating, and the other, with the aid of the customary discharger, is brought into electrical connection with the knob of the inner coating. Between the two rods is an adjustable glass plate supported also upon a glass footing. By means of this device it may be seen that a strong charge heats, fuses, vaporizes, and consumes metallic wires when placed between the knobs, and that in some cases it even bends, tears, and pulverizes them. Paper, stiff cards, and cardboard are perforated, and the holes are surrounded on both surfaces by raised edges. If the knobs do not stand directly opposite, the perforation takes place at the negative knob, as was shown by Lullin's experiment. By merely replacing the knobs of the discharger by points, wooden plates and glass disks may also be perforated, the latter with extreme ease when stearine, or paraffin, has been dropped upon them. The perforation is found to occur always at the *edge* of the spot of stearine. The discharging spark also appears under water. The liquid is thrust aside by the spark with such violence that oftentimes open glass vessels filled with water are burst asunder. Air and other gases when confined are so suddenly and so powerfully expanded by the discharge that a small bullet used to close the mouth of the *electric mortar* is hurled out with great force. To ignite gunpowder it is necessary to send the discharge through a poor conductor, such as a moist hempen cord, since with metallic connection the discharge takes place so quickly that the powder is merely blown aside. In all these effects, the energy of the electric charge which vanishes as such, is merely transformed into mechanical work and ultimately into heat.

The quantity of heat developed by the discharge was measured by Riess (1838) with his *electro-thermometer*. This consists of a simple air thermometer through whose bulb a slender, spirally wound platinum wire is drawn to transmit the discharge. The air expanding by the heat presses the liquid into a thermometer tube lying upon an adjustable inclined plane. The liquid sinks in the tube more or less according as the wire is heated to a higher or a lower temperature.

It was thus found that the heat produced is directly proportional to the square of the quantity of electricity discharged and inversely proportional to the capacity of the jar. This quantity of heat is in point of fact equal to the energy of the electric discharge— $W = \frac{1}{2} \frac{E^2}{C}$  (*vide* p. 238).

The Leyden jar is not essentially different from *Franklin's plate* (Figs. 156 and 157), a glass plate standing perpendicularly upon a glass footing and coated upon both sides with tinfoil in such manner as to leave a strip a few inches broad around the edge, to be coated with shellac varnish. Although this apparatus is less convenient than the Leyden jar for the appli-



FIGS. 156, 157 — Franklin's Plate.

cations mentioned above, it is better adapted, by reason of its simple construction, to illustrate the nature of the processes of charge and discharge. If upon either side of the plate a pith ball pendulum is stuck with a little wax, the one ball will be repelled by the first coating, which contains electricity directly from the conductor of the machine, while upon the other side, which has been touched with the finger, the pendulum hangs quietly in a vertical position. Upon the first coating, therefore, electricity is present, and that, too, upon its outer surface, while upon the second coating there is none. If now the latter coating is kept insulated and the first is touched,

the electricity escapes from its outer surface and the pendulum sinks into a vertical position. The pendulum on the second coating rises, since a portion of the negative electricity condensed on its inner surface now spreads over the outer surface. And so by alternately touching the coatings, the plate may be gradually discharged, the charge remaining upon the coatings at each contact diminishing continuously according to a law of geometrical progression. This method of successive discharge may also be shown with a Leyden jar, by providing its external coating and the knob of the internal coating, each with a pendulum, the charged jar being placed upon an insulating support.

**183. Discharging Electrometer.**—To compare the quantities of electricity in a Leyden jar, or battery, Lane's (1767) *discharging electrometer* may be used. In this apparatus (Fig. 158) the sphere, *b*, borne on the end of a horizontal metallic rod, may be adjusted micro-metrically to any given distance from the knob, *a*. This knob is connected with the outer coating of the jar to be charged while the latter is supported upon an insulating base. The induced electricity of the same kind is repelled from the coating, passes into the jar, and charges it to a potential high enough to produce a spark discharge between *a* and *b*. While the electricity is being accumulated in the battery to be charged, the electrometer continues alternately to charge and to discharge, and the battery contains at last as many times the quantity of electricity necessary to saturate the electrometer as there have been discharges of the electrometer.



FIG. 158.—Discharging Electrometer.

**184. Cascade.**—If  $n$  Leyden jars, each of capacity  $C$ , are combined into a single battery, the capacity of the latter is  $nC$ , and to charge it to the tension,  $V$ , a quantity of electricity,  $E = nCV$ , is required.

The  $n$  flasks may, however, be combined in such way that each stands upon an insulating base, and the outer coating of each is connected with the inner coating of the next following. If, then, the inner coating of the first be

charged with an electrical mass,  $e$ , an (approximately) equal mass of electricity of the opposite kind passes from its outer coating into the second jar, from the outer coating of the second jar into the third, and so on, so that each of the flasks contains the same electrical mass,  $e$ . If  $V_1, V_2, V_3, \dots, V_n$ , denote in succession the potentials of the inner coatings and, consequently  $V_2, V_3, V_4, \dots, V_n, V_{n+1}$ , those of the outer coatings, the charges of the individual jars are—

$$e = C(V_1 - V_2), e = C(V_2 - V_3), \dots e = C(V_n - V_{n+1}),$$

and, therefore, the total charge of the entire battery is given by  $ne = C(V_1 - V_{n+1})$ , or, if the outer coating of the last jar is grounded,  $V_{n+1} = 0$ ,

$$\text{and } ne = CV_1 \text{ or } e = \frac{CV_1}{n}.$$

The capacity of such a battery is, therefore, one- $n$ th of that of each single cell. Hence, to obtain a given tension only one- $n$ th part of the quantity of electricity is required as with a single jar, or, what is the same thing, with a given electrical mass,  $n$  times the tension may be obtained. Since the differences

of potential  $V_1 - V_2 = V_2 - V_3 = \dots = \frac{V_1}{n}$ , the charges are distributed in equal gradations on the individual cells, as might be illustrated by a terraced waterfall. Such a combination of cells is called a cascade (Franklin, 1784). In the former case, the jars are said to be connected *for quantity*, and in the latter, *for tension*. To obtain high tension and, consequently, also, great striking distance, it is best to charge the battery connected for quantity, and then to change it for tension into cascade.

Land's electrometer is connected in cascade with the jar whose charge is to be measured.

**185. Dielectric Constants.**—If a plate of glass, or of hard rubber, be inserted between two metallic plates (181) separated by a layer of air, one of them being charged with the tension of the conductor of an electrical machine and the other connected with the earth, the pendulum attached to the first plate sinks. The tension of the collecting plate is, therefore, diminished and its capacity increased by the presence of the dielectric plate. More electricity may now accumulate upon the apparatus and its charge is accordingly increased. The insulating plates between the coating of such a collecting apparatus are, therefore, not passive. On the contrary, they play an essential part (Franklin) in the charging of condensers, the importance of which varies with different dielectrics (Faraday). The terms *specific inductive capacity* (Faraday) and *dielectric constant* of insulating substances are applied to the ratio of the charge of a condenser when this substance separates the coatings, to the charge taken up by the condenser at the same potential when the separating medium is an equally thick layer of air.

The dielectric constant of an insulating body may also be defined as the number with which the capacity of an air condenser must be multiplied to obtain the capacity of the same condenser when the layer of air in it is replaced by the dielectric in question. The dielectric constant of air is, therefore, assumed as the unit. For some other dielectrics we have the following values due to Boltzmann (1874): paraffin, 2.32; glass, 2.6; hard rubber, or ebonite, 3.15; sulphur, 3.84; mica, 8 (Bouty). Since different gases also have different inductive capacities, that of a vacuum is taken as unity, and the dielectric constants of some gases under normal pressure are then: hydrogen, 1.0003; air, 1.0006; carbon dioxide, 1.0009; all of which differ so little from unity that the discrepancies are of no practical importance. If  $k$  denote the dielectric constant of the interposed plate of a condenser, its charge at potential  $V$ , is given by  $E = \frac{kSV}{4\pi d}$ , and its capacity is  $\frac{kS}{4\pi d}$ .

**186. Dielectric Polarization.**—Matteucci (1847) showed that nonconducting bodies are also subject to induction. A small unelectrified needle, composed of sulphur, or shellac, is electrified when brought under the influence of a charged body, the nearer end with unlike, the other with like electricity to that of the charged body. But, while conductors respond immediately to induction and lose the effect with equal readiness, with nonconductors a considerable time is necessary for its complete development, as also for its total disappearance. To explain this phenomenon Faraday assumed that a dielectric body consists of a nonconducting matrix, or *substratum*, in which conducting particles are imbedded. These particles become electrified by induction without transmitting their charges to the matrix. If, now, a positively electrified body be conceived to be brought near one end of a series of such particles, every particle will become negatively electrified at its nearer end and positively at its more remote end. Since the opposite electricities of the neighbouring particles destroy each other's external effects, these charges are effective only at the ends

of the series, where the little magnet exhibits two opposite polarities. Electrical induction upon a nonconductor, on this view, is analogous to the influence of a magnet upon a piece of soft iron. The condition into which the nonconductor is thus brought is called a state of *dielectric polarization*.

**187. Residual Charge.**—If a Leyden jar is completely discharged by bringing the inner and the outer coatings into contact, after a short time it will be found again to have a weak charge. This is called the *residual charge*. Electricity collects not only upon the metallic coatings, but also upon the glass itself. This may be proved by means of a jar fitted with coatings of tin, which may be removed. If the charged jar be placed upon an insulated support, and by means of a glass hook the inner coating be removed, when the glass vessel is withdrawn from the outer coating, the two coatings will be found to be but feebly charged. But if the glass vessel is grasped on its outer surface with one hand and touched on the inner surface with the other, the crackling sound which characterizes weak electrical discharges is heard. If after the coatings have been completely deprived of their charge, the jar is again put together, after a short time it will again give a spark. Nonconductors do not oppose an absolute hindrance to the motion of electricity through them. On the contrary, the charges of the coatings penetrate slowly into the glass and afterwards return to the coating. The dielectric polarization of the glass by the inducing action of the coatings also assists in charging the latter again.

**188. Duration of the Electric Spark—Velocity of Propagation of Electricity—Oscillatory Discharge.**—A rapidly rotating disk with painted spokes, or sectors (colour disk), when illuminated in a dark room by the spark discharge of a Leyden jar, appears to stand motionless, because the duration of the spark is so short as to show the disk in only a single position.

That the spark has nevertheless a measurable duration was found from the experiments of Wheatstone (1834) to determine the velocity of electricity in conducting wires. Six metal balls, *a, b, c, d, e, f* (Fig. 159), well insulated, were attached in a straight line upon a board. Between *b* and *c*, and between

*c* and *d*, wires each 370 m. long, were interposed. When *a* was connected with the outer and *f* with the inner coating of a Leyden jar, three sparks were seen, apparently simultaneously, at *ab*, *cd*, and *ef*. These were observed in a mirror which could be rapidly rotated about an axis parallel to the line *af*. When the mirror was at rest, the three spots appeared as luminous points, or disks, lying upon a straight line. But when the mirror was rotated 800 times to the second, the three images of the sparks appeared to stretch out into lines

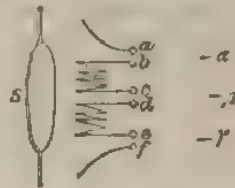


FIG. 159 — Wheatstone's Experiment

perpendicular to the axis of rotation, from the apparent length of which the duration of the spark was computed to be 0.0000421 seconds. It was also observed that the image of the middle spark was displaced with respect to the others in the direction of a retardation, whence it follows that the two outer sparks *a* and *γ* were produced simultaneously, and *β*, the middle one, a little later. Both electricities pass, then, simultaneously from the coatings and meet midway between them. From the magnitude of the displacement ( $\frac{1}{2}^\circ$ ) of the image of the middle spark and the velocity of rotation of the mirror, the velocity of propagation in a copper wire was found to be 430,000 km. By a different method Siemens (1876) found for the velocity in iron wire 240,000 km.

With moderate lengths of wires the duration of the spark is much shorter than with very long wires. With short wires the image of the spark, with 800 revolutions, had an apparent length of less than  $\frac{1}{2}^\circ$ , and its duration was, consequently, less than one-millionth of a second.

With very short wires the duration of discharge is also longer. The long ribbon of light into which the image of the spark is drawn, and which may be photographed (Feddersen, 1858), exhibits transverse stripes, alternately bright and dark, whence we infer that in this case the discharge takes place in a rapid series of sparks. The appearance of the image of the spark indicates this discharge to be vibratory, or *oscillatory*, in character (electrical vibrations), and that the jar is alternately

charged with positive and negative electricities in rapid succession.

189. **Lichtenberg's Figures** (1777) are produced by transmitting a charge of electricity by means of a metallic conductor (*e.g.* the knob of a charged Leyden jar) to some point of a nonconducting plate, such as resin, or hard rubber, the plate, having been first besprinkled with siftings from a vessel, closed with a piece of linen cloth, of a powder composed of a mixture of red oxide of lead and sulphur flowers, or lycopodium powder (electroscopic powder). The red particles of the oxide are electrified positively by friction against the sides of the meshes of the linen, and cling to the negatively electrified parts of the plate, while the negatively charged particles of the sulphur, or lycopodium, adhere to the positive portions of the plate. If positive electricity is used, the figures thus arising form a yellow star with branching rays, which proceed from the electrified point in lines radiating in all directions. If negative electricity is used, only a round red spot is produced. These dust-figures are produced by the motion of the electrified air around the conductor. They may be very beautifully displayed in the form of the brush and glow light.

190. **The Induction, or Influence Machine, or Inductor**, discovered almost simultaneously by Tœppler and by Holtz, is a source of induced electricity much more prolific than the ordinary electrical machine. The Holtz influence machine (Fig. 160) consists of two varnished disks of glass, the smaller of which, B, turns by means of a crank and endless cord, S, about a horizontal axis,  $x$ , composed of hard rubber, whose supports are borne each on a crossbar composed also of hard rubber (*kkhh*) and carried by the glass posts 1, 2, 3, 4. The larger disk, A, is fixed by means of glass rods in a permanent position just behind the rotating disk. It is provided with the two openings, *a* and *b*, diametrically opposite to each other, to whose edges paper strips (armatures), *c* and *d*, are attached, from which tongues of paper extend through the openings, *a* and *b*. In front of the rotating disk, and exactly opposite to the paper strips of disk, A, are two brass combs, or rakes (*ggii*), so placed

that their teeth point toward the disk and their brass stems project through the crossbar, *kk*, and terminate in the spheres, *f* and *e*. Through these spheres pass rods of brass, provided at their outer ends with insulating handles of hard rubber, and at their inner ends with knobs (*m* and *p*). By pushing these rods inward and outward, the distance between the knobs may be varied. If an excited disk of hard rubber (11, Fig. 161) be held behind the paper strip, *c*, and the disk, *B*, be turned in the direction of the arrow against the tongues of paper, while the knobs, *n* and *p*, are in contact, the paper strip, *c*, is negatively electrified, its positive electricity passing through the paper

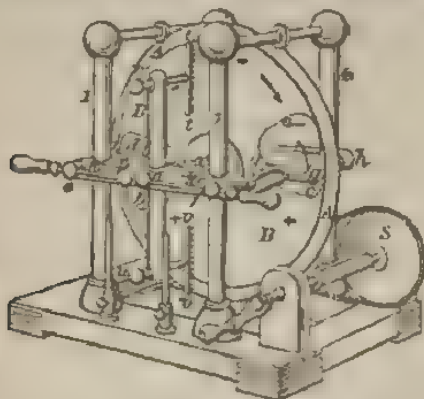
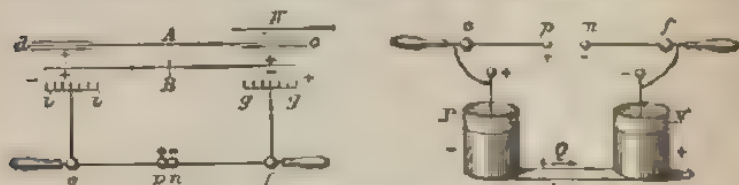


FIG. 160.—Influence Machine.

tongues toward the rubber plate while the negative electricity remains behind. So soon as this condition is attained, the rubber disk is removed. The negative electricity of the strip, *c*, acts inductively both upon the rotating glass disk and upon the comb, *gg*, attracting the positive electricity in both, and repelling the negative. The glass disk thereby assumes a positive charge upon its inner and negative charge upon its outer surface. Since, however, in the brass conducting the electricity, induction is much more rapid and perfect than in the nonconducting glass, the positive electricity, issuing from the teeth of the comb toward the disk suffices, not only to neutralize the negative electricity on the outer surface, but also to impart to this

surface a positive charge. The part of the disk which has been turned past the comb, *gg* (in Fig. 160 the lower half), is therefore charged with positive electricity upon both sides. This electricity, arriving at the paper tongue projecting into the opening, *b*, attracts negative electricity from it, is neutralized by it, and leaves the paper strip, *d*, positively electrified. The result is the same as if the positive electricity of the lower half of the disk had passed over into this strip. Since, now the positive electricity of the strip, *d*, exerts upon the rotating disk and the brass comb, *ii*, the action of induction precisely as before, compelling negative electricity to flow from the points upon the disk, the upper half becomes charged with negative electricity, which, after reaching the opening, *a*, passes over into the paper strip, *c*, augmenting its negative charge and,



FIGS. 161, 162.—For the Influence Machine.

consequently, also its inducing action. This process is repeated at every revolution, and the charge upon the paper strips rises rapidly to a definite limit. Of the electricities driven by the inducing action of the strips into the combs, the positive from the comb, *ii*, accumulates upon the knob, *p*, and the negative from the comb, *gg*, upon the bulb, *n*. Between these two knobs, called *electrodes*, the electricities neutralize each other. To enable this action to begin, on starting the machine, while the charge is very weak, the knobs must be placed in contact with each other. So soon, however, as a hissing sound indicates the presence of a sufficient charge, if the knobs are drawn apart a crackling spark will be heard between them, and it will continue to be heard at intervals so long as the disk is rotated. If one of the knobs is connected with the earth, sparks may be drawn from the other as from the conductor of an ordinary electrical machine. A Leyden jar, or battery, whose coatings

are connected with the open electrodes, may be charged in a few seconds. To convert the current of sparks into individual sparks of greater strength, each electrode may be connected with a knob of a Leyden jar, the outer coatings of the jars being first connected by a strip of tinfoil (Fig. 162). The inner coating of each jar then becomes charged with the electricity of the corresponding electrode, while the like electricity, repelled to the outer coating, passes through the strip of foil to the outer coating of the other jar, and is there accumulated. When, after a short time, the density required to produce a spark across the intervening space of air has been developed upon the electrodes connected with the inner coatings, a spark, accompanied by a loud report, passes between them, while at the same time the electricities of the outer coatings neutralize each other through the tinfoil. If, instead of using a continuous conductor between the outer coatings, the connection is effected by means of iron filings strewn between the jars upon the insulated base, the neutralizing of the two electricities becomes apparent. Whenever discharge of the jars takes place by the passage of a spark between the electrodes, a miniature streak of lightning is seen to pass through the iron filings from one jar to the other. To obtain the highest possible difference of potential, and consequently also the greatest possible length of spark, the jars must, of course, be connected in *cascade*. *Condensing tubes, or condensers*, which ordinarily accompany the machine, act on precisely the same principle as do these Leyden jars. The tubes are provided with a strip of tinfoil on their inner surfaces, and with two rings of the same material upon their outer surfaces. These rings are connected with the brass stems of the combs. They correspond, therefore, to the inner coatings of the jars above, while the interior strips of foil correspond to their connected outer coatings.

If the electrodes are separated far enough, so that the electricities collected upon them are no longer able to unite with and neutralize each other across the intervening layer of air, they will flow through the combs back upon the disks, destroying, or even reversing, the charge upon them. To prevent the cessation of action of the machine, with too great distance

between the electrodes, the supernumerary combs, *tt* and *vv* (Fig. 160), are provided, which are connected with *gg* and *ii* respectively, to take up the accumulated electricities and permit them to flow again upon the disk.

The flow of electricity from the points of the comb, whence the hissing sound originates, is perceptible to the eye in a darkened room. The positive electricity appears in the form of a brush on the points of the comb, *gg*, and upon the point of the corresponding tongue of paper it spreads out over the disk into the paper strip referred to above, while the negative electricity appears upon the teeth of the comb, *ii*, and the corresponding tongue of paper, in the form of brilliant points.

Self-exciting influence machines, first devised by Tupppler (Voss, Wimshurst), possess the peculiarity of developing for themselves the light charge of electricity necessary to start them without external excitation. The rotating disk carries upon its front surface a number of metallic buttons, which, on turning the disk, rub past two brushes of brass foil connected with the paper strips of the fixed disk. In this form of the apparatus the fixed disk has no openings, such as *a* and *b* of Fig. 160.

When the influence machine is started, a smaller resistance is perceptible before it has become charged than afterwards. The excess of labour performed in the latter case is transformed into electrical energy. When the electrodes of an influence machine in action are connected with the combs of a second machine, from which the endless cord has been removed, the movable disk of the latter takes up a rapid rotation. When in the first machine work is being transformed into electrical energy, in the second electrical energy is being converted into mechanical work.

This same principle of rise of potential is applied in an ingenious manner in the *water influence machine* (W. Thomson). From a branching tube (Fig. 163), connected with the earth, two jets of water flow through the hollow metal cylinders, A and B, in such way that the jets are converted into drops within the cylinder. The drops fall from A into a metallic cylinder, *b*, provided within with a funnel, and those from B drop into a precisely similar cylinder, *a*. A and *a* are connected

with the electrode, P, while B and *b* communicate with the electrode, N. If the cylinder, B, is feebly electrified with a negative charge, the drops of water falling through it become positively electrified by induction, give up their positive charge to *a*, A and P, and finally flow away unelectrified. The positive electricity of A charges the water drops within it negatively, these surrender their negative charge to *b*, B, and N, whereby the negative charge of B is augmented, and thus the action continues, so that finally the conductors, P and N, are charged to a far higher tension than that originally imparted to them, and sparks pass across between the electrodes.

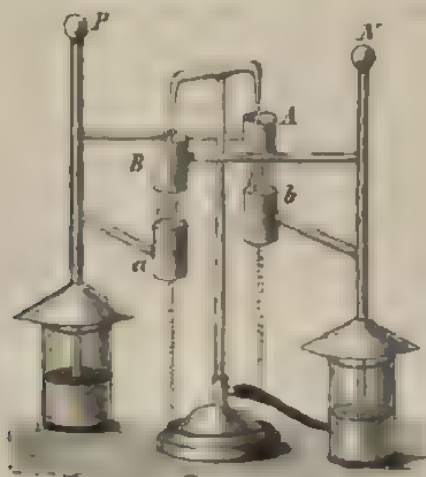



FIG. 163.—Water Influence Machine.

**191. Electrical Odour.**—In the neighbourhood of an electrical machine in action, a peculiar odour, resembling that of phosphorus, is perceptible. The electricity issuing from the points of the combs converts the ordinary, odourless oxygen ( $O_2$ ) of the air into a highly odorous, allotropic modification of oxygen, called *ozone* ( $O_3$ ). The term *active oxygen* is also sometimes applied to this modified form of the gas to indicate that it differs from the usual form in its stronger chemical activity. The presence of ozone may be shown by means of paper strips, moistened with stiff paste and a solution of calcium iodide.

The iodine withdrawn from the calcium iodide by means of the oxone, colours the paste blue.

**182. Measurement of Electrical Force. of Potential and of Capacity Electrometers.**—An electrical force is measured by holding it in equilibrium by means of a known force. Apparatus adapted for this purpose are called *electrometers*. Coulomb's *torsion balance*, previously described, is an electrometer, in which the torsional elasticity of a wire acts against the electrical force.

The same reactive force is used in the torsion electrometer of R. Kohlrausch (1847). The metallic needle, *aa* (Fig. 164), is



suspended by a glass, or quartz, fibre in the position of equilibrium with its halves on opposite sides of a curved strip of metal, *bb* (seen from above in the figure), one part of the strip being bent a little to the right, and the other a little to the left of the needle (Dellmann, 1842). When a charge of electricity is imparted to the strip, and accordingly also to the needle touching it, the electrical force will be proportional to the angle read on the torsion head above, through which the fibre must be twisted to bring the repelled needle back into its original position.

In the *sine electrometer* of R. Kohlrausch (1853), the brass needle of the former instrument is replaced by a magnetic needle swinging upon a point. When this needle is in the magnetic meridian, it touches the bent metallic strip. In this instrument the repellent electrical force is balanced by terrestrial magnetism which tends to draw the needle into the meridian. If the metal strip be again turned toward the needle until they touch, this force will be proportional to the sine of the angle of rotation, whence the apparatus receives its name.

The *quadrant electrometer* (Fig. 165) of W. Thomson (1867) contains a light, flat, broad aluminium needle cut in the form of a pair of opposite sectors, joined at their centres and swinging within a flat, cylindrical metallic box. The box is cut by a pair of perpendicular slits passing through the centre into four quadrants (in Fig. 166, seen from above). Each pair of

diametrically opposite quadrants is connected by a conductor. The needle has the so-called bi-filar suspension (32), the two silk fibres being parallel to each other when the needle is at rest. If drawn aside from this position, gravity will strive to bring it back. To protect the quadrants from external electrical influences, they are enclosed within a metallic case (*vide*, action of screens, 174), whose cover supports the glass tube enclosing the fibres, and also three insulated binding-screws connected with the quadrant pairs and the needle



FIG. 165.—Quadrant Electrometer.

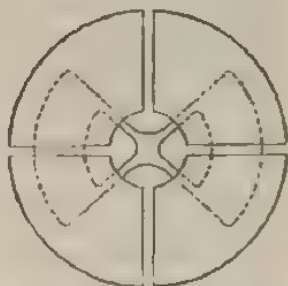


FIG. 166.—For the Quadrant Electrometer.

respectively. The binding-screw connected with the needle leads first through a platinum wire into a glass vessel situated in the lower part of the metal case and filled with sulphuric acid, into which, from the other side, dips a platinum wire. This wire forms the prolongation of the axis of the needle, and carries a pair of small transverse strips to retard, or to *damp*, the vibratory motion of the needle. To this prolongation, a small mirror is attached which is visible through a window of the case, and by the aid of a telescope and scale (143), renders the position of the needle readily observable. While in use, the

needle (of aluminium) is charged to a definite and rather high potential, by connecting it with the inner coating of a Leyden jar, or in some other similar way, one pair of quadrants being connected with the source of electricity to be measured and the other with the earth. The instrument may also be used by charging the quadrant pairs to oppositely equal potentials, and connecting the needle to the body to be investigated. For small deviations, the electrical force is in both cases proportional to the angular deflection, or to the number of graduations read from the scale.

These instruments serve also to compare electrical forces with one another, and accordingly also to measure the corresponding electrical masses and the potentials, since with unvarying capacity, these magnitudes are all proportional. The electric balance of Thomson (1867), on the other hand, gives in absolute measure, that is, by weight, the attraction of two parallel plates, one of which has a constant potential, while the other, which hangs horizontally at the end of a scale-beam, is connected with the body whose potential is to be measured. Since the electrical density upon a circular disk increases from the centre outward very slowly at first, but quite rapidly near the outer edge, the suspended disk is allowed to hang within a horizontal ring (*protecting ring*) connected electrically with it. It forms then merely the central part of a large plate over which the distribution of the electricity may be regarded as practically uniform. The other plate is now brought from below to such a distance from the latter, that the electrical attraction between the plates holds in equilibrium the force of gravity which tends to draw the suspended disk out of its protecting ring. This distance at which equilibrium is established is measurable.

To compare the capacities,  $C$  and  $C'$ , of two conductors, or condensers, we may measure by an electrometer the potential of the first due to a charge with any electrical mass whatever, and compare it with the potential,  $V'$ , of the two conductors connected with each other. We have then—

$$CV = (C + C')V', \text{ or } \frac{C}{C'} = \frac{V - V'}{V'}.$$

**193. Atmospheric Electricity.**—Even when the sky is clear the atmosphere exhibits considerable electrical tension. A point, or flame, at an elevated

position in the air and connected with an electroscope, or electrometer, is seen to take up, as a rule, a positive charge of electricity, the tension rising continually the higher the point is carried. The electroscopic effect increases toward daybreak, at first quite rapidly, and then more slowly until it reaches a morning maximum some hours after sunrise. It then diminishes and becomes weakest a few hours before sunset. After this it rises again rapidly, and attains a second maximum value some hours after sunset. During the night it diminishes again to a second minimum before daybreak, whereupon it begins again to rise. At about eleven o'clock a.m., the mean value for the entire day occurs. In winter the effect is considerably stronger than in summer. When we say that the earth is charged to a rather low (perhaps negative) potential, which increases with elevation in the atmosphere, we merely express the fact, that at the elevated point negative electricity flows off into the air while positive electricity is transmitted to the electroscope.

The cause of atmospheric electricity is unknown. It may perhaps arise in the outer layers of the air where the prevailing temperature is zero degrees, from the friction of water droplets against needles of ice (Sohncke, Lavini, 1884), the ice becomes positively and the water negatively electrified (Faraday). *Thunderstorms* are disturbances of the electrical equilibrium of the atmosphere, due to the rapid formation of heavy clouds. Storm-clouds are sometimes positively, and sometimes negatively electrical. *Lightning* is merely a gigantic electric spark passing between two clouds, or between a cloud and the earth. Franklin was the first to demonstrate the electrical nature of lightning (1752). He flew a kite provided with suitable points under a heavy cloud, and after the string became moistened by the rain and was made conducting, he was able to draw sparks from a bunch of keys suspended to the cord. The sparks were found to differ in no way from those produced by an electrical machine. The duration of lightning is so extraordinarily brief that a rapidly rotating wheel illuminated by it, seems to stand still, because during the extremely short duration of the illumination there is not time for the wheel to move appreciably. Wheatstone concluded from such experiments as this that the duration of lightning is less than 0.001 sec. According to appearance, three kinds of lightning are distinguished. *Zig-zag* lightning appears as an extremely slender line of light curved about irregularly, but never moving in a sharply broken, angular path (as is frequently represented in pictures), from cloud to cloud or from the clouds to the earth. It is often divided into forked tongues consisting of several branches, and resembles in this particular the sparks drawn from the conductor of an electrical machine. *Sheet* lightning spreads its luminosity with much less brilliance, and usually with reddish colour, over vast areas of cloud surface. It is probably only the reflection of zig-zag lightning, whose direct path is covered by a cloud. More seldom and of more enigmatical origin, is the so-called *ball* lightning, which has the appearance of fire-balls plunging from the clouds to the earth and moving so slowly as to permit the eye to follow their course and the mind to estimate their velocity. Oftentimes the ball vanishes suddenly and noiselessly, but frequently it flies to pieces with a loud report, shooting off zig-zag lightning in all directions and creating great consternation and damage.

Lightning always travels in the path which offers least resistance to electrical discharge. For this reason it prefers metals. If, in its path, it meets a body, which, by virtue of a small conductivity, opposes a resistance to its passage, it goes around it when the resistance of the circuitous route is less than that of the obstacle. When this is not the case it penetrates, or

even shatters to pieces the poor conductor. On its passage through the conductor, lightning develops a quantity of heat proportional to the resistance it overcomes. Thick plates of metal of small resistance are also slightly heated. Thin wires, or small fragments of metals, which offer too contracted a passage to the electricity, are melted and sometimes even dissipated to vapour. On the tops of mountains, the edges of jagged rocks are frequently observed to be molten and vitrified at their surfaces. When lightning strikes in sandy soil it forms branching, tube-like cavities, glazed on their inner surfaces from the fusing and flowing together, or vitrifying, of the grains of sand. These are called *lightning-tubes*, or *fulgurites*, or *sand-tubes*. They frequently attain a length of from 8 to 20 meters, and are from a few mm. to 5 cm. wide, tapering downward into a long slender point. Combustible bodies are consumed by lightning and liquids are vaporized by it. Trees, which are rendered somewhat conducting by the sap beneath the rind, are frequently struck by lightning. The rind is perforated, the bark shelled off in places, and the woody fibre split and shattered into numberless pieces. The aqueous vapour which is suddenly developed from the sap of the tree doubtless has a part in the destructive effect. Men and animals are deafened, or even killed, when lightning directly strikes their bodies, or other objects in their immediate neighbourhood. In the latter case the shock is due to the so-called reaction of the stroke. Under the action of a storm-cloud, terrestrial objects are charged by induction with electricity opposite to that of the cloud. Immediately the cloud discharges its electricity by a stroke of lightning, and the inducing action ceases, the induced electricity collected upon the body, just as suddenly passes off into the earth and has precisely the same effect as if a stroke of lightning had passed through the body. Upon the skin of the human body the trace of a direct stroke of lightning has the form of a streak, or band, with zig-zag edges, or branching arms. Many times these scars exhibit the forms of stars, or trees, formed of numberless branches. After a stroke of lightning, or even after a thunderstorm, the electrical odour (191) arising from the formation of ozone is frequently perceptible. Usually this odour is likened to the "smell of sulphur." When lightning passes near a magnetic needle it frequently deprives it of its magnetism, and sometimes even reverses its poles. Instruments made of steel are frequently magnetized by the effect of lightning.

As the spark of an electrical machine is always accompanied by a report, so is lightning always accompanied by *thunder*. Since light is transmitted almost instantly, while sound is propagated comparatively slowly, the thunder is always heard some time after the flash of lightning is seen. From the time which elapses between the flash of lightning and the clap of thunder, one may readily compute the distance from the observer to the cloud by remembering that sound traverses 340 m. per second. Although the sound arises at all points of the path of the lightning simultaneously, and like the electric spark lasts but an extremely short time in consequence of the slow propagation of sound, the peal of thunder is many times heard for 45 seconds. The path of the lightning frequently extends through 10 to 15 km. If we assume that the distances of the various points of its course from our position differ by only 1000 m., the report arising from the remotest point of the path will reach us three seconds later than that from the nearest point. During this interval the report is not uniform. Usually the thunder begins with a low rumbling, which is followed by violent cracks and claps, and finally dies away in a muffled roll. From all portions of the zig-zag path, whether directed toward or from us, a separate report reaches us, and from each point in

particular. From all points of the path, which lie transversely to the direction of propagation, the report reaches us simultaneously, and we hear a crack, or clap. A streak of lightning, all of whose points are equally distant from the ear, for example, a streak, which might be supposed to lie on the circumference of a circle whose centre is at the ear, would give a report which would be perceived as a single instantaneous clap. An abrupt curve of the path of the lightning, however, causes a sudden change in the intensity of the sound perceived, while at the same time it produces a displacement of its perception in time. In addition to this cause of the duration of a peal of thunder, a second must be mentioned, namely, the reverberations from cliff to cliff and between mountain walls, as also from the clouds themselves.

The lightning rod (Franklin, 1753) is based upon the property possessed by points of permitting electricity to flow with readiness from them. A high pointed rod of metal, connected by a good conductor with the earth, and terminating some distance beneath the surface, constitutes its essential part. The point of the iron receiving rod is gilded to protect it from rust, or it is prepared from some substance such as platinum, or silver, upon which rust does not collect. The conductor consists either of an iron rod or a cable wound from copper wire, or from galvanized iron wire, and it must not be too slender. The conductor is supported by iron brackets upon the roof and along the walls, and finally buried beneath the surface of the ground. In this way it is in connection with all the larger metallic portions of the building, *eg.* the gutters, water and gas pipes, etc.; but it is especially important that the connection with the ground shall be good. Dry earth conducts electricity very poorly. If possible the end of the conductor should be buried in moist earth, or in a pond, or brook. If, however, no large mass of water is available, the conductor must penetrate the earth to a layer which is always moist, and its branching terminal should there be imbedded in canals filled with charcoal. The latter is at once a good conductor and a preventive of oxidation. The conductor may also be made to terminate in a plate of metal with extended surfaces. If an electrified cloud passes above the point of the rod, it acts inductively upon the conductor, the like-named electricity is repelled into the earth, and the unlike is attracted to the point whence it issues toward the cloud, weakening or destroying its charge. Clouds moving over a city in which there are many such rods lose in this way, through a quiescent action, the major portion of their electricity. If the slow flow from the point is not sufficient to render the cloud harmless, the lightning will strike the rod, and if the conductor is good the electricity will pass off into the ground without injury to the building. With the ordinary arrangement of lightning rods it may be assumed that the receiving rod protects a circle whose radius is equal to the height of the rod.

A building to be protected may also be protected by a network consisting of a large number of wires connected with the earth without using the receiving rod at all (Melsen, 1865). The action here is similar to that of a coarse-meshed envelope of wire, which, as has been pointed out, protects its enclosure from the effect of external electrical influences (174). The network of telegraph and telephone wires above the houses of cities acts as an immense lightning rod, whose connection with the earth is kept in good condition by continuous use.

**194. Pyro-electricity.**—A tourmaline crystal cut into a cylindrical form becomes positive at one end and negative at the other by simply heating it. On the other hand, one end becomes negatively and the other positively electrified on cooling (Canton, 1752; Bergmann, 1767). The end which

becomes positive on heating is called the *analogous*, and the other the *antilogous*, and their connecting line is designated the electrical axis. With constant temperature the crystal is not electrical. Crystals such as turmaline, boracite, etc., which develop opposite electrical polarity at the ends by change of temperature, are called *terminal polar*. Others, such as topaz, prehnite, etc., on heating, exhibit two like-named polarities, both analogous, or both antilogous, since, by hypothesis, the corresponding opposite poles lie at the middle. These substances are on this account called *central polar* (Hankel, since 1839).

The distribution of electricity upon the surface of pyro-electric crystals may be rendered apparent by sprinkling the electroscopic powder of minium and sulphur upon them (Kundt, 1883).

**196. Piesoelectricity.**—Pyro-electric crystals become charged when subject to a pressure in the direction of the electrical axis in the same way as on cooling. When the pressure is diminished, or when the crystal is subject to a pull, the same effect is produced as if it were heated. The electrical masses developed in this way are proportional to the change of pressure (J. and R. Curie, 1881).

Non-crystalline bodies are also electrified by pressure. The two pieces of a divided cork are oppositely electrified when the surfaces of cleavage are pressed smartly against each other.

## VIII. ELECTRICAL CURRENTS.

**196. Galvanism.**—Louis Galvani, professor of anatomy in Bologna, noticed (1790) that the leg of a skinned frog jerked whenever a spark was drawn from the conductor of an electrical machine situated near it. The twitching of the muscle was manifestly due to the reactive effect of the discharge (*vide* 253); but Galvani thought he saw in this phenomenon a confirmation of his favourite view of the existence of an electricity peculiar to the bodies of animals. He accordingly devoted himself with great zeal to a further study of the observed facts. On one occasion he happened to suspend several frogs' legs by wire hooks upon the iron railing of his balcony to test the effect of atmospheric electricity upon them. He noticed a vigorous twitching of the muscles whenever he touched one of the frogs' legs to the iron railings. He was soon convinced that the phenomenon had nothing whatever to do with the electricity of the air, but that it occurred whenever the nerves or the spinal column of the frog came into metallic connection with the muscles. According to Galvani's view, the leg of the frog may be regarded as a sort of Leyden jar, whose opposite electrical coatings are the nerves on the one side and the muscles on the other, and that these coatings discharge into each other when brought into metallic connection. The observation made by Galvani himself that the twitching became more violent when the connecting metal was composed of two different substances, caused Alexander Volta, professor of physics in Pavia, to seek for the electrical source in the metals rather than in the frog's leg. When Volta completely denied that electricity is developed in the bodies of animals, he of course went too far, for it has been shown more recently that

the tip of a freshly prepared muscle is in reality negatively charged and the end attached to the brass positively (Du Bois-Raymond, 1848). His view, however, led him to the important and fruitful discovery that *two metals of different kinds, when brought into contact with each other are oppositely electrified*. Electricity produced in this way is called *galvanic*, or *contact* electricity.

**197. Fundamental Experiment of Volta.**—This fact may be demonstrated by the following experiment. A zinc and a copper disk, fitted with insulating handles, are brought together so that their clean metallic surfaces are in contact, and are then slipped apart parallel to each other. The electroscope then shows the zinc plate to be positively and the copper plate negatively electrified. Since the electrical mass developed by a single contact is too small to act strongly upon the electroscope, Volta used the condenser (*vide* 169), devised by himself, which he screwed to his electroscope to increase the density of its charge. To prevent contact with other metals, one plate of the condenser was made of zinc and the other of copper. Both were varnished upon their adjacent sides so that, when one was set upon the other, a thin insulating layer of resin separated them. When the zinc and copper disks mentioned above were taken apart after contact, the former was touched to the zinc and the latter to the copper plate of the condenser, the same process being repeated some sixteen times. The upper plate was now lifted off, and the capacity of the condenser so diminished, that the electrical mass collected upon the lower plate raised the tension of the electroscope high enough to spread the gold leaves apart, with positive electricity, when the condensing plate screwed to the electroscope was composed of zinc and with negative, when it was composed of copper.

**198. Volta's Contact Series.**—Volta investigated other metals in this way, and arranged his results so that the metals form a series of such character that each preceding metal, when touched with the following, becomes positively electrified, and each following is negatively electrified in contact with the preceding. The following are the most important series if

this series : zinc, lead, tin, iron, copper, silver, gold, platinum, to which may be added, as non-metallic bodies, carbon, and some metallic oxides, *e.g.* manganese peroxide and lead peroxide.

**199. Volta's Law.**—The difference of electrical tension, or of potential, produced by the contact of any two of these bodies is a definite magnitude depending upon the constitution of the bodies, but independent of their form and magnitude. To charge the metals to their characteristic potential, it is only necessary for them to touch at a single point. The quantity of electricity developed may, however, be very different, since the capacity of the two bodies is determined by their form, size, and relative position. If the zinc and the copper discs are placed with their surfaces in contact, they form, together with the atmospheric layer adhering to them, a condenser of high capacity, which, of course, requires a large quantity of electricity to charge it to a definite difference of potential. But if the surface of one disc is touched with only the edge of the other, the capacity is very small, and to charge the apparatus to the same difference of potential, a far smaller quantity of electricity is required.

The difference of tension between two substances is greater the farther they stand apart in the contact series. By means of a straw electroscope, so arranged as to permit the divergence of the straws to be estimated in degrees, Volta found the following numerical values for the differences of potential :—

Zinc - lead	...	...	5	Copper - silver	...	1
Lead - tin	...	...	1	Zinc - silver	...	12
Tin - iron	...	...	3	Tin - copper	...	5
Iron - copper	...	...	2	Zinc - iron	...	9

If now the first five values, from zinc - lead to copper - silver, are added, we have  $5 + 1 + 3 + 2 + 1 = 12$ , which is precisely equal to the difference of potential between the first and the last metals, *i.e.* between zinc and silver as given by observation. We find also that tin - iron + iron - copper = tin - copper and zinc - lead + lead - tin + tin - iron = zinc - iron. It follows, then, that the difference between two substances equals the sum of the potential differences between the terms of the potential series lying between those

substances. This proposition is called Volta's law of tension, or of *difference of potential*. Volta called those substances which follow this law, viz. the metals in particular, including mercury, *conductors of the first class*; the conducting liquids, however, especially ordinary water, acids, alkalis, salt solutions which do not conform to this law, *conductors of the second class*.

**200. Contact of Metals with Liquids.**—Volta assumed that electricity is produced only by contact of metals with each other. Later investigations, however, have shown that metals are also electrified by liquids, which act chemically upon them, and that the difference of potential produced is greater the stronger the tendency of the liquid to enter into chemical combination with the metal. In contact with dilute sulphuric acid, most metals are negatively electrified, while the acid is equally strongly electrified positively. But zinc, which has a great affinity for sulphuric acid, attains about as great a negative tension as copper, which possesses a much smaller tendency to combine with it. Platinum, which is not attacked by sulphuric acid, is not at all, or at most, very feebly electrified by it.

**201. Electromotive Force.**—By Volta's discovery a fact was established which seemed to contradict experience, so far as it had gone at that time, with reference to the nature and behaviour of electricity. Two different conductors, when in contact, become charged with opposite electricities, which, in spite of their mutual attraction, do not combine, but during the entire time of contact remain separate and at a constant difference of potential. To effect this separation work must be done, the amount of which equals the potential energy of the difference of potential attained, and the necessary work is measured by this same difference of potential. This quantity of work is called the *electromotive force*.

**202. Voltaic Pile.**—Any number of pairs of metallic plates (conductors of the second class) may be placed one upon the other, in which case the difference of potential between the terminal plates, by virtue of the law of tension, remains the same as though the terminal plates were in immediate contact. By making use of liquids also (conductors of the

second class), Volta succeeded in raising the feeble electromotive force to a high difference of potential.

If a disk of paste-board, or felt, saturated with a dilute solution of sulphuric acid, be laid upon an insulated copper plate, and upon this a zinc plate, at the surfaces of contact of the metals with the liquid, electromotive forces act, each of which repels negative electricity to the metal and positive electricity into the liquid and to the other metal in conducting connection with it. But since the electrical excitation between zinc and sulphuric acid is about four times as great as that between copper and sulphuric acid, about four times as much electricity passes from the zinc to the copper plate, as the copper plate, by virtue of its own action with the sulphuric acid, is able to absorb of negative electricity. In the zinc plate there is developed the same excess of negative electricity over the positive electricity led to it from the copper plate. The copper plate, therefore, becomes positively charged, and the zinc plate is just as strongly charged negatively, both with a difference of potential equal to the difference of the electromotive forces of copper-sulphuric acid and zinc-sulphuric acid, or, what is the same thing, equal to the sum of copper-sulphuric acid and zinc-sulphuric acid. In the combination copper-liquid-zinc (CLZ), called a *voltic*, or *galvanic element*, or *pair*, we have, accordingly, an apparatus whose continuously acting cause repels positive electricity into the copper and negative into the zinc plate. This action continues until a definite difference of potential is reached, which occurs, however, in an extremely brief time, and this difference of potential is then maintained under all circumstances.

The electromotive effect which, with a single pair, is very small, rises to a high difference of potential if, as was done by Volta (1800), several elements are placed upon each other in the form of a column, and in the order CLZ, CLZ . . . the same electromotive force is effective in each element, expelling the electricities toward opposite directions, the positive to the copper end, and the negative to the zinc end. The terminal plates must, therefore, attain a difference of potential as many times higher than that of a single element as there are

elements in the column. The copper end will be positively electrified, and the zinc end just as strongly negatively electrified, while the middle of the column remains neutral, because here equal and opposite tensions (potentials) meet and neutralize each other.

For simplicity, in the foregoing discussion, the electromotive effect due to the contact of the metal plates with one another was neglected. The final result is, however, not altered in any qualitative particular by this procedure. The entire effect of this latter modifying circumstance merely requires the difference of potential due to the action of the liquid to be augmented by that due to the mutual contact of the metallic plates.

Fig. 167 shows the *voltaic pile* in its original form. It is contained between glass columns let into the varnished wooden



FIG. 167 —  
Voltaic Pile.

plates, *a* and *b*. The ends of the pile are called its poles, the copper end, the positive, and the zinc, the negative pole. Volta, who regarded the contact of the metals alone as productive of the electromotive effect, and the fluids as merely passive conductors, constructed his pile of double plates composed of copper and zinc disks soldered together in the order ZCL, ZCL . . . , so that, in this case, the positive end was zinc and the negative, copper. If copper wires are attached to the terminal plates, the poles appear to be displaced to the ends of these wires (*electrodes*). So long as the wires are not allowed

to touch, the pile is *open*, and exhibits to electromotive tests measurable phenomena of potential. If  $V$  denote the difference of potential, produced by an element, and  $n$  the number of elements,  $nV$  will denote the difference of potential of the entire pile. If one end be connected with the earth, it is brought to the potential zero, and the other end has the potential  $\pm nV$ ; hence, at the middle of the pile, the potential is  $\pm \frac{1}{2}nV$ . If the pile is now insulated, the potential at the positive end becomes  $+\frac{1}{2}nV$ , at the negative end  $-\frac{1}{2}nV$ , and at the middle zero, so that the difference of potential is again  $nV$ . This difference remains unchanged, no matter how

the value of the potential at the poles themselves be changed by the application of electricity from without.

**203. The Dry Pile of Zamboni** (Ritter, 1802; Behrens, 1806; Zamboni, 1812) is a voltaic pile of from one thousand to two thousand pairs of plates, in which air-dried paper takes the place of the moistened disks of felt, copper-bronze; or binocide of manganese and tin replace the metallic copper and zinc respectively. To prepare a dry pile, leaves of gilded and silvered paper are stuck together on their paper surfaces. Disks are cut from this, and piled one above the other in a glass tube in such order that the gilded surface of the one disk lies against the silvered surface of the preceding. The tube is then closed by means of brass terminals cemented over its ends. The moisture, which is always present in the air-dried paper, acts upon the metals with the same electrical effect as does the liquid of an ordinary voltaic pile. The terminals, or poles, of the insulated pile, become charged, therefore, with opposite electricities, the gilded pole, positively, and the silvered, negatively, to a difference of potential proportional to the number of pairs of plates. Since the electromotive force, acting incessantly in each element, or pair of plates, maintains this difference of potential, immediately replacing any electricity which may perchance be withdrawn, the poles will remain oppositely electrified with undiminished strength for many

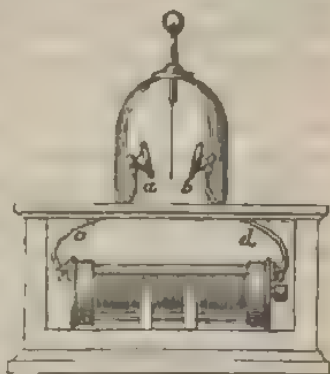


FIG. 168.—Dry Pile Electroscope.

years. In Fig. 168, a horizontal dry pile is represented. By means of the conducting wires, *c* and *d*, the poles are displaced to the plates, *a* and *b*, one of which is, therefore, always positively, and the other negatively electrified, and both, too, with the same intensity. Over the pole-plates, a glass cover is inverted, from the apex of whose arch a thin sheet of gold-leaf is suspended between the poles, by the aid

of a metallic rod, terminating above in a knob. If the gold-leaf be brought into contact with one of the poles, for example, with the positive pole, so soon as it becomes charged with positive electricity it will be repelled and drawn to the negative pole. After having surrendered its positive charge to this pole and received in its place a negative charge, the leaf will be repelled from and then attracted to the positive pole, and so it will continue to move back and forth between the pole-plates incessantly. One of these so-called "perpetual-motion electrical machines" may continue to act for many years. If the pole-plates are separated so far that the motion of the gold-leaf ceases, it will remain at equilibrium midway between the two poles, since it is equally strongly influenced by both of them. The apparatus then forms a highly sensitive electroscope, called the *dry-pile electroscope* (Fig. 168; Behrens, 1806; Bohnenberger, 1817; Fechner, 1829). If a feebly charged body be brought near the knob, the gold-leaf will be attracted by the oppositely electrified pole of the pile, and reveal by its motion not only the *presence*, but also the *kind* of electricity upon the body to be tested. The dry pile is also frequently applied in the quadrant electrometer (*vide* 192) to keep the needle of the instrument at a constant and high potential.

**204. The Electric, or Galvanic Current.**—If the electrodes of a voltaic pile are brought into contact, the pile being thereby closed, the electricities accumulated upon the terminal plates of the open pile will neutralize each other through the connecting wire of the terminals, since the positive electricity passing from the gilded end of the pile through the connecting wire to the silvered end, unites with the same quantity of negative electricity passing from the silvered to the gilded end. This so-called *electric*, or *galvanic current*, flows continuously, because the electromotive force acting within the elements of the pile, continually striving to maintain a definite difference of potential, constantly repels positive electricity toward the gilded end, and negative toward the silvered end, and thence through the closing wire. The closed pile is itself also traversed by an electric current, forming with the closing

wire an unbroken circuit, within which, after a brief interval following the instant of closing, a stationary condition is reached. When this state of dynamic equilibrium has been reached, through each section of the circuit equal quantities of opposite electricities pass in opposite directions during equal times. The quantity of electricity passing through any arbitrary section of the circuit, during one second, is called the *strength of the current*, while the *direction of flow of the positive electricity* is called the *direction of the current*. We say, therefore, that the galvanic current flows in the connecting wire from the gilded to the silvered pole, and, in the pile, from the silvered to the gilded pole. In these apparatus, zinc and copper plates are frequently used instead of the silvered and gilded surfaces respectively, and it will be more convenient to use the terms zinc and copper poles, zinc and copper plates, etc., when we refer to the negative and positive electrodes, than the more cumbrous terms referring to the substances used in *Zamboni's pile*. For this reason, we shall use the nomenclature of the zinc-copper pile. Since, in each element positive electricity flows from the zinc plate through the liquid to the copper plate, zinc is called the *electro-positive*, and copper (or its representative) the *electro-negative* metal.

**205. Galvanic Battery.**—In consequence of the many difficulties attending the construction of a pile with moistened felt, or pasteboard, the voltaic pile, in its original form, is now seldom used in the production of currents. A voltaic element may be more conveniently formed by placing a plate of copper and another of zinc into a glass vessel containing dilute sulphuric acid, and, since it is unnecessary for the zinc and

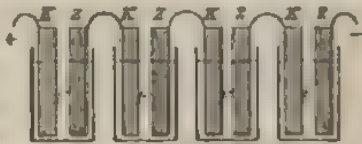


FIG. 169.—Voltaic Cells.

copper plates of two adjoining elements to touch throughout their entire extent of surface, a "pile," or "chain," or "battery," may be constructed of such elements by simply connecting the copper of each element with the zinc of the next following through a wire, or strip of copper (*Voltaic Cells*, Fig. 169). Binding-screws are therefore provided to make any desired

mode of connecting the plates easily possible. A battery such as this, composed of copper and zinc in ordinary water, and called a "*water battery*," is especially well adapted for charging the quadrant electrometer.

A high difference of potential may be obtained by using with zinc a plate (conductor of the first class) of platinum, or carbon, or any other substance less highly influenced electrically by the liquid than is the copper. Smee's element, for example, is composed of a plate of zinc, and another of platinum coated with platinum dust, or of a plate of silver immersed in a rarefied solution of sulphuric acid.

The more convenient and powerful Bunsen cell (Fig. 170) contains two carbon plates in the condition of coke, in metallic



FIG. 170.  
Plunge Battery.

connection with each other, immersed in a solution of chromic acid, or in a mixture of potassium bichromate and sulphuric acid, filling the lower and larger portion of the flask. Between the carbon plates is a plate of zinc, which, by means of a metallic rod extending through the cover of the vessel, may be lowered into the liquid when the cell is in use. One of the two brass binding-screws upon the cover is connected with the two carbon plates and the other with the zinc plate. The chromic acid *plunge-battery* of Bunsen has the zinc and carbon plates attached to a common frame of wood, by the aid of which they may be simultaneously immersed in the liquid chromic acid contained in the lower part of the vessel.

The elements thus far described contain but a single liquid and give at the outset a strong current, though the current strength diminishes very rapidly, because during the passage of the current through the liquid, certain chemical changes occur which reduce the electromotive force. Such elements are for this reason called *inconstant*. This diminution of strength may be prevented by immersing each of the plates in a specially prepared liquid, and the so-called *constant element* is then obtained. The latter furnishes a current which flows for a considerable time with constant strength.

*Daniell's* (1836) element (Fig. 171) consists of zinc in dilute sulphuric acid, or of a solution of zinc sulphate, and of copper immersed in a dilute solution of copper vitriol (copper sulphate). The dilute sulphuric acid is contained in a cylindrical vessel, *T*, of porous earthenware, and the solution of copper vitriol is contained in the glass vessel itself. The porous wall of the jar prevents the mixture of the liquids but permits the passage of the current, since after the walls are drenched with the liquid they become conducting. The zinc plate, *Z*, and the copper plate, *K*, are bent into a cylindrical form to conform to the shape of the containing vessel. The copper strips, *m* and *p*, soldered to the plates, and the binding-screws, are used to connect the plates with the neighbouring cell, or with the wires to be used as poles.

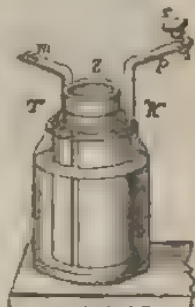


FIG. 171.—  
Daniell's Element.

*Meidinger's* (1859, Fig. 172) element embodies a practical modification of Daniell's. A cylindrical zinc plate, *XX*, to which is soldered a conducting wire, *cc*, is supported against a contraction, *bb*, in the glass wall of the vessel, *AA*. Within the glass vessel, *dd*, soldered to the bottom of the vessel, *AA*, is a curved copper plate, *a*, to which a copper wire, *fg*, insulated by a rubber coating extends. From the cover of the vessel, *AA*, extends a wide, open-mouthed glass tube, *h*, entirely within the vessel, *dd*. This tube, *h*, is filled with crystals of copper vitriol, and the vessel, *AA*, contains a solution of magnesium sulphate. The copper sulphate dissolves, forming a solution, which, by reason of its great specific gravity, remains in contact with the copper plate, within the vessel, *dd*, while the zinc plate is surrounded by the lighter magnesium sulphate solution. And thus, without the use of the wall of a porous earthen jar, the salt solutions are kept apart sufficiently well.



FIG. 172.—  
Meidinger's  
Element.

*Grove's* element (1839) consists of a piece of zinc immersed

in dilute sulphuric acid and of platinum immersed in concentrated nitric acid contained in a porous earthen cell. In Bunsen's element (1842) the platinum is replaced by the equally effective carbon. Fig. 173 represents a battery composed of three Bunsen elements,



FIG. 173.—Bunsen's Elements.

in which the carbon in the form of thick bars is immersed in the nitric acid of the porous cells, and the zinc is surrounded by dilute sulphuric acid contained within a glazed earthen vessel outside of the porous jar.

With the element of *Leclanché* (1868, Fig. 174) a carbon plate, K, is surrounded by a mixture of powdered manganese and carbon contained within a porous cell, while outside of the jar a solution of sal ammoniac surrounds the bar of zinc, Z.



FIG. 174. Leclanché's Element.

In all of these elements, the zinc is amalgamated, i.e. coated with mercury until the surface is covered with a compound of zinc and mercury called zinc amalgam. This is done to protect the zinc from the direct action of the sulphuric acid when the battery is not in use.

In the case of *dry elements*, the liquid is replaced by an artificially prepared substance drenched in a suitable solution and then hardened. These prepared substances are usually made of gypsum, calcium hydrate, chalk, clay, etc. The dry Daniell's element due to Beetz (1884) is prepared by pouring

a mass of gypsum into a solution of zinc sulphate and another into a solution of copper sulphate, the mixtures being contained in the branches of a U-shaped glass tube. A wire of zinc is inserted into the former, and a wire of copper into the latter solution, which, after the gypsum hardens, are rigidly

connected with the prepared substances. With the dry elements used in commerce, the outer covering of zinc serves as one of the exciting plates, while the other is usually prepared from coke.

**206. Switches, Commutators, or Gyrotroves,** are used to open, close, and change at will the current of the galvanic circuit. Of the numerous forms, the following will serve as types. The commutator of Pohl (1828, Fig. 175) consists of a wooden base carrying six mercurial cups, the four corner cups being connected in pairs by diagonal wires. Two branching pieces of metal, each having three prongs, are connected by a brass rod into the form of a rocking frame. The middle prongs dip into the two middle cups. The poles of the battery wires are connected with these cups, while the ends of the conductor in which the current is to be turned, dip into the two corner cups on the right. If the rocker lies on the right side as shown in the figure, the current flows in the wire connecting these cups in the direction of the arrow. If, however, the rocker be turned so that its arms lie in the corner cups on the left, the current flows through the wire in the opposite direction.

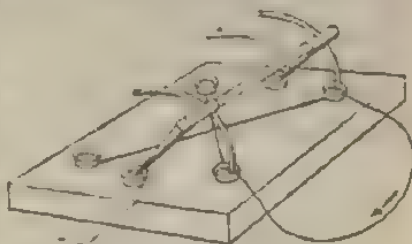


FIG. 175.—Pohl's Commutator.



FIG. 176.—Ruhmkorff's Commutator.

The commutator of Ruhmkorff (1846, Fig. 176) consists of an ivory cylinder provided with two diametrically opposite curved brass strips and borne by a metallic axis. This axis, instead of passing continuously through the cylinder, consists of two pieces, one of which is connected with one of the brass strips and the other with the other strip. The parts of the axis are connected through their brass supports with binding-screws for the reception of the battery terminals, while the two binding-screws

in which the connecting wire terminates in both directions are in metallic connection with the brass springs, which press against the cylinder. If the cylinder is turned by means of the milled head so that the brass strips attached to it come in contact with the springs, the current passes through the conductor in one direction, while, if the cylinder be turned 180 degrees, the direction of the current in the conducting wire is reversed. When the brass strips do not touch the springs, as is the case in the figure, the circuit is open.

**207. Electrolysis.**—In the year 1800, Ritter discovered that liquid conductors undergo chemical decomposition on the



FIG. 177.—Apparatus for Decomposing Water.

passage of the galvanic current through them. If two platinum plates connected with the poles of a galvanic battery (the electrodes) are immersed in water containing a little sulphuric acid, bubbles of gas are seen to rise from the plates, although no bubbles are noticeable in the liquid between the plates. By means of the apparatus represented in Fig. 177, the gaseous masses developed at each plate may be collected in separate vessels. Acidulated water is placed in a funnel-shaped glass vessel through the bottom of which pass two insulated conducting wires,  $f$  and  $f'$ , which carry the platinum plates. Above each platinum plate a glass tube, closed at its upper end and filled at the outset with the liquid in the funnel, is inverted in such way that the bubbles

of gas rising from the terminal plates collect in the upper parts of the tubes at H and at O. It is soon found that the gas rising from the negative (−) pole occupies twice as much space as that developed at the positive (+) pole. The former may be ignited, and it burns with a feebly luminous flame, while the latter does not burn, but kindles a feebly glowing splinter of wood inserted in it to a brilliant flame. From these phenomena it is inferred that the former gas is hydrogen (H), and the

latter, oxygen (O). These two elements are the constituents of water, and it is known that exactly two parts by volume of hydrogen ( $H_2$ ) unite with one part of oxygen (O) to form two parts of water vapour ( $H_2O$ ). The current, passing through the liquid between the electrodes, decomposes the water, the hydrogen collecting, as the electro-positive constituent (*cathion*<sup>1</sup>), at the negative electrode (*cathode*), and the oxygen as the electro-negative (*anion*) constituent, at the positive electrode (*anode*). Both elements, it will be noticed, are decomposed in the same relative quantities in which they combine with each other to produce water.

That the water is not decomposed by the current alone, however, is seen from the fact, that if the apparatus be filled with perfectly pure distilled water, no gas collects on the platinum plates, and no current passes between them. Pure water is, therefore, not an *electrolyte*. The substance decomposed by the current must then be sulphuric acid ( $H_2SO_4$ ). This consists of hydrogen on the one hand, and sulphur in combination with oxygen on the other, and is decomposed in such manner that the hydrogen ( $H_2$ ) collects at the negative pole, and the residue ( $SO_4$ ) at the positive pole. This sulphuric acid residue cannot exist, however, in an isolated state, but is immediately retransformed into sulphuric acid by extracting from the water the hydrogen necessary to its formation, and at the same time liberating a corresponding quantity of oxygen, which collects at the positive electrode ( $SO_4 + H_2O = H_2SO_4 + O$ ). The oxygen is then produced, not by immediate electro-chemical decomposition, but by the indirect effect, or secondary action, of the liberated acid residue upon the *solvent*

<sup>1</sup> For conveniently designating the ideas occurring in electro-chemical decomposition, Faraday introduced a nomenclature which has now found universal acceptance. According to him, the process itself is called *electrolysis* and a chemical compound which may be decomposed by means of the galvanic current is called an *electrolyte*. The pole plates through which the current enters and exits are called the *electrodes* ("electrical paths"), the positive electrode is called the *anode* ("upward path"), and the negative the *cathode* ("downward path"). The decomposed constituents are called *ions* (more correctly *iontes*, "the migrators", that particular constituent collecting at the anode being called the *anion* ("the ascending ion"), and that collecting about the cathode, the *cathion* ("the descending ion").

of the sulphuric acid, i.e. upon the water. The final result is the same as if the water had been decomposed, and the sulphuric acid had remained wholly unaffected. For this reason the entire process may with propriety be designated the decomposition of water. In the same way as with sulphuric acid, salts which are soluble in water are also decomposed by the galvanic current. Glauber's salt (sodium sulphate,  $\text{Na}_2\text{SO}_4$ ), for example, is to be regarded as sulphuric acid in which the hydrogen has been replaced by sodium (Na).

The first action of the current upon this salt, therefore, is to collect the sodium at the negative pole and the sulphuric acid residue ( $\text{SO}_4$ ) at the positive pole. The sodium then extracts oxygen from the water and forms sodium hydroxide ( $\text{NaHO}$ ), more familiarly known as *caustic soda*. The chemical action may be formulated thus:  $\text{Na}_2 + 2\text{H}_2\text{O} = 2(\text{NaHO}) + \text{H}_2$  and the sulphuric acid residue is converted to sulphuric acid by withdrawing hydrogen from the water and liberating the oxygen.



FIG. 178.—U-shaped  
Decomposing  
Apparatus.

Oxygen and hydrogen gas are therefore developed at the positive and negative poles as before, with the distinguishing characteristic that, in the former case, free sulphuric acid was formed, and, in the latter, free caustic soda. These latter substances may be rendered apparent by staining the colourless solution of Glauber's salt with a greenish vegetable matter and subjecting it to electrolytic action in a U-shaped vessel (Fig. 178). The liquid at the positive pole will then be coloured red by the acid, and at the negative pole, green by the alkali.

When the metal contained in the salt solution can exist in contact with the water without decomposing the latter, no hydrogen will be developed at the negative pole, but the metal itself will then be deposited upon the pole plate. This occurs when the galvanic current is passed through a solution of copper-vitriol (copper sulphate,  $\text{CuSO}_4$ ). The cathode becomes coated with a coherent layer of metallic copper, while at the anode, free sulphuric acid and oxygen gas appear. If the anode be

composed of copper, the sulphuric acid residue is transformed into copper vitriol by uniting with the copper, and no decomposition of the water, and, consequently also, no development of oxygen occur. Copper is merely dissolved at the anode, and at the same time an equal portion is deposited at the cathode. To prevent such an action of the decomposed constituents upon the pole-plates, they may be made of platinum, because this metal is least attacked by chemical action.

Many metals, such as silver and lead, are separated from their solutions in a crystalline form. The process of separation may be rendered visible to a large number of spectators at the same time by projecting an image of the decomposing cell, containing the solution of lead acetate and provided with parallel glass walls, upon a screen, the two electrodes being composed of lead. At the cathode, the crystals of lead appear in beautiful branching forms (lead trees, *arbor saturni*). These may be dissolved and reformed at the other electrode by reversing the current.

The alkalis and the earths were supposed to be undecomposable, until Davy, in 1807, succeeded in obtaining from moistened calcium hydroxide (KOH) brilliant little globules of the metal calcium (K and H at the cathode and O at the anode), of the lustre of silver.

Molten chlorides of metals give chlorine at the anode, and the metal at the cathode. Magnesium, and especially aluminium, are prepared in wholesale quantities by electrolysis.

If the poles of a battery are placed upon a piece of paper moistened with a solution of calcium iodide and starch paste, a blue spot forms at the positive pole by reason of the deposition of iodine at this place. Such a piece of paper may be used to distinguish the electrodes. The observation of Soutzter (1760), made long before Galvani's discovery, that when two pieces of metal, copper and iron, are brought into contact with each other, and their free ends are placed beneath the tongue, a peculiar sensation of taste is experienced, is due to electrolytic action. The nature of the poles may, in fact, be distinguished by the taste, the positive pole placed upon the tongue having an acid, and the negative pole, an alkaline taste.

**208. Electrolytic and Metallic Conduction.**—A liquid, therefore, conducts an electric current in a manner quite different from a metal. While a metallic conductor undergoes no chemical change upon the passage of the current, any liquid, which is not itself a metal such as mercury, transmits the current only when accompanied by chemical action. Liquids which cannot be decomposed by the current do not conduct it. Such, for example, are pure water, alcohol, petroleum, and carbon disulphide. Volta's distinction, therefore, between conductors of the first and the second class is well founded. The former comprises the metals and the carbons, and the latter the electrolytes.

**209. Electrometallurgy (Galvanoplastics).**—Jacobi observed, in Dorpat, in 1837, that the copper coating deposited upon the negative pole-plate by the electrolytic action of the copper vitriol solution, may be readily dissolved off and the irregularities of the plate thereby perfectly copied. He based upon this principle a method of reproducing medals, engraved plates and other objects in copper, deposited chemically, and called the method *galvanoplastics* (*electrometallurgy*). To obtain an electrolytic reproduction of a medal, or any other suitable object of art, an image, or *form*, of the object is prepared in wax, stearine, gutta-percha, gypsum, or other similar substance, and this *form* is given a conducting surface by coating it carefully by hand with finely powdered graphite. The *form* is then connected with the negative pole of a galvanic battery, or other source of current, and immersed in a copper bath, while a copper plate is attached to the other battery wire as an anode. As soon as the current begins to pass, copper is deposited from the solution upon the surface of the form, while at the same time enough copper is dissolved from the anode to keep the solution at a uniform degree of concentration.

Metallic coatings deposited electrolytically upon perfectly clean metallic surfaces adhere with extreme closeness. The process of coating objects made of cheaper metals with a thin, firm coating of a precious metal—electrolytic silvering, nickel-ing, gilding, etc.—and called "*electro-plating*," is based upon this principle. In this process, the object to be plated is

connected with the cathode, and immersed in a properly prepared solution of the metal to be used, while a plate of the same metal is used as anode.

**210. Faraday's Electrolytic Laws.**—If a number of apparatus for decomposing water (177) be connected in series in the same circuit, the same amount of hydrogen will be developed in all, no matter how the form, size, and distance of the electrodes may be varied. Precisely the same quantity of gas is furnished when the current is permitted to pass during the same interval of time, whether all, or but a part, or even a single one of the apparatus be used. A given current, therefore, always decomposes the same quantity of water (Faraday, 1833).

If several decomposing vessels containing various electrolytes (for example: water acidulated with sulphuric acid, hydrochloric acid, lead chlorate, and copper sulphate) be placed in turn in the same circuit, that containing the first-mentioned electrolyte will furnish two parts by weight of hydrogen and 16 of oxygen; the second, two parts of hydrogen and 71 of chlorine; the third, 207 of lead, and 71 of chlorine; and the fourth, 63 of copper and 16 of oxygen. Or, if three such decomposing apparatus, one containing hydrochloric acid ( $\text{HCl}$ ), one water ( $\text{H}_2\text{O}$ ), and the third, liquid ammonium ( $\text{NH}_3$ ), precisely the same quantity of hydrogen gas will be developed in all, and in addition to it, in the first, an equal volume of chlorine gas; in the second, half as great a volume of oxygen gas, and in the third, three times as great a volume of nitrogen gas. By one and the same current, therefore, with different electrolytes, the constituent parts separated out have quantitative relationships identical with those under which they combine chemically, *or the weight-relations of the constituents separated by a given current are as the chemical equivalents (combining weights)*. It may be said also that the same quantity of electricity is necessary and sufficient to separate the equivalent of any arbitrary product of decomposition. This law, discovered by Faraday in 1833, is called the *law of fixed electrolytic action*.

If the external connecting wire of the battery be divided at

the point A (Fig. 179) into two exactly equal branches, which again unite at the point B, and if one of three exactly equal decomposing apparatus,  $M$ ,  $M_1$  and  $M_2$ , be inserted into each of the branches of the circuit, and the other, into the unbranched circuit, it is clear that through  $M_1$  and  $M_2$  only half as

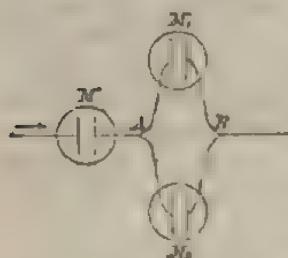


FIG. 179.—Faraday's Experiment.

much electricity will pass as through  $M$ , or that the strength of the current at  $M_1$  and  $M_2$  will be only half of that at  $M$ . It is also found that the quantities of gas developed at  $M_1$  and  $M_2$  are equal to each other, and each to one half of the quantity developed at  $M$ . From this it follows that the amounts of the electrolytes decomposed in the same time are proportional to the strength of the current.

**211. Theory of Electrolysis.**—To explain the process of electrolysis, and especially the fact that the products of decomposition collect only at the electrodes, each molecule of the electrolyte may be imagined to be composed of an electro-positive and an electro-negative constituent (Benzlius, 1819), the former of which is attracted by the negative and the latter by the positive pole. In case of the sulphuric acid, the Glauber's salt, copper vitriol, for example, hydrogen, sodium, copper, form respectively the electro-positive, while the sulphuric acid residue ( $SO_4$ ) constitutes the electro-negative constituent. According to the kinetic theory of aggregation (Clausius, 1857), the molecules of a liquid are in incessant motion, crowding one another together, gliding beside one another, and frequently coming into close contact, thereby producing collisions. During these motions oppositely electrified constituents of molecules closely-crowded together must, at times, be torn asunder and new molecules of the same kind formed, while, at the same time, the liberated molecular constituents reform into molecules, either by combining with other free molecular constituents of the opposite kind which they meet, or by tearing apart the neighbouring molecules, and uniting with the opposite electrical constituent. The molecules of a liquid, according to this view, would be in a condition of incessant decomposition and reformation, even before being subjected to electrical action, their oppositely electrical constituents (*ions*) moving aimlessly about in all possible directions. By immersing the poles of a galvanic battery in the liquid this motion is rendered more or less regular, as the electro-negative constituents are then attracted toward the positive pole and the electro-positive toward the negative pole. At the poles themselves the constituents are liberated; but between the electrodes occurs an incessant separation and immediate recombination of them. Since, by the electromotive force of the latter, the electrical masses withdrawn at the electrodes are being continually replaced, this motion of the "ion" becomes continuous, the current at the same time being transmitted through the liquid. According to this view, the electrolytic law of equivalents

signifies that the molecular constituents of different electrolytes are charged with equal quantities of electricity, and hence it follows also that the electrical masses decomposed are proportional to the quantity of electricity transmitted in equal times, i.e. to the strength of the current.

**212. Voltameters.**—Since, according to Faraday's laws, current-strength is proportional to the quantity of the electrolyte decomposed in a definite time by determining this quantity, the strength of the current may be measured. Apparatus used for this purpose are called *voltameters*. The one most generally used is the *oxy-hydrogen voltameter* (Fig. 180). Two insulated wires pass through the air-tight stopper of a glass vessel, to the terminals of which platinum plates are attached as electrodes. The vessel is filled with water, acidulated with sulphuric acid. The gas developed on the plates, composed of one part by volume of oxygen and two parts of hydrogen, mix in the upper part of the vessel forming oxy-hydrogen gas. This escapes through the bent tube and is caught in a graduated glass tube over a vessel of water (93). With the voltameter it may be easily proved that the current-strength is the same throughout the circuit; for at whatever point the voltameter is inserted the quantity of oxy-hydrogen gas developed is the same during a given interval of time.

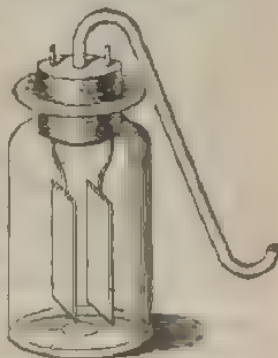


FIG. 180.—Oxy-hydrogen Voltameter.

Jacobi (1839) selected as the unit of current-strength that of a current which develops one ccm. of oxy-hydrogen gas at  $0^{\circ}$  C. and 760 mm. pressure in one minute. The unit of strength now generally used is called the *ampère*, and furnishes 1044 ccm. of oxy-hydrogen gas, or 6.96 ccm. of hydrogen gas in one minute.

Instead of using water, the current to be measured may be passed through a solution of copper sulphate (*copper voltameter*), or of silver chloride (*silver voltameter*), and the quantity of metal deposited at the negative pole determined by weighing.

Since, during a given time, a given current decomposes equivalent masses of different electrolytes, it is easy to compute the quantity of oxy-hydrogen gas corresponding to these masses. The copper, and especially the silver voltameter, furnish more accurate results than the oxy-hydrogen voltameter. A current-strength of 1 ampère deposits during one minute 19.68 mg. of copper and 67.09 mg. of silver, or in one hour such a current-strength would precipitate 4.025 g. of silver.

The amount of an "ion" separated by the unit current (one ampère) in the unit of time (one second), is called its *electrochemical equivalent*. That of silver is 1.118 mg. The quantity of electricity which flows in electrolytes with 1.118 mg. of silver, or with the equivalent weight of any other ion, is consequently selected as the unit of electrical quantity and is designated the *coulomb*.

**213. Deviation of the Magnetic Needle.**—Oersted discovered in 1820 that a magnetic needle suspended upon a point, when brought near a conductor traversed by a current, is deflected from its normal position of equilibrium in the magnetic meridian, and drawn aside into a new position of equilibrium. A conductor, therefore, acts upon the needle as a couple, under the influence of which it is turned until the couple originating from terrestrial magnetism holds it in equilibrium. When the effect of the earth is balanced by means of a magnet, *i.e.* when the needle is rendered *astatic* (138), it assumes a position at right angles to a rectilinear conductor passing horizontally above or below it. The current, therefore, strives to bring the needle to a position perpendicular to its own direction, or the couple-action of the current upon the needle is perpendicular to the plane through the current and the pivot of the needle.

**214. Ampère's Rule.**—To determine the direction of this deviation, Ampère gives the following practical rule:—*Imagine a small human figure swimming in the circuit with its face turned toward the needle, the north pole of the needle will then always be deflected toward the figure's left.*

If the conducting wire is curved about the needle in a vertical plane passing through it, we see from this rule that all parts of the current strive to deflect the needle in such

direction as to turn the south pole toward that side from which the current would appear to encircle the needle in the direction of the hands of a watch.

**215. Galvanoscope.**—A copper strip provided with binding-screws at its ends and bent about a magnetic needle suspended upon a point may therefore be used to determine not only the presence, but also the direction of the current, into the circuit of which this *galvanoscope* (rheoscope) has been inserted. The fact of deviation determines the former, and its *sense*, the latter.

**216. Galvanometers, or Multipliers.**—The force with which a current tends to deflect the magnetic needle is proportional to its strength. For, if a magnetic needle be suspended by a wire provided at its top end with a torsion circle, above a conductor placed horizontally in the magnetic meridian, the angular twist through which this wire must be turned to bring the deflected needle into its original position is proportional to the quantity of oxy-hydrogen gas, which would be developed in a voltameter placed in circuit with it.

By observing the deviation of the magnetic needle, one may thus compare and, hence, measure the strengths of currents. Apparatus to be used for this purpose are called *galvanometers* (*rheometers*).

To measure currents of low intensity (weak currents), the conducting wire is wound about the needle a large number of times, to augment the effect of the current upon it (Fig. 181).

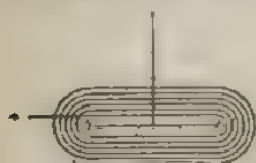


FIG. 181.—Multiplier with Simple Needle.

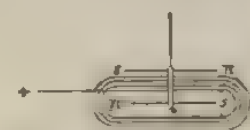


FIG. 182.—Multiplier with Astatic Needle.

The coils of wire are usually covered with silk, or some other nonconducting material to insulate them from each other. Since the windings are all in the same sense, the effect upon the needle is multiplied as many times as there are coils. Such an

apparatus is called a *multiplier* (Schweigger, Poggendorff, 1821). The magnetic needle is suspended by a silk fibre so as to be easily movable.

To secure greater sensitiveness, the *astatic pair* (Nobili, 1825) is used (Fig. 182). In this, two magnetic needles, *ns* and *sn* (cf. 138), are rigidly connected by a light bar. The poles are placed in opposite directions, the one being suspended within and the other without the coils of the multiplier. If the needles are of nearly equal magnetic strengths, the effect of the earth's magnetism is neutralized almost completely. The needle is then held in the magnetic

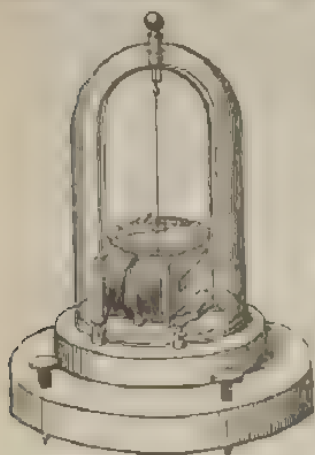


FIG. 183.—Astatic Galvanometer.

meridian with a very feeble force, and may, therefore, be deflected from it by a very weak current. This current may, of course, be weaker the greater the number of windings in the coil, since, according to Ampère's rule, the effect upon both needles is in the same sense. Fig. 183 represents such a galvanometer fitted with such an astatic pair. The lower needle swings within the hollow of a small wooden frame about which the coils of the multiplier are wound. The upper needle plays above a circle, graduated to degrees, from which the angular deviation may be read. To prevent the disturbing effect of atmospheric influences a wide glass bell is inverted over the instrument. In front of the bell two binding-screws are visible. The latter are connected with the ends of a wire of the coil, and are used for the reception of the conducting wires.

Still greater sensitiveness may be obtained by the use of the *mirror galvanometer*. In the instrument represented in Fig. 184, the magnetic bar is suspended by a silk fibre within a fixed copper case, about which the multiplier is wound, the separate layers being insulated from each other. By means

of the binding-screws shown at the left, the conducting wire may be connected in various ways with the wire of the coil. Above the magnetic bar, and fixed rigidly to it, is a small mirror, which, as with the magnetometer, is used in connection with a telescope and graduated scale to measure the deflection.

The angular deflection does not immediately furnish a measure of the current-intensity; for the deflecting force, while proportional to the intensity when the needle is fixed, varies when the needle changes its position relatively to the current. The intensity is therefore dependent upon the angle of deflection, but the nature of this dependence varies with the construction of the instrument. Since, however, a definite deviation corresponds to each intensity, it would be possible to construct experimentally with currents of known intensity, a table for each galvanometer, from which, for any observed angle of deflection, the corresponding current-intensity could be read.

If the angle of deflection be kept small, so that the magnet does not appreciably alter its position with respect to the current, the intensity may be regarded as proportional to the deviation. This condition of things actually obtains with mirror galvanometers, where the current-intensity is, without perceptible error, proportional to the number of the graduations read by the telescope.

217. *Sine Galvanometer* (Pouillet, 1837).—If the frame of

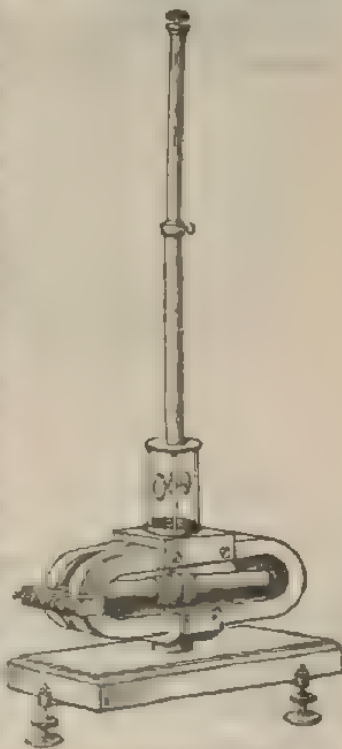


FIG. 184. —Mirror Galvanometer.

the multiplier be movable about the vertical axis of the galvanometer, and it be turned so as to follow the deflected needle until the plane of its windings coincides again with the direction of the needle, the position of the needle with respect to the coils will be the same at every observation, and consequently, the deflecting force of the current perpendicular to the plane of the windings is proportional simply to the intensity, " $J$ ," of the current.

If  $\alpha$  (Fig. 185) denote the angle read from the horizontal graduated circle, through which the frame must be turned, and



FIG. 185.—For the Sine Galvanometer.

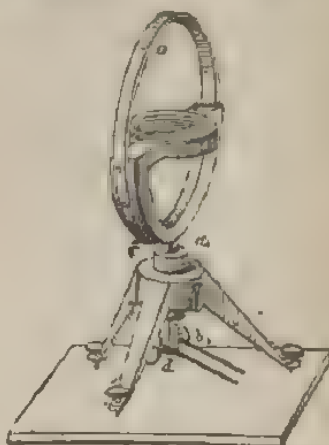


FIG. 186.—Tangent Galvanometer.

$M$ , the moment of rotation due to the action of the earth's magnetism upon the needle, the component  $M \sin \alpha$  of this moment perpendicular to the needle, holds in equilibrium the deflecting force  $kJ$ . We must then have  $kJ = M \sin \alpha$ , or  $J = K \sin \alpha$ , where  $K (= \frac{M}{k})$  is constant for any given instrument.

We see thus that the current-intensity is proportional to the sine of the angle of deflection. A galvanometer so constructed, is for this reason called a *sine-galvanometer*.

**216. Tangent Galvanometer (Pouillet, 1843).** If the conductor be given the form of a circular ring in a vertical plane (Fig. 186), and the needle, free to turn at the centre of this

ring in a horizontal plane, be made so small that the variations of its positions with respect to the current need not be considered, the deflecting force, which is perpendicular to the plane of the circle, is again dependent only upon the intensity and proportional to it. The ring may be turned about in its footing, so that its plane coincides with the position of rest of the needle (i.e. with the magnetic meridian). The instrument is kept in this position during observations, and the angle of deflection,  $\alpha$ , is read from a graduated circle about whose centre the needle swings. The deflecting force of the current,  $kJ$  (Fig. 187), is in this case perpendicular to the plane of the ring, i.e. perpendicular to the magnetic meridian. Its component perpendicular to the needle,  $kJ \cos \alpha$ , holds in equilibrium the component,  $M \sin \alpha$ , of the magnetic moment, also perpendicular to the needle. We have, therefore,  $kJ \cos \alpha = M \sin \alpha$ , or  $J = K \tan \alpha$ , i.e. the current intensity is proportional to the tangent of the angle of deflection, wherefore the instrument is called the *tangent galvanometer*. Since the circle is at considerable distance from the needle (its radius is made at least 0.2 m), this instrument can be used only with currents of considerable strength.

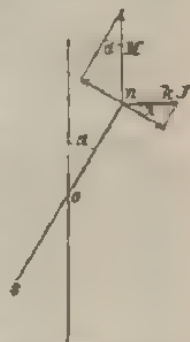


FIG. 187.—For Tangent Galvanometer.

When several sine and tangent galvanometers are placed successively in a given circuit, they usually show different deflections; since the deflection of the needle depends in each instrument upon the structure of its particular conductor, as also upon the strength of the magnetic needle, so that, with one and the same instrument, current-strengths may be accurately compared, though they cannot be measured in absolute units. A voltmeter, on the contrary, however constructed, indicates for the same current during the same time always the same quantity of gas. A tangent galvanometer may be adjusted to an absolute scale, by connecting it in a circuit simultaneously with a voltmeter. The deflection,  $\alpha$ , must then be read from the former and the intensity,  $J$ , in amperes, from the latter. From the

equation  $J = K \tan \alpha$ , in which  $J$  and  $\alpha$  are now known, the "*reduction-factor*,"  $K$ , of the tangent galvanometer is easily determined, so that henceforth by the use of the galvanometer alone the intensity of the current may be obtained directly in amperes, and more conveniently than with the voltameter.

**219. Galvanic Polarization.**—If the current of a galvanic battery be passed through dilute sulphuric acid by means of platinum electrodes (*e.g.* through a voltameter, or any other water-decomposing apparatus), so that at the negative plate hydrogen is deposited, and at the positive, oxygen, and if the current is then broken and the platinum electrodes are connected by a conducting wire, a galvanometer inserted in the circuit shows the presence of an electric current directed oppositely to the original current (Ritter, 1803).

The hydrogen covering the first pole-plate strives by virtue of its chemical attraction to unite again with the other constituent ( $\text{SO}_4$ ) of sulphuric acid. The electro-negative molecular constituent ( $\text{SO}_4$ ) moving about in the liquid, is attracted toward this plate and, combining with hydrogen, surrenders its electricity to the plate. In the same way, the oxygen, covering the other plate, attracts the electro-positive water-molecule to this plate, which, on the formation of water, becomes equally strongly charged positively. This process, however, can continue after the plates are connected, only until each plate has attained an electrical tension sufficient to hold in equilibrium the tension of the corresponding electrical constituent. The plates, therefore, attain a definite difference of potential. If now the plates be connected externally so that this difference of tension may become equalized, the gases liberated, following their natural tendency, return into their former compounds. The same sort of motion, therefore, obtains in the liquid between the poles' plates, which previously during decomposition transmitted the current of the battery (*vide* theory of electrolysis) only in the opposite direction. Or, stated differently, an electrical current, opposite to the original, arises which lasts until both gases have disappeared by returning into their original compounds.

During this process, the decomposing apparatus acts as a galvanic element, in which the platinum plates charged respectively with hydrogen and oxygen, play the parts of the positive and the negative metal. To characterize this antithesis, the plates are said to be *polarized*, and the current arising from the electromotive reaction is termed the *polarizing current*.

**220. Storage Battery — Accumulator.** — By connecting polarized plates of the same metal with one another as in the

voltaic pile, very efficient batteries, called *storage batteries* (Ritter, 1803), or *secondary batteries*, may be constructed. Their names are due to the circumstance that they must be charged by passing through them the (primary) current of an ordinary galvanic battery, or other source of galvanic electricity. The secondary element of Planté (1860) consists of two lead plates rolled one upon the other, and insulated by strips of rubber, immersed in a vessel of dilute sulphuric acid. If the plates are connected for a considerable time with the pole of a Bunsen's element, the surfaces in contact with the sulphuric acid become coated with lead sulphate, and the sulphuric acid residue deposited on the positive plate forms with the lead sulphate, in the presence of water, sulphuric acid and lead peroxide,<sup>1</sup> which adheres to the plates as a brown coating, while the hydrogen developed at the other plate transforms the lead sulphate into lead sponge, forming at the same time sulphuric acid. The plates are now "formed" or polarized, since lead acts with the same electromotive effect with respect to lead peroxide as it does with respect to copper. The charged element is now capable of furnishing a current opposite to that of the primary current, until the chemical change accompanying the discharge has caused the plates to return to their former state. Since in the secondary element the work of the current in the primary battery is, in a sense, stored up for subsequent use, the element is called an *accumulator*. If a number of secondary elements are desired, they may advantageously be charged by a battery of comparatively weak electromotive force by connecting them in *multiple*, that is, all positive plates connected with each other and so with all the negatives. Before discharging them, however, they should be connected in *series*, that is, each positive should be connected with the next following negative plate. In this way, as was the case with the voltaic pile, the electromotive force is multiplied in the ratio of the number of elements, and it is thus possible by the aid of accumulators to convert low into high electrical tension.

**221. Non-polarizing Electrodes.**—The electromotive reaction of polarization in a decomposing cell acts not only after

<sup>1</sup> According to the equation  $\text{PbSO}_4 + \text{SO}_4 + 2\text{H}_2\text{O} = 2\text{H}_2\text{SO}_4 + \text{PbO}_2$ .

the primary current has ceased, but also during its passage. Its effect is to weaken the primary current, since it diminishes the electromotive force proportionally to the reaction. The polarized cell in the circuit acts in the same way as an opposing galvanic element. The polarization in a cell may, however, be avoided, by so selecting the electrodes and the electrolyte that both remain unchanged during the passage of the current. For example, zinc plates in a concentrated solution of zinc sulphate form *non-polarizing* electrodes, because the same amount of zinc sulphate is decomposed, liberating zinc at the cathode, as is reformed during the same time, at the anode (Du Bois-Raymond, 1859).

**222. Constant Galvanic Elements.**—Since, in a closed galvanic pair, the liquid is decomposed, the reactive force of polarization must also be considered. While, for example, in the element zinc sulphuric acid platinum, the formation of zinc sulphate on the zinc plate is the source of the electromotive force, which sets in motion in the liquid an electrical current in the direction from the zinc to the platinum plate, the hydrogen liberated at the same time at the platinum terminal strives by virtue of its tendency to return into its original compound with  $\text{SO}_4$  or with oxygen, to produce an opposite polarization-current. Soon after being closed, therefore, the pair furnishes an enfeebled current, which is due to the difference of the opposite electromotive forces. No considerable current-strength is observed excepting for a short time after immersion. The atmospheric oxygen absorbed in the liquid combines with the liberated hydrogen to form water and retards chemical decomposition and, accordingly also, delays polarization. So soon as this oxygen is consumed, the current sinks to the lower intensity corresponding to the above-mentioned difference and finally ceases entirely, when the metallic zinc begins to form from the zinc sulphate upon the platinum plate. The combination, zinc-sulphuric acid-platinum, is consequently a *variable* element. Instead of maintaining its original intensity, it declines very rapidly. To prevent the diminution due to polarization, as far as possible, it is only necessary to keep a supply of oxygen around the platinum plate, which by

combining with the hydrogen, prevents deposition. This may be accomplished by placing the platinum plates in a porous earthen jar containing concentrated nitric acid. This acid, which is rich in oxygen, has the property of readily giving up a portion of its oxygen to any substances, for which it has a chemical affinity (as, for example, to hydrogen). The combination, zinc in dilute sulphuric acid, platinum in concentrated nitric acid, forms therefore a *constant element* in which the hydrogen separated by electrolytic action immediately oxidizes to water, and accordingly prevents polarization. This so-called Grove's element furnishes a steady current for a considerable length of time. Nitric acid acts similarly in the Bunsen element, which differs from the Grove's element, in that carbon takes the place of platinum. In the constant Daniell's element (zinc in dilute sulphuric acid, copper in copper sulphate-solution) polarization is avoided by the separation, due to the hydrogen, of metallic copper from the copper sulphate, with the simultaneous production of sulphuric acid. This metallic copper is then deposited upon the copper plate in the place of hydrogen. The element of *Latimer Clark* (mercury—mercury paste and zinc sulphates—pure zinc) possesses also a constant electromotive force.

**223. Resistance—Conductivity.**—When a tangent galvanometer is inserted into the closed circuit of a galvanic element, it is found that the current is weaker the longer the connecting-wire, or circuit. This enteeblement of the current is ascribed to a *resistance*, which the wire opposes to the passage of the current, analogous to the resistance opposed to water when flowing with constant current through a tube at whose ends different pressures prevail. It is assumed that with the former resistance, as with the latter, for uniform cross-section, it is proportional to the length of the conductor.

With wires of the same material, it is found that current-strength remains constant, provided lengths and cross-sections are increased in the same ratio. With constant length, therefore, the resistance in the conductor is inversely proportional to cross-section. The form of the cross-section is of no consequence. If a cylindrical wire be rolled flat, without

changing its length, the resistance to the passage of the current will not be altered.

With the same length and cross-section, different materials have different resistances. A german-silver wire, for example, may be replaced by one of copper of the same section, and 13.5 times as long, without any alteration whatever in the current-strength. The resistance of the copper wire, therefore, equals that of the german-silver, or with the same length and cross-section, the *specific resistance* of copper is only 1 : 13.5 of that of copper, or its *specific conductivity* is 13.5 times as great as that of german-silver. To recapitulate, the resistance of a linear conductor (*e.g.* of a wire) varies directly with its length and inversely with its cross-section and specific conductivity.

If the length of a wire be denoted by  $l$ , its cross-section by  $q$ , its specific conductivity by  $k$ , and its resistance by  $r$ , we shall have—

$$r = \frac{l}{kq}.$$

Since the value,  $r$ , denotes the length of a conductor of cross-section and specific conductivity equal to unity, and possessing the same resistance as that of the given conductor, it is also called the *reduced length*.

The specific resistance is the reciprocal of the specific conductivity and equals  $l : k$ .

By means of comparisons similar to the one just mentioned of german-silver and copper, though much more accurately made, by methods to be described later, the following values for the specific conductivity of some conductors of the first class, that of mercury being taken as unity, have been obtained—

Mercury ... ..	1	Brass ... ..	13
Manganite (Ni. Mn. Cu.)	2	Phosphoric bronze ...	13
German-silver ... ..	4	Aluminium ... ..	32
Lead ... ..	5	Gold ... ..	46
Iron ... ..	8	Silicium bronze ...	48
Platinum ... ..	8	Copper ... ..	55
Aluminium bronze ... ■		Silver ... ..	64

According to G. Wiedemann and Franz (1853), the electrical

conductivity of metals is proportional to their thermal conductivities (125).

The conductivity of alloys does not always lie intermediate between the conductivities of their constituent parts. The conductivity of a metal is often considerably modified by a slight admixture of another metal. Changes of the internal structure of a substance also exert an influence upon the conductivity. For example, the conducting power of steel increases with its degree of hardness.

The layer of liquid contained between the electrodes of a decomposing cell, or between the plates of a galvanic element, may also be regarded as a linear conductor, a liquid wire, as it were, whose length equals the distance between the plates measured along the path of the current, and whose cross-section equals the surface of the plates. The conductivity of liquids, or, as we say, of conductors of the second class (electrolytes) is much smaller than that of metals. For example, the conductivity of dilute sulphuric acid (30 per cent.) is only the 69 millionth part (0·0000069) of that of mercury, that of concentrated silver-nitrate is 0·000020, and that of a saturated solution of copper sulphate 0·000004.

The specific resistance of metals rises with the temperature. The increase for 1° C. or the *temperature-coefficient* is for mercury 0·000095, for the solid simple metals about 0·000366 (almost equal to the coefficient of expansion of gases), for german-silver 0·0004, and still less for niccolite and manganite. With coke and graphite, the resistance diminishes with rising temperature. Crystallized selenium conducts better when rays of light are incident upon it.

With conductors of the second class (the electrolytes), the resistance diminishes rapidly on heating; with a concentrated solution of zinc-sulphate, for example, by 0·04 for each degree Centigrade.

**224. Unit of Resistance.**—W. Siemens selected as the unit of resistance, that of a thread of mercury of 1 m. length and of 1 sq. mm. cross-section under a temperature of 0° (1 Siemens). By international agreement, however, another unit, selected from a theoretical point of view, called the ohm, was established. (The unit was named in honour of G. S. Ohm, the discoverer of the laws of the galvanic current. A (legal) ohm is the resistance of a mercurial thread 106 cm. long, and 1 sq. mm. in cross-section at 0° C. (or more exactly of 106·3 cm. in length, and since the cross-section is determined in practice by weighing, of 14·4521 g. weight).)

**225. Rheostats—Rheochord.**—In establishing the unit of resistance, mercury was selected, because it may be easily obtained at any time, and in a perfectly pure state by distillation. For executing measures, the liquid metal, however, would be very inconvenient. Wires of solid metal are for this reason prepared in such way that their resistances are equal to 1 ohm, or to some integral number of ohms. For the construction of these wires, german-silver (54 per cent. copper, 28



FIG. 188. —Wheatstone's Rheostat. Front view.

per cent. zinc, 18 per cent. nickel), or still better, manganite may be advantageously employed, because these alloys have a very high specific resistance, and are at the same time but little influenced by changes of temperature.

Apparatus used to insert resistances of a known magnitude into an electric circuit, or to cut them out at pleasure,

without interrupting the current, whether for the purpose of securing a desired current-strength, or of comparing unknown with known resistances, are called *rheostats*.

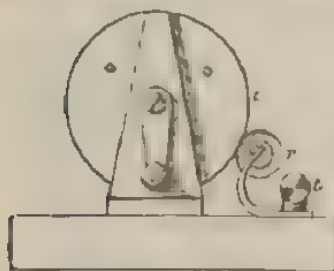


FIG. 189.—Wheatstone's Rheostat. Side view.

Wheatstone's rheostat (Figs. 188 and 189) consists of a stone cylinder (serpentine, or marble), movable about its axis, and with spiral grooves cut in its surface, in which german-silver wire is wound. One end of the wire is passed through the cylinder and connected with the metallic axis.

The axis does not extend continuously through the cylinder. Upon a horizontal metal bar, *ab*, lying parallel to the axis of the cylinder, is an adjustable metal roller, *r*, whose periphery carries a single groove, which is pressed against the spirally-wound wire by means of a spring. If the cylinder is turned by the crank, *h*, the little roller, following the coils of the wire, is slipped along the metal bar so that the current, entering the axis through the binding screw, *s*, must traverse the coils of wire to the roller. From the roller it passes through the metal bar, whose resistance, as also that of the axis, by reason of their large cross-sections, is inappreciable, and returns finally to the binding screw, *t*. The number of coils, which measures the interposed resistance, is read from a scale placed along the bar, *ab*. Subdivisions are read by means of the pointer, *i*, to the hundredth part of a circumference of the roller.

The resistance-box of Siemens' (Fig. 190) consists of a series of doubly-wound spools of wire (266) enclosed in a wooden



FIG. 190.—Siemens' Rhoostat.

box. The resistances of the spools (266) amount to 1, 1, 2, 5, 10, etc., ohms respectively, and as with the weights of a balance, any desired number of units may be compounded of them. Above each spool, on the cover of the box, is placed a thick metal plate (*a, b, c, . . .*), the first, *a*, being connected with the binding screw, *k*, and the last with the binding-screw, *k'*. One end of each spool is connected with the plate above it, and the other is soldered to the plate next following. The plates are provided on their opposite edges with semicircular notches, into which brass plugs, *s*, may be placed. If the plugs are all in circuit, the current passes from *k* to *k'* through the thick metal plates without appreciable resistance. If, however, one

or more of the plugs be withdrawn, the current will traverse the corresponding spool and suffer a corresponding resistance.

To obtain still smaller resistances Poggendorff used the so-called *rheochord*. It consists of two platinum wires (Fig. 191) stretched upon a horizontal board in such manner that the box, *k*, which is filled with mercury and penetrated by the platinum wires, effects the electrical connection between them. The current, entering at the metal plate, *c*, passes along the path, *ckd*, to the other plate, *d*, passing from one wire to the other through the mercury in *k*. The little box may be moved back and forth so that shorter, or longer, portions of the wire may be

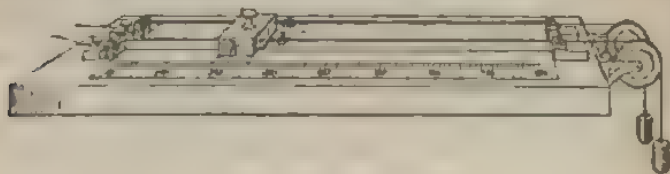


FIG. 191.—Rheochord.

traversed by the current, the lengths of these portions being read from a millimeter scale along which the box slides.

**226. Ohm's Law.**—Let a constant galvanic element be closed by means of a Wheatstone's rheostat and a tangent galvanometer, whose copper ring, together with the conducting wires, is so thick as to make its resistance inappreciable relative to that of the rest of the circuit. The magnetic needle of the galvanometer will be deflected through a definite angle. If a second element, precisely equal to the former, be now added, the elements being connected in series after the manner of a voltaic pile, the electromotive force will be doubled, but at the same time the resistance within the elements is also doubled, since now the liquid pile to be traversed has twice its former length. To make the galvanometer indicate the same deflection, or to keep the current strength the same as before, double the length of rheostat wire must be inserted, thereby doubling the resistance of the circuit. The current strength, therefore, does not change when the ratio of the electromotive force to the total resistance of the circuit remains constant. But, if for the

original element two equal elements of the same kind are substituted, the plates being of the same size and only half the former distance apart, the electro-motive force will be again doubled, while the resistance remains unaltered since the total length of the liquid to be traversed has suffered no change. The galvanometer now indicates double the current strength. The current-strength is, therefore, proportional to the electro-motive force.

G. S. Ohm (1826) found, from experiments similar to these, the important law which bears his name: *The current-strength is directly proportional to the electromotive force, and inversely proportional to the total resistance of the circuit.*

The unit of electromotive force has been conventionally decided to be that electromotive force which, in a circuit of 1 ohm resistance, produces a current of 1 ampère intensity. In honour of Volta, the unit has been called the *volt*. With this system of units, Ohm's law may be also stated thus: The current-strength ( $J$  in ampères) equals the electromotive force ( $E$  in volts) divided by the total resistance ( $R$  in ohms), or  $J = E : R$ .

Since the electromotive force corresponds to the difference of potential at the poles of the open battery-circuit, and is measured by this difference, it may be obtained by means of an electrometer, such as Thomson's quadrant electrometer, by connecting one pole with one of the quadrant pairs, and the other,  $a$ , as also the other quadrant-pair, with the earth. The angular deflection of the instrument, due to the unknown electromotive force, is compared with that of a constant normal element, whose electromotive force is known in volts. Daniell's element (1.104 volts), or the element of Latimer Clark (1.434 volts), is ordinarily used as the *normal* element. Daniell's element itself is frequently used as a *practical* unit of electro-motive force (called a *daniell*).

When a galvanic element is closed by a wire, the potential diminishes along the wire from the positive pole, because a flow of positive electricity can occur only from points of higher to those of lower potential. Electrical measurements indicate that the potential falls, by equal amounts, at points separated by

portions of the circuit within which the resistance is the same, or that the *diminution of potential is always proportional to the resistance of the part of the circuit traversed*. The quotient of the difference of potential at the ends of the conductor, divided by its resistance, is therefore constant for any given circuit, and this constant is the current-strength. We know, from former considerations, that this current-strength is the same at all places of a stationary circuit. Ohm's law, "current-strength equals difference of potential divided by resistance," holds then, not only for the entire circuit, but also for each of its parts in particular.

The difference of potential at the ends of any portion of the conductor is found, accordingly, by multiplying the current-strength into the resistance of the conductor. As a special case, the difference of potential between the terminals of the closing wire, or between the binding-screws of a closed battery, may be found by multiplying the current-strength by the resistance of the closing wire. This latter difference of potential of the closed battery is always smaller than its total electromotive force, which equals the product of the current-strength by the total resistance. The two quantities, however, approach each other more and more nearly as the resistance of the closing wire is increased, and become equal when the battery is open, i.e. when the resistance of the closing wire becomes infinite.

**227. Applications of Ohm's Law.**—In all practical applications of the galvanic current, Ohm's law is of inestimable value, since by its aid it is possible to determine how a battery must be constructed to accomplish a definite purpose. The resistance in any circuit is composed of two parts, viz. of the resistance to be overcome by the current while traversing the liquid within the element, or of the *internal* resistance, and of the *external* resistance to its passage from pole to pole through the closing wire. If a number of elements (e.g. ten) are connected in series, not only does the electromotive force become ten times as great, but the internal resistance also increases tenfold. If now the external resistance is so small as to be negligible in comparison with the internal; if, for instance, the

battery is closed by a short thick metal wire, the augmented electromotive force is neutralized by the increased resistance, and the ten-celled battery gives no stronger current than would a single one of its elements. In case the external resistance is very small, therefore (with a "short circuit"), it will be advantageous to use a single element with plates as large as possible. If, for instance, the plates of the element are made ten times as large, the electromotive force will remain unchanged, but the internal resistance will be reduced to one-tenth its former value, because the cross-section of the liquid conductor between the plates has been multiplied tenfold. With ten times as large an element, therefore, ten times as great an effect is secured. From this we have the following rule that, with small external resistances, no advantage is obtained by using several elements connected *in series*, but that the use of a single large element is of great practical advantage. It is, however, possible to prepare a single element with tenfold pole-plates from the ten individual elements. This is accomplished by merely connecting all the positive (*e.g.* zinc) plates with one another, and so also all the negative (*e.g.* copper or platinum) plates, or by connecting the ten elements, not into a pile, but into *one* element (*in parallel*). If, on the other hand, the external resistance is very great, as, for instance, in the case of a long telegraph wire, more and more current-strength will be obtained by combining, *in series*, an increasing number of elements into a single battery; because, while the electromotive force increases with the number of elements, the total resistance is not perceptibly altered. The internal resistance becomes of less and less consequence the greater the external resistance, or with large external resistance it is immaterial whether small or large plates are used. The same result may be obtained in this case with small elements as with large and more expensive ones. When a number (*e.g.* ten) of elements are at disposal, they may be compounded in various ways, *viz.* into one element with tenfold plate area, into a column of two elements with fivefold plate area, or of five elements with double the plate area, or finally into a column of ten elements of the same plate area as any element itself possesses. To the

question, which of these combinations furnishes the greatest current strength. Ohm's law furnishes an answer: *That in which the internal resistance is most nearly equal to the given external resistance.* An apparatus which makes possible rapid combinations of this sort, and permits of ready transformation from one to the other so that the most advantageous arrangement may be readily selected, is called a *pachytrope*.

Ohm's law also furnishes a rule for the most advantageous construction of galvanometers. This rule is so to select the wire that the resistance of the multiplying coils will, as nearly as possible, equal the resistance of the rest of the circuit. If the current to be measured has a high resistance outside of the galvanometer, as with liquids, the multiplier is made of the greatest possible number of coils of a very slender wire, for in this case the deflection of the needle is approximately proportional to the number of coils. But if the resistance of this portion of the circuit is small, as, for example, with the thermopile, a few windings of a thick wire are more effective, for, in this case, the current-strength is proportional to the cross-section of the multiplier, and independent of the number of coils.

**228. Constant Galvanic Elements.**—If a galvanic element is closed by means of a thick wire with a tangent galvanometer in the circuit, the current strength indicated is  $J = \frac{E}{R}$ , if  $E$  denote the electromotive force and  $R$  the resistance of the element. If now a known resistance,  $r$ , is added by means of a rheostat, the current-strength is diminished to  $J' = \frac{E}{(r + R)}$ . From these equations in which the intensities  $J$  and  $J'$ , and also the resistance,  $r$ , are known, the two unknown quantities  $E$  and  $R$ , i.e. the electromotive force and the internal resistance of the element, are readily computed. These quantities are called "constants of the galvanic element." This method, due to Ohm, is applicable only to constant elements. Other methods of determining electromotive force will be discussed later.

The electromotive forces of some galvanic elements are—

	Volts.
Daniell	1.104
Bunsen and Grove	1.9
Chromic acid element	2.0
Moodinger	1.0
Leclanché	1.3
Latimer Clark	1.434
Lead accumulator	1.9-2.0

**229. Branched Circuits—Shunts.**—Thus far we have considered only simple circuits. We shall now assume that the connecting wire of the element, or of the battery,  $E$ , divides at the point  $a$  (Fig. 192) into two branches, which again come together at the point  $b$ . Since at each branching point, to maintain dynamical equilibrium in each cross section, the same amount of electricity must flow out as flows in, the sum of the current-strength in the branches  $amb$  and  $anb$  must equal the current-strength in the unbranched portion,  $bEa$ , of the circuit. Since, furthermore, the same difference of potential, viz. that between the points  $a$  and  $b$ , set the two branched currents in motion, the loss of tension, or the *fall of potential* along  $amb$  and along  $anb$ , i.e. the products of the current-strengths into the corresponding resistances, must be equal. In other words, the current-strengths in the branches are inversely as their resistances. Suppose, for example, the resistance in the branch  $anb$ , technically called the *shunt*, is 99 times as great as in the branch  $amb$ , the current-strength in  $amb$  will then be 99 times, and the strength of the entire current 100 times as great as that in  $anb$ .

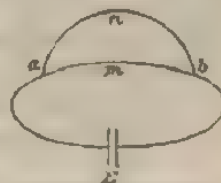


FIG. 192.—Branched Circuits (Shunts).

This principle is used to measure strong currents with galvanometers designed for use with weak currents only, the galvanometer being inserted in the shunt,  $anb$ . If the resistances of the coils of the galvanometer, and of  $amb$  are known, the current-strength in  $amb$  is as many times greater than the indication of the galvanometer, as the resistance of the galvanometer is greater than that of the branch  $amb$ . The

strength of the main current is then equal to the sum of the strengths in  $amb$  and in  $anb$ .

A galvanometer of high resistance may also be used in the shunt to determine the difference of potential between the points  $a$  and  $b$  where the shunt is connected with the main current. For this difference of potential the current-strength is multiplied by the resistance of the galvanometer wire, and is, therefore, proportional to the current-strength in the galvanometer. When the resistance of the galvanometer is known in ohms, and the current-strength in amperes, for every deflection of the needle, the difference of potential between the points,  $a$  and  $b$ , is given in volts by the product of these two magnitudes. Or, if the instrument is provided with a scale graduated in volts, the difference of potential may be read off directly. Such instruments, provided with a large number of windings of a slender wire and used in shunts, are called *volt-meters*. Similar apparatus with short thick wires, to be used in the main current, which give the current-strength in amperes, are called *ampère-meters*, or better, *ammeters*.

**280. Wheatstone's Bridge.**—If the branches  $amb$  and  $anb$  (Fig. 193) of the current are connected by a cross wire,  $mn$ ,

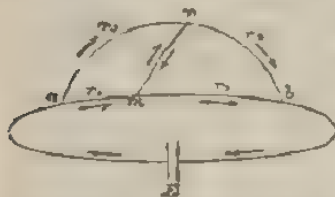


FIG. 193.—Wheatstone's Bridge

called a "bridge," two currents flow in opposite directions in the bridge. These branch off from the currents in  $am$  and in  $an$ , at the points  $m$  and  $n$ . If these currents have equal strengths they neutralize each other and no current passes through the

bridge. But when no current is flowing in the wire,  $mn$ , the potentials at its ends  $m$  and  $n$  are also equal. Since now, along  $amb$  and  $anb$  the potential sinks gradually with the resistance, from its value at  $a$  to its value at  $b$ , in exactly the same way as though the wire  $mn$  were not present, the points  $m$  and  $n$ , where equal potentials exist, must be so situated that  $r_1 : r_2 :: r_3 : r_4$ , where  $r_1$ ,  $r_2$ ,  $r_3$ , and  $r_4$ , denote the resistances in the segments,  $am$ ,  $mb$ ,  $an$ ,  $nb$ , respectively.

Wheatstone (1843) used this bridge to measure the

resistances of conductors. When the conductor whose resistance is to be determined is inserted at  $r_1$ , and a rheostat is placed at  $r_2$ , the resistance of the latter may be varied until a galvanometer placed in the bridge points to 0. The resistance sought will then be to that of the rheostat, in the ratio of the known resistances,  $r_2$  and  $r_1$ . If the latter have been made equal, the resistance sought equals that of the rheostat. The needle of the galvanometer plays the part of the pointer of a balance which, by pointing to zero, indicates the resistances in the branches  $r_1$  and  $r_2$  to be equal.

The bridge process may be modified by stretching between  $a$  and  $b$  a wire along the edge of a scale graduated to mm., and sliding upon this the end  $m$  provided with a contact-button, until the galvanometer inserted in the bridge points to zero. The desired resistance,  $r_x$ , is then to the known resistance,  $r_s$ , in the ratio of the part,  $mb$ , of the measuring wire to the distance,  $am$ .

In the *universal galvanometer* of Siemens, bridge, galvanometers, measuring-wire and a known resistance are ingeniously combined into a single apparatus.

The *Bolometer* of Langley (1881) consists of a Wheatstone's bridge into both of whose branches a number of slender wires of steel and platinum are inserted. These wires have equal resistances at the same temperatures. But, if one portion of the wires is heated, the resistance to conductivity is increased. The galvanometer placed in the bridge, which was at rest when the two sets of wires were at the same temperature, is now deflected in consequence of the difference in intensity of the currents flowing in opposite directions in the bridge. The instrument is thus capable of indicating extremely small changes of temperature.

**231. The Process of Compensation.** — If two electromotive forces act against each other in a simple circuit, a current arises which corresponds to their difference, no current at all arising when the electromotive forces are equal. It is found, for example, that a battery of seventeen to eighteen Daniell's elements must be made to act against a battery of ten Bunsen's elements to make the galvanometer-needle indicate 0. Hence it results that the electromotive force of one Bunsen's element is 1.7 to 1.8 of a Daniell's. In this way the electromotive forces of various elements may be roughly compared.

A more accurate comparison is obtained by means of the following branched

circuit (Poggendorf, 1841). At the points,  $m$  and  $n$  (Fig. 194), of the circuit, in which at  $E$  and  $\epsilon$  the elements to be compared are inserted so as to act in opposite directions, i.e. with like poles connected, a cross-wire,  $mn$ , with a rheostat is attached. In the portion of the circuit containing the element of smaller electromotive force,  $\epsilon$ , a galvanometer,  $G$ , is connected up. The resistance of

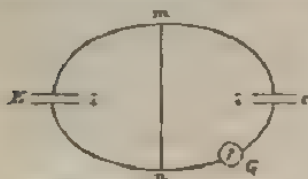


FIG. 194.—Compensation.

the rheostat is now varied until the galvanometer points to zero. In the branch,  $mn$ , there is then no current, and the current in the closed circuit,  $EmaE$ , must be the same as though the branch,  $mn$ , were not present. If  $R$  denote the resistance of the branch,  $mn$  (composed chiefly of the internal resistance of the element,  $E$ ), and  $r$ , that of the cross-wire together with the rheostat, according to Ohm's law, the current-strength in this circuit is

$J = \frac{E}{(R + r)}$ . To be able to bring the intensity of the current in the branch,  $mn$ , to zero, the difference of potential at the points,  $m$  and  $n$ , expressed by the product of the intensity,  $J$ , into the resistance,  $r$ , of the cross-wire must be equal to the opposite electromotive force,  $\epsilon$ , i.e.  $Jr = \epsilon$ . Hence it appears that the electromotive forces,  $E$  and  $\epsilon$ , are to each other as the known resistances,  $R + r$  and  $r$ . If the internal resistance,  $R$ , of the element,  $E$ , is not known, a resistance,  $R'$ , must be added in the branch,  $mn$ . The resistance of  $mn$  must then be changed to  $r'$  to make the galvanometer-needle play, and in addition to the equation  $Er = \epsilon(R + r)$  we have also  $Er' = \epsilon(R + R' + r')$ , whence we determine both the ratio of the electromotive forces and the internal resistance,  $R = \frac{R'r}{(r' - r)}$ .

Siemens' universal galvanometer is also arranged for this process of compensation to determine electromotive forces and internal resistances.

**232. Kirchhoff's Laws.**—The problem of branched circuits in general may be treated according to the same principles as are involved in these special examples.

At every branch-point and at every instant, just as much electricity flows to the point as from it. If the strength of the entering current be regarded as positive and that of the outgoing negative, we have this proposition—

1. *At every branch-point the sum of all the current-strengths equals zero.*

Every branched circuit may be decomposed into a number of simple circuits, each closed in itself. In Fig. 193, for instance, we have the following closed circuits:— $EambE$ ,  $EanbE$ ,  $anbma$ ,  $anma$ ,  $mnbm$ . In each of these circuits, the sum of all the losses in tension (potential) must equal the sum of all the electromotive forces acting in the circuit, and in

particular, this sum must equal zero if the circuit considered, as with the three last-mentioned, contains no electromotive force at all. We have, therefore, this second proposition—

2. *In every closed portion of a branched circuit, the sum of the products of intensities and corresponding resistances equals the sum of the electromotive forces acting in the respective parts.*

These two laws, formulated by Kirchhoff, which are the most general expression of Ohm's law for linear conductors, always furnish as many equations as are required to determine the current-strengths of the individual branches, when their resistances and electromotive forces are given.

233. *Flow of Currents in Conducting Bodies.*—As in the linear conductors hitherto exclusively considered, so also in conducting surfaces (e.g. in metal plates), and in conducting bodies connected in battery-circuits, a stationary condition of the current is reached, in course of time, by the passage of electricity from places of higher to places of lower potential. Between points of equal potential, on the contrary, no current can flow. If two points of a plate traversed by a current be touched with the poles of a galvanometer, a



FIGS. 195, 196.—Lines of Flow.

deflection results, whenever the points touched have different potentials. By moving one end of the galvanometer about, a series of points may be readily found, where the galvanometer-needle remains at rest. At these points the potential is the same as at the point touched by the other galvanometer pole. In their aggregate they form a line of equal tension, or of equal potential, passing through the latter point. Similarly in extended conducting bodies, surfaces of equal tension, called *equipotential surfaces*, may be found. From one equipotential surface to the next lower, the electrical flow passes in lines (called lines of flow), perpendicularly to each of the surfaces, corresponding to the lines of force in electrostatic potential, and to every current limited by such lines of flow Ohm's law applies.

For example, upon a rectangular plate of metal to whose opposite sides, *ab* and *cd* (Fig. 195), the wires of a galvanic battery are soldered, the lines of equal potential run parallel to these sides, and the lines of flow are parallel to the other pair of sides. If the poles of a battery be attached to the points *a* and *b* (Fig. 196) of the circumference of a circular disk, the equipotential lines are circles, whose centres lie harmonically to *a* and *b*, upon the connecting line *ab*, and the lines of flow perpendicular to these circles are themselves circles passing through the points *a* and *b*.

**234. Heat of the Current—Joule's Law.**—It was observed, soon after the discovery of the voltaic pile, that conductors are heated when traversed by currents, and that, with sufficient current-strength, wires may even be made to glow and melt.

By immersing a coil of wire traversed by a current in a calorimeter, which was filled with a non-conducting liquid (e.g. alcohol, benzine, oil of turpentine, etc.), of known specific heat to prevent shunting, Joule (1841) found the law which bears his name:—*The quantity of heat developed in a conductor in a unit of time is proportional to the resistance of the conductor and also to the square of the current-strength.*

Joule's law could have been found without experiment, from the following considerations, based upon the principle of the conservation of energy. While the quantity of electricity flowing through the wire in a unit of time, or, while the current-strength,  $J$ , sinks from the higher potential at one terminal to the lower at the other, it performs a quantity of work equal to the product of this electrical mass into the difference of potential at the ends of the wire. According to Ohm's law, this difference,  $E$ , equals the product of the strength,  $J$ , and the resistance,  $R$ , of the wire, or equals  $JR$ . The work performed by the current in the wire equals, then,  $JJR$  or  $J^2R$ . This work is transformed in the wire into an equivalent quantity of heat,  $W$ . If, therefore, we select as the unit of heat, the quantity of heat equivalent to the unit of work (*erg*), we have Joule's law,  $W = J^2R$ , or, since  $JR$  equals  $E$ ,  $W = JE$ . The heat developed in a segment of the conductor in one second, or the work per second corresponding to it, called the *effect* of the current, is accordingly measured by the product of the difference of potential ( $E$ ) of the segment in volts into the current-strength in amperes. The result appears, therefore, in terms of a unit called 1 *volt-ampere*, or also 1 *watt*. One watt equals ten million ergs per second, equals 0.1019 kgm. per second, equals  $\frac{1}{7.38}$  horse-power.

From Joule's law it is evident that metal wires are heated by the current to higher temperatures, when they are of small cross section and when the conducting-power of the metal is low. For illustration, when a current is passed through

a chain of equally thick wires alternately of silver and of platinum, the platinum wires are heated more strongly than are the far better conducting silver wires. The platinum wires may even be heated to incandescence, while the silver remains perfectly dark (Children, 1815).

Edison's (1879) incandescent electrical lamp depends upon the heating action of the current. A charged filament of hemp, or cotton, of high resistance (*e.g.* 140 ohms) and bent into the form of a horseshoe, is enclosed in an exhausted air-tight glass globe to protect the filament from burning, while a current of about 100 volts passing through it, heats the filament to incandescence, giving it an intensity of approximately fifteen candles. This same principle of the heating effect of the current is made use of in blasting in mines. A cartridge is prepared, in which a slender platinum wire terminates. The wire becomes heated by a current led to it through a thick insulated copper wire to a temperature high enough to explode the cartridge. In medical science glowing platinum wires, heated by electrical currents, are used to burn out ulcers about which the wire is looped (cauterization, *galvano-caustics*).

**235. Galvanic Sparks.**—Even with galvanic batteries, which furnish currents of great strength, the potential at the poles of the open circuit is so small that they must be approached to infinitesimally small distances before a spark can pass. A spark sufficiently strong to pass through the atmospheric layer between the poles can only be obtained with batteries consisting of a large number of elements. Gassiot found that the poles of a battery of 3000 elements must be brought to a distance of 0.2 mm. of each other before a spark would pass.

If, on the contrary, the poles of a galvanic battery are connected, the little particles upon the surfaces of contact form, as it were, a thin wire of very high resistance. A very considerable rise of temperature, therefore, occurs here. If the wires are separated again, at the point where the current is broken, luminous sparks, called *galvanic sparks* (*opening sparks*), are produced. The heated air between the electrodes, which is

filled with metallic vapours and particles, transmits the current for a time, until the distance, and hence the resistance, has become too great.

**236. Davy's Arc-Light.**—When Davy (1821) brought the points of two carbon bars connected with the poles of a strong battery in contact and then separated them slightly, they shone out with a blinding white light and a less luminous current of light, resembling a flame, was seen between them. When the connecting line of the poles was placed horizontally, this flame assumed the form of an arc curved upwards (Fig. 197), and it was therefore called the "voltaic arc." This arc, which formed a conducting bridge between the incandescent carbon points and acted as a movable conductor, is formed by hot air mixed with gases and vapours of combustion and glowing particles of carbon, torn loose from the carbon points. The greater



FIG. 197.—Arc-Light.

quantity of these particles proceeded from the more strongly-heated positive pole. As a result, the positive carbon became blunted and even hollowed out, while the negative remained always pointed. Since, moreover, both carbons burn away at their glowing points, they are gradually consumed, the positive carbon about twice as rapidly as the negative. The arc resists the passage of the current, and with greater strength the greater the distance of the points. In consequence of their gradual consumption, the strength of the current diminishes, until it is no longer able to form the arc. The circuit is then broken and the light extinguished. For purposes of illumination, the electric "arc light" must be so arranged that the carbon points are kept automatically at the proper distance apart. Apparatus for accomplishing this are called regulators, or *arc-lamps*. The mechanism of these regulators will be explained later.

**237. Thermo-electricity.** — In 1821 the elder Seebeck discovered that an electric current may be produced by simple

heating. When a bent copper plate, *mn* (Fig. 198), is soldered to a plate, *op*, of bismuth, and one of the soldered junctions, *o*, is heated or the other, *t*, is cooled, a needle, *a*, suspended between the plates indicates by its deflection that an electric current exists which circuits the rectangular space between the plates. At the more highly heated junction the current flows from the bismuth to the copper, and at the cooler, from the copper to the bismuth. A current produced in this way is called a *thermo-electric* current. Replacing the bismuth plate by a bar of antimony, the current at the warmer junction is found to flow from the copper to the antimony. Testing various metals thus, it is possible to arrange them in a series, called a

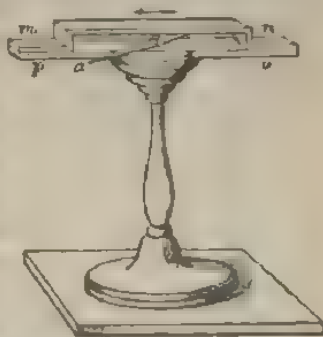


FIG. 198.—Closed Thermo-electric Element.

*thermo-electric potential series*, of such character that, at the warmer junction, the thermo-electric current flows from the metal standing higher in the series to the one below it. This series is: bismuth, nickle, mercury, platinum, lead, copper, gold, silver, zinc, iron, and antimony. A few sulphuretted and antimonuretted metals, as also a few oxides, *e.g.* copper and arsenic pyrites, blue lead, manganese, etc., stand above bismuth, while an alloy of two parts antimony and one part tin stands even below antimony. With the same differences of temperature at the junctions, the differences of potential are greater, the wider the metals are separated in the series.

For small differences of temperature the electromotive force is proportional to these differences. With larger differences, on the contrary, the electromotive force is seen to be dependent upon the absolute temperature. The metals shift their places in this series under certain circumstances and even interchange them; the electromotive force at a definite temperature (the so-called *neutral point*) becoming zero, and beyond this point becoming reversed.

A circuit composed of two metals soldered together, as in Fig. 198, is called a *closed* thermo-electric element, or a *thermo element*. Two bars of different metals, *e.g.* bismuth and

antimony, soldered together at but one end and left free at the other, the free ends carrying the conducting wire, form a so-called *open element* (Fig. 199) which becomes a closed element on bringing the ends of the wires in contact. The tension is heightened ( $n$ -fold) by connecting several ( $n$ ) elements after the manner of a voltaic pile, into a *thermo-electric pile*, or *thermopile* (Nobili, 1831), as is shown in Fig. 200. In this apparatus several small bars, or plates, whose enclosures are filled with a non-conducting material, are united into a single bundle and enclosed within a case, P (Fig. 201), in such way

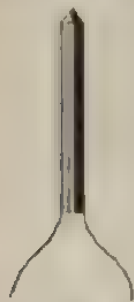


FIG. 199.—Open Thermo element.



FIG. 200.—For the Thermopile.

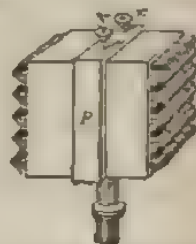


FIG. 201.—Thermopile

that the terminal plates are connected with the binding-screws  $x$  and  $y$ . Such a thermopile, connected with the multiplying coil of a galvanometer, becomes a *thermo-multiplier* (Melloni, 1841), which furnishes a delicate means of demonstrating and measuring feeble thermal effects, especially such as are due to radiant heat. With larger thermopiles, designed to replace galvanic batteries, one series of junctions is heated by a gas flame, while the other is cooled either by simple radiation or by immersion in water, or ice (Marcus, 1864; Noé, 1870; Clamond, Guelcher).

**239. Peltier's Effect.**—Peltier found, in 1834, that a galvanic current, transmitted through a thermo element, produces at the junction a change of temperature opposite to that required to develop a thermal current in the same direction. If, for instance, the current passes from bismuth to antimony, the junction cools. It rises in temperature also when the current

passes from antimony to bismuth. This heat, developed at the junction in the former case, and consumed in the latter, is called *Peltier's heat*, to distinguish it from the heat due to ordinary current flow. The latter, since it follows Joule's law, is called *Joule's heat*. While Joule's heat is proportional to the square of the current-strength, Peltier's heat is proportional to the first power of the current-strength. The former does not depend upon the direction of the current, but the latter is positive (heating) or negative (cooling) according as the current flows in the one direction or in the other.

Peltier's effect may also be proved directly by the following experiment. Through the glass bulb of a differential air-thermometer passes a bar of antimony to both sides of which a bismuth bar is soldered, so that the junctions are at the centre of the bulb. When a current is sent through the composite bar the Joule's heat developed in both bulbs in the same quantity is augmented by Peltier's heat in the one bulb and diminished by an equal amount in the other. The liquid in the U-shaped glass tube connecting the bulbs must therefore sink in the former bulb and rise equally in the latter. Lenz filled a small hole made in the bar of bismuth and antimony near the junction with water and cooled the bar with ice to  $0^{\circ}$ . When a current was then passed through the junction from the bismuth to the antimony, the water froze in a few moments. Peltier's cross furnishes an indirect demonstration of this same effect. This cross consists of a bar of bismuth soldered to another of antimony, as shown in Fig. 202. If the current of a galvanic element passes through the one pair of cross-arms from bismuth to antimony, after removing the element, a galvanometer inserted in circuit with the other arm, indicates a thermal current from antimony to bismuth, and consequently also a cooling at the junction corresponding to this current.



FIG. 202.—Peltier's Cross.

Any current flowing through a thermo element as also the thermal current itself, by cooling the warmer junction and

heating the cooler, produces a thermal current of opposite direction, which weakens the original current, or Peltier's effect causes an electromotive reaction similar to that produced by the chemical effect in a galvanic element. This has already been referred to as the reaction due to polarization. During this process heat is continually transferred from the warmer to the cooler junction, and in order that a thermal current of uniform intensity may exist, the one junction must be kept constantly at a higher temperature by the continual addition of heat, and the other at a lower temperature by a continual extraction of heat. The difference of these quantities is transformed into work, and appears throughout the entire circuit as Joule's heat. The proposition that when heat is transformed into work, a corresponding quantity of heat passes from a warmer to a cooler body is again confirmed by this phenomenon.

The higher heating of the positive carbon of the voltaic arc-light is explained by Peltier's effect (Wild). From this carbon an electromotive reaction (Edlund, 1868) of 35 to 40 volts arises, which works against the original current. In order that the arc maintain itself continuously, an electrical source must be applied which will maintain a potential difference at the carbon points of at least 40 volts.

**239. Electro-magnets**—Soon after Oersted's discovery of the deflection of the magnetic needle, Arago (1820) observed that the copper closing wire of a galvanic battery becomes covered with iron-filings when dipped into them, but that the filings fall off immediately when the current is broken. Iron bars become magnetic in the neighbourhood of a current, so long as the current continues to flow across them, and steel needles become permanently magnetic. A far more powerful effect is obtained when a copper wire, covered with silk, wool, or other insulating substance, is wound about a bar of soft iron several times (Sturgeon, 1835), and a current is passed through the coils. The iron bar is instantly converted into a strong magnet capable of attracting and holding other pieces of iron. It loses its magnetism, however, almost completely when the current is broken. Such an iron core surrounded by coils of wire, which may be magnetized and demagnetized at will by closing and opening the current, is called an *electro-magnet*.

Instead of winding the wire immediately upon the iron core, it is more convenient to wind it about a spool, forming what is called a *magnetizing, or induction, coil* (Fig. 203), into the hollow of which the iron core may be inserted. To make both poles lie beside and mutually strengthen each other, the electro-magnet is sometimes given the form of a horse-shoe (*abc*, Fig. 204), about whose branches the coils *a* and *c* are wound. For the purpose of testing the sup-



FIG. 203.—Magnetizing Coil.

porting-power of the magnet, an *armature, de*, connecting the poles and "closing" the magnet, is made to support a scale-pan for the reception of weights. Much more powerful forces may be obtained by means of electro-magnets than are possible with steel magnets. Steel bars inserted into the coil become also strongly magnetic, but maintain a considerable portion of their magnetism after the current is broken. Permanent magnets may also be obtained in this way. The best method of preparing strong steel magnets is by stroking one-half of the steel bar from the middle over the north pole ten or twenty times, and the other half the same number of times over the south pole of a powerful electro-magnet. The former end thus becomes a south pole, and the latter a north pole.

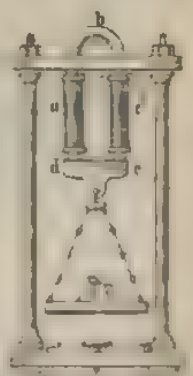


FIG. 204 — Horseshoe Electro-magnet.

The magnetic effect of the current is explained upon the idea (207) already presented, that the molecules of the unmagnetized iron and steel are little magnets movable about their centres of gravity, which, however, being irregularly disposed, exert no external effect. By means of the current in the coil, the little magnets are turned, according to Ampère's rule, so that their axes lie perpendicularly to the plane of the coils, or parallel to the axis of the iron bar, in such manner that their north poles lie toward the left of the figure swimming in the current, and facing the iron core. A portion or all of

these molecular magnets having become thus arranged, the bar becomes a magnet with its north pole toward the left of the Amperian swimmer, or, what is the same thing, with its south pole toward that end from which the current appears to flow in

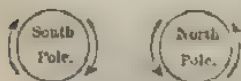


FIG. 205.—Direction of Current at the Poles

the opposite direction to the hands of the watch (Fig. 205). This process may be rendered clearer by means of a series of small magnets turning about their centres, which carry paper disks with the halves representing the north poles coloured black, and which are visible through the coils of an enveloping spiral (Fig. 206). Irregularly disposed at the beginning, they arrange themselves upon the passage of a current through the spiral, with the blackened semi-circles toward one side,



FIG. 206.—Model of Molecular Magnet.

and are reversed when the current is sent in the opposite direction.

The highest degree of magnetization, or "saturation," is reached, when all the molecular magnets have the same direction. On gradually increasing the current strength, magnetization increases at first almost proportionally to the strength, but on approaching saturation, the increase becomes slower and slower. When the latter condition is reached, no current-strength can carry the magnetization further.

At the moment of magnetization, the bar of iron suffers a slight elongation, and gives out a perceptible sound. Its volume, however, remains unchanged.

When the processes of magnetization and demagnetization are repeated in rapid succession, the bar becomes heated, because the *coercive force* (134) acts as internal friction.

**240. Solenoids.**—Under the action of a circular current, a magnetic needle, even when its point of rotation does not lie

at the centre of the circle (as with the tangent galvanometer), but anywhere upon a straight line perpendicular to the plane of the circle at its middle point (axis of the circular current), is always deflected according to Ampère's rule, so that its south pole turns toward the end from which the current appears to circulate in the direction of the hands of a watch. The needle assumes a position parallel to this line, and consequently perpendicular to the current, in case the influence of the earth's magnetism has been eliminated (Fig. 207). The circular current acts, therefore, upon the needle like a short bar magnet, lying at the centre of the needle and perpendicular to the plane of the circle, the bar having its south pole toward the side whence the current appears to flow clockwise. In other words, the circular current behaves with respect to a magnet, as though its plane were south-magnetic upon the clockwise side and north-magnetic upon the counter-clockwise side of a doubly magnetic surface.

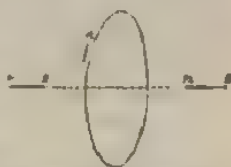


FIG. 207.—Circular Current.

The effect of a magnetic needle is intensified by placing a number of circular currents, flowing in the same sense, with their centres upon a common axis. Such an arrangement, called by Ampère a "solenoid" (from *σωλήν*, tube), has the same effect as a series of small magnets, with like poles all turned in the same direction. It comports itself, therefore, as a magnet with its south pole at the clockwise end.

A solenoid is very approximately realized in a single spirally-coiled wire (Figs. 203 and 208). A movable magnetic bar is repelled, or attracted, by a pole of the solenoid, according as the approached poles are like, or unlike. In the latter case the magnet is drawn into the coil until its centre coincides with that of the solenoid, and with a force which is proportional both to the intensity of the magnetism and to the current-strength. A bar of soft iron under the influence of the solenoid becomes an electro-magnet,



FIG. 208.—Solenoid

whose nearer pole is unlike that of the solenoid, and is therefore always attracted into it. Since in an electro-magnet, the intensity of magnetization for weak currents is proportional to the current-strength, the force with which the iron is drawn into the coil is proportional to the square of the current-strength.

With a fixed magnet and a movable conductor, the latter would be set in motion by reason of the equality of action and of reaction. Under the influence of the earth, a circular current, or a coil of wire, with movable suspension, assumes a position such that the axis lies in the magnetic meridian, the clockwise side turning so as to face the south. Such currents comport themselves when under magnetic influence precisely as though they were magnets.

Ampère's stand (Fig. 209) offers a ready means of obtaining a movable suspension for conductors. The two brass columns, *v* and *t*, standing upon the baseboard, *A*, are bent near their upper ends at right angles, and they carry at their ends small steel cups, *y* and *y'*, filled with mercury, the former lying

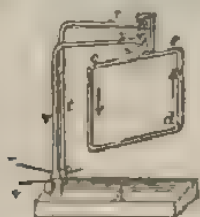


FIG. 209.—Ampère's Stand.



FIG. 210.—Circular Conductor

just above the latter. The conductor, which is a wire, *etc.*, curved to the form of a rectangle (or the circular current,



FIG. 211.—Magnetic Field about a Current.

Fig. 210, or the solenoid, Fig. 208) of copper, or, still better, of aluminium, is suspended by means of steel points soldered to its ends, in the mercurial cups, and is thus easily movable about an axis through the points.

#### 241. Magnetic Field about a Current.—

Every current produces a magnetic field about its path. A long, rectilinear current rising perpendicularly to the plane of the drawing (Fig. 211) from

its middle point, according to Ampère's rule would place a small movable magnetic needle, freed from the earth's influence, everywhere perpendicularly to the plane determined by the conductor and the centre of the needle. The position of the *astatic* needle would always be such that its north pole would point toward the left of the observer swimming in the current. A large number of small magnetic needles, situated at the same distance from the conductor, would assume positions along the circumference of a circle about the conductor. This circumference will then represent a line of force of the field due to the current. Such lines of force surrounding the conductor in concentric rings become apparent on passing the wire perpendicularly through a card upon whose surface iron-filings have been sprinkled. Under the influence of the current the filings are converted into little electro-magnets, which arrange themselves in continuous rings about the wire (Fig. 212). The above-mentioned (239) experiment of Arago depends upon the fact that the magnetized iron-filings about the immersed terminal arrange themselves thus in closed rings. The equipotential surfaces corresponding to these circular lines of force are planes passing through the rectilinear conductor.



FIG. 212.—Magnetic Lines of Force about a Rectilinear Current.

Figs. 213 and 214 show in the same way, by means of iron-filings, the lines of force of a circular current and of a solenoid.



FIG. 213.—Lines of Force about a Circular Current.



FIG. 214.—Lines of Force of a Solenoid.

The similarity of the magnetic field of a solenoid to that of a magnetic bar is worthy of note.

The equipotential lines (233) of a plate traversed by a current are at the same time the magnetic lines of force due to current-flow. These equipotential lines may therefore be made visible by sifting iron-filings upon the plate (Lommel, 1892).

**242. Electro-Magnetic Rotation.**—When a linear current acts upon a single magnetic pole, the latter must revolve about the conductor along a line of force, in a *dextrorsum*, or a *sinistrorsum* direction (to an observer swimming with the current and facing the pole), according as this pole is north, or south. This may be shown by the apparatus of Fig. 215. Two parallel, vertical magnets *ns* and *n's'*, with like poles directed upward, are held by a cross-bar to a piece of brass, *d*, which is suspended by a delicate fibre attached to its upper end, while its lower

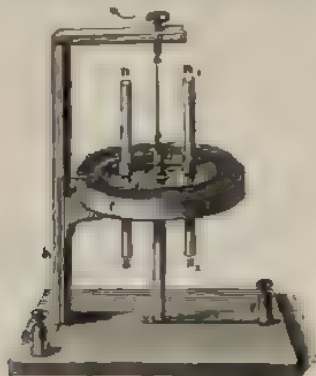


FIG. 215. — Rotation of a Magnet about a Current.



FIG. 216. — Rotation of Current about a Magnet.

end dips, by means of a platinum point, into the mercury-cup, *b*. The cup is borne on the top of a metallic bar, *ab*, which conducts the current from *e*. A horizontal wire, *c*, with a platinum point bent downward, conducts the current into the circular basin of mercury, whence it returns, through a wire, *h*, by way of the other pole, *g*, of the current-source. The current flowing in the little metallic column, which exerts almost its entire effect upon the lower and nearer magnetic pole, puts the magnet in continuous rotation in a direction opposite to that of the current.

In consequence of the effect of reaction a movable

conductor, such as the bent metallic rod (Fig. 216), free to turn about its point of support which dips into a steel mercurial cup while its ends project downward into a circular trough of mercury, must also rotate under the influence of a fixed magnet, while the cup is connected with one pole and the trough with the other pole of a source of current.

A flexible metallic cable, or ribbon which hangs limp beside a vertical bar-magnet, when traversed by a current begins to rotate and finally winds itself in a spiral about the magnet. When the current is reversed, it unwinds and rewinds in the opposite direction.

*Barlow's wheel* (Fig. 217), a copper disk movable about a horizontal axis, and provided with slender teeth dipping into a trough of mercury, rotates in one direction or the other under the influence of a magnetic pole brought near the trough, according as the current flows from the axis toward the trough, or in the reverse direction.

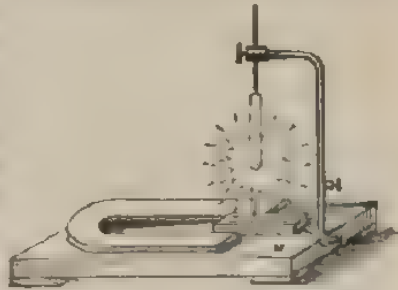


FIG. 217.—Barlow's Wheel

In these experiments, the energy of motion is attained at the expense of the energy of current-flow. When the movable part of the apparatus is fixed, the entire energy of flow is transformed into Joule's heat. When, however, it is allowed to move, the heat developed in a conductor is diminished by an amount equal to the energy produced.

**243. Law of Biot-Savart.**—The force exerted by a linear conductor upon a magnetic pole perpendicularly to the plane passing through the current and the pole, must obviously diminish with the distance of the pole from the conductor. Let us consider now a very long rectilinear current flowing vertically upwards, piercing the horizontal plane of the drawing (Fig. 218) at the point A, and a magnetic needle, *nm*, lying in this plane and directed toward A, the needle being attached to a lever-arm. *Am*, movable about A.



FIG. 218.—BIOT-SAVART'S LAW

The current acts upon the poles,  $n$  and  $s$ , with the forces,  $f$  and  $f'$ , respectively, tending to rotate the lever in opposite directions. Since no rotation occurs, however sensitive the lever and however great the current-strength be made, the moments of rotation must be oppositely equal. If, then, the distances of the poles from the conductor, i.e. the lever-arms,  $An$  and  $As$ , are denoted by  $r$  and  $r'$  respectively, we must have  $fr = f'r'$ , or what is the same thing,  $f : f' :: r' : r$ , i.e. the force exerted by a current upon the pole of a magnet is inversely proportional to the distance of the latter from the conductor.

This law was discovered by Biot and Savart (1820), who confirmed it experimentally by means of the pendulum-like vibrations of short horizontal magnetic needles freed from the earth's magnetizing effect, under the influence of long vertical currents. From the experiments it was found that the square of the number of vibrations, and consequently, also, the accelerating forces, are inversely proportional to the distances from the current. It was furthermore found that the force is proportional directly to the current-strength, and to the strength of the magnetic pole.

**244. Current-Elements.**—A linear conductor may be conceived of as subdivided into an indefinitely large number of small parts, or "current-elements," and it may be furthermore imagined that the effect of a current upon a magnetic pole is compounded of the effects of all these current-elements. In order that Biot-Savart's law may apply to a long rectilinear conductor, it must be assumed that the force exerted by a current element of length,  $\sigma$ , having a current strength,  $i$ , upon a magnetic pole of strength,  $m$ , is proportional to the expression (Laplace),  $\frac{m\sigma \sin \alpha}{r^2}$ , or with a proper choice of the unit of current-strength, that the force equals this expression, where  $r$

denotes the length of the line connecting the current-element with the pole and  $\alpha$ , the angle of the current-element with this line.

**245. Computation of the Effect of a Circular Current upon a Magnetic Pole.**—

Let Fig. 219 represent in perspective a circular current of radius,  $R$ , and intensity,  $i$ , acting upon a magnetic pole of strength,  $m$ , lying upon the perpendicular erected to the plane of the circle at its middle point. Let  $p$  denote the distance of  $m$  from the middle point, and  $r$  its distance from the element  $\sigma$ , of the current. The force  $f$  exerted by the current-element  $\sigma$ , upon  $m$ , is

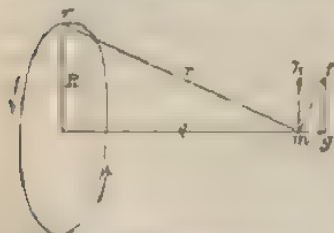


FIG. 219.—Effect of a Circular Current on a Magnetic Pole.

directed perpendicularly to the plane through  $\sigma$  and  $m$ , and therefore, also perpendicularly to  $r$ . Since  $\sigma$  is perpendicular to  $r$ , and accordingly  $\alpha = 90^\circ$  ( $\sin \alpha = 1$ ), we have—

$$f = \frac{m\sigma i}{r^2}.$$

The force  $f$  may be resolved into the components  $g$  and  $h$ , the former along the line  $r$ , and the latter perpendicular to it. The latter is destroyed by an equal and opposite component, arising from the diametrically opposite current-element, leaving the component  $g$  alone effective. Since  $g : f = R : r$ , there results—

$$g = f \cdot \frac{R}{r} = \frac{m\sigma i R}{r^2}.$$

The total force,  $K$ , exerted by the entire current upon  $m$ , in the direction of  $g$ , is the sum of all the components,  $g$ , arising from all the elements of the circumference, and replacing  $\sigma$  by the entire circumference  $2\pi R$  in the expression for  $g$ , we obtain—

$$K = \frac{2m\sigma i R^2}{r^2},$$

or, since  $\pi R^2 = F$ , the area of the circle, this becomes—

$$K = \frac{2miF}{r^2}.$$

It has already (147) been found that the force,  $K$ , exerted by a short bar magnet of moment,  $M$ , upon a magnetic pole,  $m$ , lying in its prolongation at the relatively great distance  $r$ , is given by  $K = \frac{2mM}{r^2}$ . By comparing the latter two expressions, it is seen that the effect of a circular current upon a magnetic pole may be replaced by the effect of a short magnetic bar placed perpendicularly through the plane of the circle, and that its magnetic moment equals the product of the intensity by the area of the surface circuiated by the current.

#### 246. Absolute Electro-Magnetic Unit of Current-Strength.—

In reference to the laws of the magnetic effect of currents, W. Weber (1842) selected, as the absolute unit of current-strength, that current which, flowing around the unit of surface (1 cm.<sup>2</sup>), produces the unit of magnetic moment, or, what amounts to the same thing, that current which, flowing in a circle of radius 1 (1 cm.) through the arc of length 1 (1 cm.), exerts upon the unit of magnetism (or of pole-strength) at the centre of the circle the force 1 (1 dyne), or produces at this centre the field-intensity 1.

For measurements according to this absolute standard, the tangent galvanometer (218) may be used. When, as is the case with this instrument, the magnetic needle is very small in comparison with the radius of the circular circuit, the poles remain during deflection almost at the centre of the circle, and the force exerted by the current upon a pole is found from

the foregoing equation, by putting  $r = R$ , to be  $K = \frac{2\pi ni}{R}$ . If, furthermore,  $H$  designates the horizontal intensity of terrestrial magnetism, the force acting at the magnetic pole parallel to the magnetic meridian is  $Hm$ . If now the needle is in equilibrium at the deflection  $\alpha$ ,  $\frac{2\pi ni}{R} \cos \alpha = Hm \sin \alpha$ , or,  $i = \frac{HR}{2\pi} \tan \alpha$ . The reduction factor of the tangent galvanometer for absolute measures is accordingly  $\frac{HR}{2\pi}$ , and to obtain it, the horizontal intensity of terrestrial magnetism (in absolute units) at the place of observation, and the radius of the circle (in cm.) must be measured.

For most practical applications, however, the absolute unit of current is too great. The Electrical Congress convened at Paris in 1881, therefore, selected the tenth part of this unit as a practical standard for current measurements, and called it the "Ampère," although historical justice would have required the new unit to be named for Weber, the originator of the methods of absolute measurement of electrical quantities.

**247. Electromagnetic Telegraphy**—The magnetic effects of a galvanic current, as also the deflection of the magnetic needle and the magnetization of soft iron, find an important application in the rapid transmission of signals and characters through long distances (telegraphy). *Needle-telegraphs* depend upon the deflection of the magnetic needle. By sending to a remote station a current, which there circuits a movable magnetic needle through several coils of wire, with a suitable device for reversing the direction of the current, the needle may be deflected at will toward the right, or left. From these two designations, "right" and "left," a set of conventional signals representing all the letters of the alphabet, or other symbols, is formed. The first electromagnetic telegraph of this kind was constructed by Gauss and Weber (1838) between the Observatory and the Physical Institute at Göttingen. With subterranean telegraphic cables, also, the transmission of signals is made by means of the deflection of the magnet of a very sensitive mirror galvanometer, the magnet being suspended within a coil of wire.

Other electromagnetic telegraphs are based upon the application of electromagnets. As an example of this form of

instrument, the *recorder* of Morse (1837), still in use upon many telegraphic lines upon the European continent, may be cited. In Fig. 220, AA are the two spools of an electro-magnet above whose poles lies the iron armature, *a*, borne by the brass lever, *cc*. The other end of the lever carries a steel stylus, *d*, which, when the armature is attracted by the magnet, presses against the paper strip, *bb*, as it slowly unwraps from the roller, *B*. The strip of paper is drawn along with uniform velocity between two small rollers by means

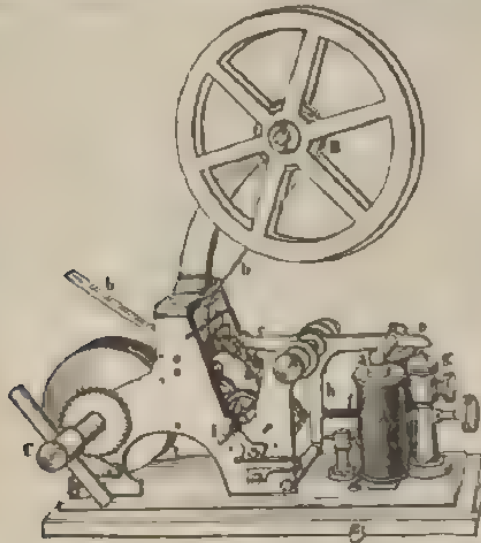


FIG. 220 —Morse's Recorder

of clock-work. That the paper may receive impressions from the point, the upper roller, *e*, is provided with a shallow groove. When the lever descends, its right end strikes against the screw, *g*, to prevent the armature from coming in contact with the pole of the magnet and adhering to it. When the magnetism disappears on breaking the circuit, the spring, *f*, acting on the arm, *h*, of the lever, *cc*, draws the point, *d*, again downward. The arm, *C*, is used to wind the clock, and the crank, *i*, to release and stop it. To close and open the circuit, a *signal key* (Fig. 221), is used. It consists of a brass lever, *ED*,

movable about the brass supports,  $BC'$ , screwed to the wooden base,  $AA$ . This support is connected with the telegraph wire leading to the next station, while the metal pillar,  $a$ , is connected with one pole of the battery. While at rest, the point,



FIG. 221.—Key of Morse's Telegraph.

$b'$ , is pressed by the spring,  $cc'$ , against the metal cone,  $b$ , and between  $a$  and  $a'$  contact is then broken. If, however, the pillars,  $a$  and  $a'$ , are brought in contact by pressing upon the button,  $F$ , the current

flows along the path,  $aa'BC$ , through the conducting-wire around the electromagnet at the other station, and presses the point against the paper strip drawn along by the clock producing a depressed point, or line, according as the key was pressed down but for an instant, or for a considerable time. From these *dots* and *dashes* the entire alphabet is constructed. The Morse signal code introduced and now in use in Germany is the following :—

a	..	k	.	tl	....	7	....
ä	...	l	...	v	...	8	....
b	....	m	...	w	...	9	...
c	....	n	..	x	....	0	....
ch	...	o	..	y	..	.	....
d	...	ö	...	z	..	.	....
e	..	p	...	1	....	.	....
f	....	q	...	2	....	.	....
g	...	r	...	3	...	?	....
h	....	s	...	4	....	!	...
i	..	t	...	5	....		
j	...	u	...	6	....		

In the earliest forms of telegraphic instruments, to operate the signal-senders two wires were necessary, through one of which the current was sent and through the other returned. But, in 1838, Steinheil discovered that the *return* wire could be dispensed with by soldering copper plates to the terminals and burying them in the earth (*earth circuit*). When a galvanic battery is connected in this circuit, the opposing electricities flow from its poles through the copper plates into the earth as into a great reservoir, and the current then flows through

the wire just as though the conductor were closed by means of the earth. To make communication in both directions possible, each of them must be equipped with both a *receiver* (e.g. a *recorder*) and a *sender* (*key*). The course of the current between two Morse stations is indicated in Fig. 222, when the key, *c*, of the sending station is pressed, the current passes from the battery, *b*, by way of *a*, into the conductor, through the closed key, *c'*, of the receiving station, around the electromagnet, *a'*, of the recording apparatus, then into the earth toward the plate, *d'*, and from the plate, *d*, about the electromagnet, *a*, at the first station, and finally

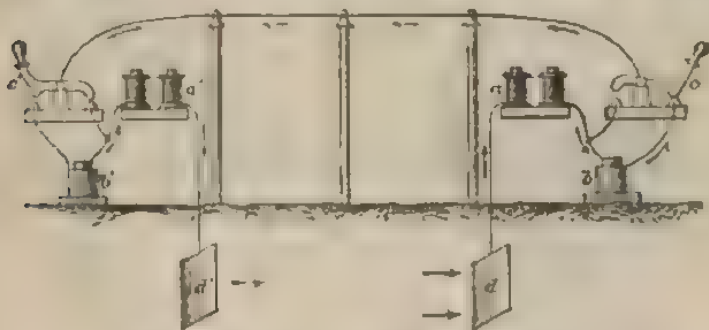


FIG. 222 — Course of Current between Stations.

to the other pole of the battery. The Morse apparatus needs no special calling device. The rattling of the armature suffices both to call the operator at the receiving station, and to release the clock-work. In reality, the electromagnets of Fig. 222 must be thought of as constituting parts of the so-called "relay," and not of the writing apparatus. By reason of the great resistance of long conductors, the "line-current," coming from the sending station, is too much enfeebled to operate the recording lever with sufficient force. This current is, therefore, only used to excite an electromagnet, *C* (Fig. 223), with a very sensitive armature, *a'*. When the attracted armature presses the lever-arm, *bc*, against the screw, *d*, it closes the local battery, *A*, at the receiving station, which sends its more powerful current through the

coils of the electromagnet, B, shown in the drawing. This apparatus, devised by Wheatstone (1830), is called a transmitter, or a *relay*.

**248. The Magnetic Hammer** (Wagner, 1839) is an apparatus for automatically breaking and making the circuit (Fig. 224). The current passes from the battery to the binding-screw, *a*, through a metal strip to the metal post, *b*, through a platinum point upon a small platinum strip soldered to the brass spring, *p*, thence into the brass column, *d*, then from *d* to *e* by means of a closing wire, then through the coils of the electromagnet, M, leaving by way of *f* for the negative pole of the battery.

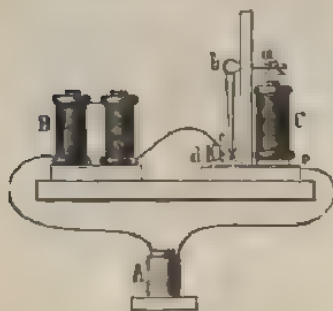


FIG. 223.—Relay

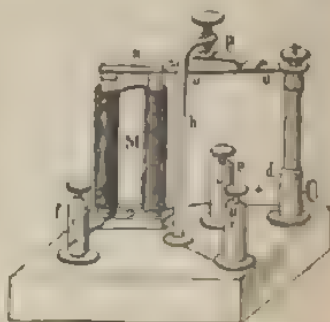


FIG. 224.—Magnetic Hammer

So soon as the current passes through the coils of the magnet, the latter is energized, attracts the iron armature, *a*, attached to the brass spring, *oo*, and, bending this spring downward, breaks the circuit at the platinum point, *p*. In consequence of this, the magnetism of the iron core is quenched, the spring flies back, restores the connection at *p*, whereupon the same process is repeated in rapid succession.

**249. The Electric Bell** consists of a magnetic hammer, whose spring terminates at its free end in a knob which may be made to strike rapidly against a bell, as soon and as long as a current is sent through the conductor by pressing the key. The arrangement of the conductor for an electric bell is shown in Fig. 225. From the pole, C, of the battery, AC, the conductor passes to the bell, B, and continues beyond the bell

to whatever stations the keys may be desired. A wire from the other pole, A, runs parallel beside the former, and is insulated from it. From each of these wires, branches extend to keys, so that the current may be closed and the bells set in action from any one of them.

**250. Electric Clocks** are devices which, by the aid of an electric current, may be moved in exact agreement with a precise clock (*master clock*). The mechanism consists of a wheel with sixty teeth, into which a steel lug fastened to the armature of an electromagnet engages in such manner as to allow the wheel to advance by the distance of one tooth, whenever the armature of the electromagnet is attracted. This electromagnet

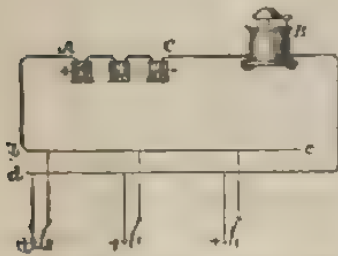


FIG. 225. — Electric Bell.

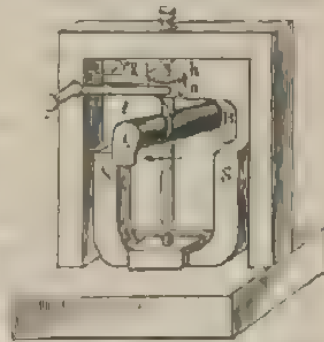


FIG. 226. — Ritchie's Electromagnetic Machine (Motor).

is connected in a circuit, which is closed every minute by a device operated by the master clock. The minute hand attached to the axis of this wheel at the end of each minute jumps forward by one sixtieth of the circumference of the dial-plate (*minute-jumper*). Such electrical clocks may be inserted into the same circuit in any number and at any desired distances, so that a single master clock may operate them all, keeping them in complete agreement with itself and with each other. Such clocks are known as *secondary clocks*, *minute-jumpers*, etc.

**251. Electromagnetic Motors.**—The powerful effect of electromagnets soon suggested the application of electromagnetism as a moving force to drive machinery. Fig. 226 shows a small

electromagnetic motor devised by Ritchie (1820). A horse-shoe magnet of steel is fixed rigidly to a baseboard with the poles N and S directed upward. Midway between the branches of the magnet is a vertical axis carrying a horizontal electromagnet, AB, whose terminal surfaces during rotation move just over the poles of the steel magnet. When the current passes through the coils of the electromagnet, so that end A becomes a south pole and B a north pole, A will be attracted by N, and B by S, and a rotation will consequently ensue in the direction of the arrow. This rotation, however, would cease as soon as A arrived above N, and B above S, were it not that at this instant the direction of the current in the coils is automatically reversed, so that A is converted into a north pole and B into a south pole. Since A is then repelled by N, and B by S, the rotation continues in the former direction. The reversion of the current at the proper instant is effected by means of a so-called *commutator*, *hi*. It consists of a metal ring fixed to the axis of rotation and insulated from it, which is divided at two opposite points by insulating spaces into two separate halves, the one of which is connected with the end, *o*, and the other, *i*, with the other end of the coil. Upon the circumference of the ring, two brass springs, *g* and *s*, slide with gentle friction, the outer ends of the springs carrying binding-screws (+ and -) to receive the terminals of the battery wires. In the position represented in the figure, the current passes through the spring, *g*, to the portion, *h*, of the ring and through the terminal, *o*, into the coil, passing over upon this to the portion, *i*, of the ring, and returning finally through the spring, *f*, to the negative pole of the battery. At the instant when A passes over N and B over S, the insulating spaces, *h* and *i*, pass under the springs, the positive spring, *g*, coming upon *i*, and the negative, *f*, upon *h*. The current now flows through the coils in the opposite direction, and the poles of the electromagnet are reversed. The steel magnet, NS, may be replaced by a fixed electromagnet, whose coils are traversed by the same current as is used with the movable coil. Other electromagnetic motors, known as *electromotors*, cannot be discussed until later.

**252. Electrical Arc Lamps.**—To make the electrical arc light (236) subserve the purposes of illumination, the carbon bars, or points, must be pushed together automatically, as rapidly as they are consumed, so that the arc will preserve a constant length, and that the same resistance may prevail. This is accomplished by means of carbon light-regulators, or electrical arc lamps. The current producing the arc is passed around an electromagnet, which, so long as the points are at the proper distance and the current is of the proper strength, attract an armature, and hold a toothed wheel mechanism, which, if not arrested, strives to crowd the carbon points close against each other. When, however, from the consumption of the carbons, the length of the arc increases and the current-strength diminishes, the armature of the enfeebled electromagnet is released, the mechanism liberated, and the points approach each other until the current becomes strong enough, whereupon the electromagnet, which is also intensified, again arrests the mechanism.

The earlier lamps (of Foucault-Duboscq, Serrin, Hefner-Alteneck) constructed on these principles, required each its own current. If several of them were put in the same circuit, they did not work, because the operation of each depended not merely upon the resistance of its own arc, but upon the sum of the resistances of all the arcs. The "distribution of the electric light," i.e. the operating of a number of lights in the same circuit, as is required in practice, was not possible with these primitive regulators. They were then, and are still, used only for purposes of demonstration, for projecting images upon screens, etc. The latter problem was first solved by Jablochhoff (1876), by the invention of the electric candle, with which, in the simplest way and without any mechanism whatever, a constant distance is kept between the carbon points. The bars of carbon (Fig. 227) are placed in parallel positions, and are separated by a non-conducting layer of gypsum, or of kaolin. Between the upper ends of the bars a flame is generated, the non-conducting



FIG. 227 —  
Electric Candle

layer melts and volatilizes at the rate of consumption of the carbon. The arc is started by a small piece of carbon, or of zinc, which connects the points, but burns away on the passage of the current. Jablochkoff's arrangement of candles is, however, still imperfect, because all the lights are extinguished when one fails to operate, and the light is not again automatically started, and finally because the arc, which is filled with the vapours of the insulating substance, has a disagreeable, reddish-blue colour, and flickers badly.

The problem of transmission of light is much more perfectly solved by *differential lamps*, the action of each of which, no matter how many are "in series," depends entirely upon the processes going on between its own binding-screws.

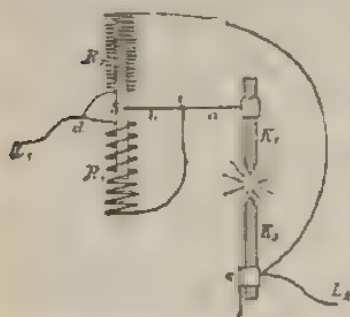


FIG. 228.—Electric Differential Lamps.

The arrangement of the differential lamps of Hefner-Alteneck (Siemens and Halske) is represented schematically in Fig.

228. The upper carbon,  $K_1$ , is attached to one arm,  $a$ , of a lever, movable about  $c$ , and a vertical bar of iron,  $z$ , is attached to the other arm,  $b$ . The lower end of the iron bar extends into a solenoid,  $R_1$ , wound from thick wire, while the upper end con-

tinues into a spool of slender wire,  $R_2$ . The latter solenoid is connected at  $d$  and  $e$ , as a shunt of great resistance, with the main circuit  $L_1, dR_1, cK_1, K_2, eL_2$ . If now the carbon points  $K_1$  and  $K_2$  are too widely separated, the current will pass wholly through the spool of slender wire, since passage over the other spool is obstructed by the resistance between the points. The solenoid,  $R_1$ , draws the bar,  $z$ , into it (240), the arm,  $b$ , of the lever rises, and the arm,  $a$ , allows the other carbon to sink until the points touch. At this instant, the current almost ceases to flow through the shunt, in which the spool,  $R_1$ , is connected, while in the spool,  $R_2$ , whose resistance is small, a powerful current is now flowing. This again draws the iron bar downward, raises the upper carbon, and the arc begins to form. In consequence of

the resistance of the arc, the current in  $R_1$  is again weakened, while in  $R_2$  it increases until, at a definite resistance (*i.e.* corresponding to a definite length of arc), the attractions of  $R_1$  and  $R_2$  upon the bar,  $S$ , hold each other in equilibrium.

**253. Voltmeters—Ammeters.**—To measure the strong currents, applied in electro-technics for illuminating and other similar purposes, the attraction exerted by a solenoid upon a movable iron core is again used. The very extensively used current-meter, or *ampèremeter* (*ammeter*), whose arrangement is shown in Fig. 229, contains a spool of thick wire enclosed within a box. A slender light iron core, suspended from one arm of a bent lever, is drawn more deeply into the spool, the stronger the current flowing through it, and the amount of rotation of the lever is indicated by a pointer attached to its axis and playing along a graduated scale. Tension-meters, or *voltmeters*, are constructed in a similar way, excepting that their spools are composed of many windings of a slender wire, and that they are inserted in a shunt.

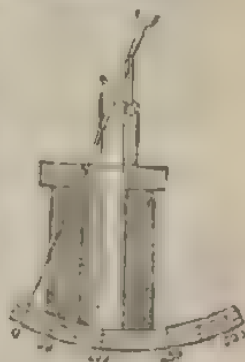


FIG. 229.—Ampèremeter (Ammeter).

With the current and tension meters of S. Kohlrausch the iron core, upon which the coil acts, is suspended to a spiral spring, and a pointer fastened to the iron core indicates its position on a vertical scale. The construction of these apparatus, *i.e.* the graduation of their scales into ampères, or volts, as the case may be, is effected by passing currents of measured intensities (*e.g.* upon an oxy-hydrogen-voltameter) through them, either directly or in a shunt.

**254. Electrodynamical Action**—Since a solenoid comports itself with respect to a magnet, or bar of soft iron, as a magnet, it was early suspected that two solenoids would act upon each other magnetically, and that currents in general, since each produces a magnetic field about it, would act magnetically upon one another. The truth of the suspicion was proved experimentally by Ampère in 1823. If the rectangular copper

wire, *ab* (Fig. 230), through which a current is transmitted by means of the wires *f* and *h*, is brought near to a similarly bent conductor (of copper, or aluminium wire) *cde*, suspended movably on an Ampère's stand (240), the portion, *de*, of the

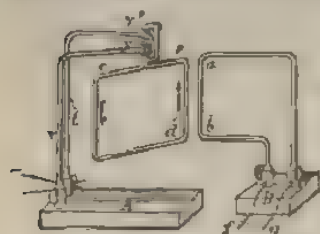


FIG. 230.—Ampère's Stand.

current is attracted by the parallel portion, *ba*, when the current flows in the same direction in both, and the movable conductor seeks a position of stable equilibrium with respect to the fixed conductor, whither it returns after a series of vibrations, whenever it is drawn aside from this position. But when the current in *ab* is made to flow in a direction opposite to that in *de*, by the aid of a com-

mutator connected in the circuit, *ab*, these portions of the current mutually repel each other, and the movable conductor turns from its initial position of unstable equilibrium through 180° into the stable position, where the opposite side, through which the current flows in the direction of that in *ba*, lies as nearly as possible to *ba*. The two circuits act in both cases upon each other like two bar magnets, placed with their axes perpendicular to the surfaces of the rectangles, so that their south poles are turned toward their clockwise surfaces, or like two magnetic double surfaces (240).

The magnetic effect of the earth exercises a disturbing influence upon these experiments, since a movable circuit, such



FIG. 231—Astatic Conductor.

as *cde*, tends to assume a position with its axes in the magnetic meridian, and the earth's magnetism tends to hold it in this position. If, however, the wire of the movable conductor is bent, as shown in Fig. 231, the conductor is freed from the influence of terrestrial magnetism, or it is "astatic"; for it must now act like two equally strong magnets rigidly connected, whose poles point in opposite directions (*astatic needle*).

When the conductor, *rs* (Fig. 232), passes below or above a

conductor,  $pq$ , movable about  $a$ , e.g. under the horizontal portion,  $d$ , of the rectangle (Fig. 230) suspended on an Ampère's stand, so that the conductors intersect, the currents tend to assume parallel and like-directed positions. It may also be readily verified, that between those parts of the conductors, in which both currents move toward, or from, the point of intersection,  $o$ , attraction occurs; while between two portions of the conductors, in one of which the current flows toward the point of intersection and in the other away from it, repulsion takes place.

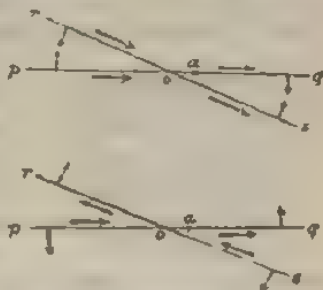


FIG. 232.—Intersecting Currents.

Summarizing the results of these experiments, it may be said that parallel currents attract, or repel, one another according as their directions are the same, or opposite, and that intersecting currents strive to assume parallel and like-directed positions. Ampère called this reciprocal magnetic action between conductors *electrodynamic action*.

Ampère found that the force,  $R$ , exerted by two current elements of lengths,  $l$  and  $l'$ , and of intensities,  $i$  and  $i'$ , upon each other along the connecting line,  $r$ , of their mobile points, is given by the equation—

$$R = -\frac{ii' l l'}{r^2} \left( \cos \epsilon - \frac{3}{2} \cos \delta \cos \delta' \right)$$

when the directions of the elements form with each other the angle  $\epsilon$  and with the line  $r$  the angles  $\delta$  and  $\delta'$  respectively.

**255. Other Electrodynamic Experiments.**—Since parallel like-directed currents attract, the adjacent windings of a spiral wire, suspended from a metallic stand, with a point at its lower end immersed in mercury, must mutually attract, when a current passes through the spiral to the mercury. Hereupon the spiral contracts, raises the point out of the mercury, and breaks the circuit. The circuit then being broken, the attraction ceases; the spiral lengthens by reason of its own weight, and again restores the circuit. This alternate contraction and extension of the spiral wire gives rise to a series of rapid vertical vibrations (Roget, Petrina).

By virtue of the reciprocal action of two intersecting currents, a wire traversed by a current may be set in continuous rotation. This is exemplified in the apparatus devised by Garthe and represented in Fig. 233. Within a fixed wooden frame, AB, about whose periphery several layers of copper wire are stretched, is a light wooden frame,



FIG. 233 — Electrodynamic Rotating Apparatus.

CD, also wound with insulated wire, and easily movable about a vertical axis. If now the poles of a galvanic battery are connected with the binding-screws, *f* and *g*, the movable frame will rotate until the current in its coils is parallel and

like-directed with that in the coils of the fixed frame. To prevent the frame, CD, from remaining in this position, its current is reversed by means of a commutator (*cf.* Fig. 226), attached to the lower end of its axis, so that the portions of the current, which formerly attracted, now repel each other, and thus continue the rotation in the former direction. This rotating apparatus differs from the electromagnetic apparatus described above (251), only in that the former steel magnet, as also the electromagnets, are here replaced by coils of wire, or *solenoids*.

In the rotating apparatus of Fig. 216, the magnet may also be replaced by a solenoid.

**256. The Electrodynamometer** (W. Weber, 1846) is a galvanometer, whose magnet is replaced by a solenoid, which is suspended bifilarly (32 and 142) to the two slender conducting-wires within a fixed multiplier. The force with which the movable coil is deflected is proportional to the product of the current-strength in the two coils, or if the same current flows in both coils, to the square of the current strength. The electrodynamometer of Siemens and Halske (1880), designed for the measurement of the strong currents used in electrotechnics, consists of an inner fixed, and an outer movable coil. The latter has but a single turn, and is, therefore, almost independent of the effect of terrestrial magnetism. The current is admitted to the movable wire through two mercurial cups lying one above the other, in the axis of rotation. The movable wire is suspended to a spiral spring (torsion-spring), through whose rotation, by means of a torsion head at the top of the instrument, the deflected wire is again brought into its position of equilibrium. The torsion head carries a pointer, indicating upon a graduated circle the angle of rotation, which furnishes a measure for the deflecting force. Since the latter is proportional to the square of the current strength, the instrument furnishes precisely the magnitude which the experimenter wishes particularly to know, namely, the

amount of light, or heat, produced by the current, or the work performed by it; for, according to Joule's law, this is also proportional to the square of the current-strength.

**257. Ampère's Theory of Magnetism.**—Since the phenomena of magnetism may be reproduced without the use of steel, or iron, by the electrodynamic effects of galvanic currents, Ampère sought to explain the magnetism of iron and steel by assuming the presence of electrical currents in these substances. He supposed the molecules of iron to be surrounded by little circular currents, flying incessantly about, but without electromotive effects; because, upon its path about the molecule, the current encounters no resistance. In an unmagnetized iron bar, the planes of these molecular currents have the most varied positions, and on this account their reciprocal external effects are mutually destroyed. But when an electric current is passed around the bar of iron, it sets the molecular currents to flowing parallel and like-directed to itself and hence also to each other. Consequently, all the axes of the molecular currents thus become parallel to that of the bar. The little currents circuiting the inner molecules of the bar can exert no external effect, because each is neutralized by its neighbour. Those currents which circuit the molecules at the circumference of the bar, however, in the external portions of their path, are not neutralized by neighbouring currents. In their totality these unneutralized fragmentary currents will coalesce and form closed currents encircling the entire bar. The bar must, therefore, act like a spiral wire traversed by a current (*i.e.* like a solenoid). It must exhibit the phenomena of attraction and repulsion peculiar to solenoids and called magnetic, or it has become an *electromagnet*, whose south pole is directed toward the side whence the magnetizing current, as also the molecular currents of the iron, appear to flow in the clockwise direction. While the molecular currents of soft iron are easily movable about the centre of gravity of the molecules, and, after the cessation of the magnetizing effect, readily return to their former disorganized condition, the more difficultly movable molecular currents of steel (*coercive force*) maintain for a considerable time any arrangement which may be imparted to them. A

steel magnet comports itself, therefore, as a spiral wire traversed continuously by electrical currents. The law of repulsion of like, and of attraction of unlike poles is now explained, as a glance at Fig. 234 will show, by the tendency of currents in the



FIG. 234. Ampère's explanation of Magnetism.

two magnets acting on each other, to direct themselves parallel and similarly to each other. A magnetic needle is deflected by a current, because the Amperian currents surrounding it seek positions

parallel and like-directed to the magnetizing current. According to this view terrestrial magnetism is only the effect of electrical currents flowing about the earth incessantly in a direction from east to west.

**258. Induction.**—In the year 1831 Faraday discovered that, when a magnet, or a conductor traversed by a current, is moved in the vicinity of a closed currentless conductor, electrical currents are produced, which last only so long as the conductor, or the magnet, is in motion. He called this process "induction," distinguishing between the two species of induction by calling that due to the current "voltaic induction," and that to the magnet, "magnetic induction." The currents thus arising were called by him "induced" or "induction" currents.

**259. Voltaic Induction.**—A wire, insulated with silk and wound upon the spool, A (Fig. 235), is connected with the binding-screws, *ab*, of the coils of the galvanometer, G, the current being thus completely closed. A second spool, B, is then inserted within the spool, A. The terminals of the coil, B, are connected by means of the binding-screws, *c* and *d*, with the poles, *n* and *p*, of the galvanic element, E, so that a current may be sent through the coils, B. If now the coil, B, when traversed by a current, be inserted quickly into the spool, A, the deflection of the needle of the galvanometer indicates that a current has arisen in the wire, A, which flows in the *opposite* direction to that in B. This current, which is produced in the latter by bringing the coil, B, within the coil, A, or which was *induced*,

lasts, however, only during the short interval of approach, and ceases immediately so soon as the coil, B, comes to rest within A. This is shown by the fact that the needle of the galvanometer returns to its position of equilibrium immediately after the coil, B, has been inserted. If the coil, B, be quickly drawn out, or if its windings are removed from the coil, A, the needle is deflected toward the opposite side, and then returns immediately to its position of rest, showing thereby that in the coil, A, an instantaneous electrical current was excited, having the same direction as the exciting current.

Instead of bringing the *primary coil*, B, near the *secondary*

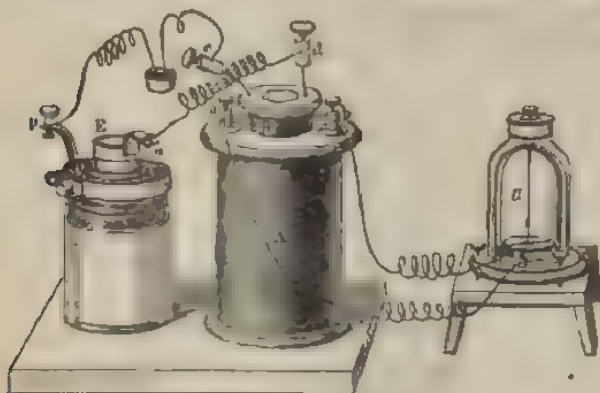


FIG. 235.—Voltage Induction.

coil, A, or of removing the former from the latter, or, instead of inserting the primary coil into the latter and then withdrawing it, it is more convenient to leave the primary within the secondary coil and alternately to open and close the primary circuit. Closing the primary circuit has exactly the same effect as though it were brought instantaneously from an infinite distance and inserted into the secondary coil, and opening it, as though it were suddenly removed beyond the sphere of their mutual action. On closing the primary circuit, therefore, a *closing-current*, or *current on closure*, opposite to the primary current arises in the secondary coil, and on opening

it, an *opening current*, or *current on opening*, in the same direction as the latter is awakened.

The making and breaking of the primary current may be effected, as in Fig. 235, by means of a mercurial cup connected with the end, *c*, of the primary wire, by immersing and withdrawing the wire coming from the pole, *p*, of the galvanic element : while the other pole remains connected with the other end, *b*, of the primary coil. To produce a rapid succession of oppositely-directed induced currents, or an *alternating current*, in the secondary coil, a special device (*rheotome*) for interrupting the current is inserted in the primary circuit. This is most advantageously accomplished by using an automatic *interrupter* such as Wagner's hammer (248).

**260. Magnetic Induction.**—Since a magnet, NS (Fig. 236),

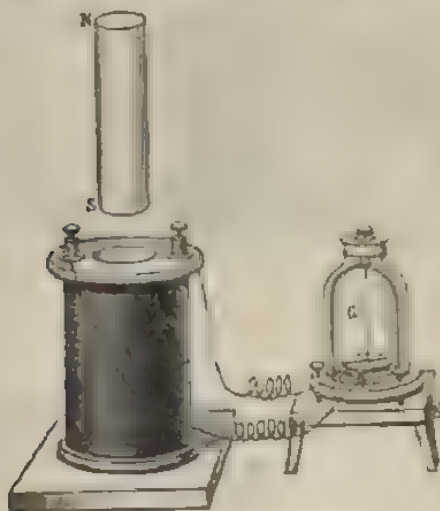


FIG. 236 — Magnetic Induction

acts as a *coil* (257) traversed by a current when inserted into and withdrawn from the coil, A, which is closed with a galvanometer, G,\* in circuit, it must also induce currents,

\* The galvanometer must be at such a distance that the magnet, NS, does not act directly upon its needle.

oppositely directed on approach, and like-directed on withdrawal, to the inducing current, regarded as flowing around the magnet according to Ampère's theory. Instead of inserting and withdrawing the magnet, a bar of soft iron may be placed permanently within the spool and alternately magnetized and demagnetized by approaching and withdrawing a magnetic pole. In both cases electrical currents are generated by *magnetic induction*, without the use of a galvanic element, by merely moving a magnet in the vicinity of a closed circuit.

In the case of both voltaic and magnetic induction it is noticed that induced currents arise only while the magnetic field produced by the primary coil, or the magnet, within which the secondary coil is situated, is being developed, or destroyed, or is undergoing some other change. While the solenoid, or the magnet, remains quietly in the coil, its field remains unaltered and the circuit remains currentless. But, on withdrawal, its lines of force intersect the coils of wire and arouse within them the like-directed Amperian currents.

**261. Lenz's (1834) Law.**—Since like-directed currents mutually attract and opposite repel, when approached to the induced current, the inducing current, or magnet, is repelled by it; and when withdrawn, it is attracted. We may then say with Lenz that, if the relative position of two conductors, A and B, of which A is traversed by a current, or of a magnet and a conductor, be changed, a current is induced in B in a direction such that by its electrodynamic action on the current in A, it would have imparted to the conductors a motion of the contrary kind to that by which the inducing action was produced. Closing, or strengthening, the primary current is here to be regarded as equivalent to an approach; while interrupting, or weakening it, is equivalent to a withdrawal.

By means of Lenz's law, the direction of the induced current is readily determined in all cases. The following rule given by Faraday, is also very convenient for this purpose:—*Let the observer imagine himself swimming in the direction of the line of force, with his face in the direction of the motion in the conductor, the induced current is then always directed from left to right (dextrorsum).*

**262. Electromotive Force of Induced Currents.**—According to Lenz's law, the induced current opposes to the instantaneous motion of the inducing body a resistance, to overcome which a definite quantity of work must be performed; which according to the principle of the conservation of energy (Neumann, 1845; Helmholtz, 1847) reappears as energy of the induced current. By the process of induction, therefore, work is transformed into an equivalent quantity of current-energy. The electromotive force of the induced current is here proportional to the degree of rapidity with which the magnetic field about the secondary conductor is altered, or, inversely proportional to the time within which the change takes place.

When two circular currents are brought from an infinite distance into given relative positions, by virtue of their electrodynamic action, a certain quantity of work is necessarily consumed. If for the case in which the current-strength unity prevails in both conductors, this work be denoted by  $M$ , when the current in the primary conductor has an intensity,  $J$ , and in the secondary, the intensity 1, the amount of work required will amount to  $JM$ . This work  $JM$  is called the "electrodynamic potential" of the currents upon each other. If now the currents are brought into different relative positions, and at the same time the current strength changes in the primary, the potential is at the same time changed to  $J'M$ , and the work expended equals the difference of the potentials,  $J'M - JM$ . If then in the secondary conductor, the intensity,  $i$ , of the current induced in it obtains, this work will equal  $i(J'M - JM)$ .

If now,  $\tau$  denote the short time during which the change of potential was taking place and  $e$ , the electromotive force (or tension) of the induced current,  $e\tau$  denotes the current energy (244) developed in the secondary conductor in the time,  $\tau$ . But this must equal the work consumed, i.e. we must have  $e\tau = i(J'M - JM)$ . From this we find the electromotive force of induction 
$$e = \frac{(J'M - JM)}{\tau}.$$
 Consequently, the electromotive force of induction is proportional to the change of potential of the currents upon each other (to the change of magnetic field), and inversely proportional to the duration of this change.

If the intensity of the induced current during its motion relative to the inducing current remains unchanged ( $J' = J$ ), the electromotive force 
$$e = \frac{J(M' - M)}{\tau}$$
 is proportional to this intensity, and to the rapidity with which the relative position of the currents is changed.

If, on the contrary, the intensity in the primary conductor changes, while this conductor remains in the same position relative to the secondary conductor ( $M' = M$ ), the electromotive force 
$$e = \frac{M(J' - J)}{\tau}$$
 is proportional to the velocity with which the mutual positions are changed.

The magnitude,  $M$ , depends only upon the form and position of the conductors, and is called the *reciprocal coefficient of induction*.

As a special example, let us consider the inducing effect of a magnetic pole upon a circular conductor. If a north magnetic pole of intensity,  $m$ , lying upon the axis of a circular conductor of radius,  $R$ , and near the centre of the circle, is moved away from this centre, an induced current of intensity,  $i$ , arises, which repels the magnetic pole with a force,  $\frac{2\pi m^2}{R}$ . If  $v$  denote the velocity, with which the pole is moved toward the conductor (or the conductor toward the pole),  $\left(\frac{2\pi m^2}{R} v \tau\right)$  will be the work performed in the time  $\tau$ . But this work must equal the energy of the simultaneously induced current, or we must have,  $e\tau = \left(\frac{2\pi m^2}{R} v \tau\right)$ , where  $e$  denotes the electromotive force of this induced current. This force is accordingly given by—

$$e = \frac{2\pi m^2}{R} \left(2\pi R \cdot \frac{m}{l^2} \cdot v\right)$$

In this,  $2\pi R = l$  is the length of the circular current, and  $\frac{m}{R} = T$  the force with which the pole,  $m$ , would act upon a magnetic pole of intensity,  $l$ , situated on the circumference of the circle, or  $T$  also indicates the intensity of the magnetic field prevailing there. The electromotive force of the induced current may therefore be expressed thus,  $e = l^2 T v$ . This expression holds also even when  $l$  denotes the length of one of the small portions (current-elements) of the conductor,  $v$ , whose sum makes up the circumference of the circle, or an arbitrary portion of the circumference, or finally a rectilinear conductor, which moves perpendicularly to the lines of force of the field. Hence, in all parts of the conductor of equal length, the same force operates.

When a rectilinear conductor of length,  $l$ , moves with the uniform velocity,  $v$ , perpendicularly to the lines of force of a homogeneous magnetic field of intensity,  $T$ , and parallel to itself, during 1 sec. it sweeps over the surface,  $lv$ , and  $lvT$  is the number of lines of force (45 li. cuts). The electromotive force induced in the conductor is therefore equal to the number of lines of force cut perpendicularly by it in a unit of time. If the conductor forms another angle,  $\phi$ , than a right angle with the lines of force, only the component  $T \sin \phi$  perpendicular to the conductor must be considered. The electromotive force is accordingly 0 when the conductor is parallel to the lines of force ( $\phi = 0$ ).

**263. — Absolute Electromagnetic Unit of Electromotive Force.**—W. Weber selected as an absolute unit of electromotive force, that force arising in a rectilinear conductor of length 1 ( $l = 1 \text{ cm.}$ ), when the latter is moved in a magnetic field of intensity 1 ( $T = 1$ ) with the velocity 1 ( $v = \frac{1 \text{ cm.}}{\text{sec.}}$ ) parallel to itself and perpendicularly to the lines of force. The practical unit of electromotive force, already familiar to us as the *volt*, is one hundred million times ( $10^8$ -fold) this absolute unit.

**264. Extra Current—Self-Induction.**—If, as heretofore assumed, the primary wire is wound upon a spool, at every alteration of current-strength each turn of the primary wire acts inductively upon the adjacent turns, and on closing, or increasing the primary current, it awakens a current opposite to this, while on opening, or diminishing, the primary current, a like-directed current is induced. Faraday called these induced currents in the primary wire itself, *extra currents*. They act always against any change of the original current.

This so-called “self-induction” acts not only in a wire wound into a coil, but also in a straight wire; for the wire may be conceived of as composed of longitudinal strands, each of which acts inductively upon its neighbours. The electromotive force of self-induction is proportional to the rate of change of intensity of the primary current.

The electromotive force of self-induction is  $\epsilon = L \cdot \frac{J' - J}{t}$ , where  $L$  denotes a magnitude depending upon the form of the conductor, analogously to the reciprocal coefficient of induction,  $M$ , and it is called the coefficient of self-induction.

**265. Difference between Closing and Opening Current.**—Since the extra current (“counter-current”) arising on closing the primary coil is directed oppositely to the primary, it weakens the latter and causes it to attain its full strength gradually and after a short interval of time. The effect of the extra current on opening, on the other hand, is readily recognized from the following facts, which may be easily verified. With the straight wire of a galvanic battery, a feeble spark is obtained on breaking the current; but if the same wire is wound into a coil, self-induction acts much more strongly. A powerful extra current, like-directed with the primary, forms beside it, thereby strengthening it, so that the spark on opening is much stronger, and forms, at the point of interruption, a conducting bridge for a short time. On interruption, therefore, the primary current suddenly disappears, but it sinks in a brief time gradually from its full intensity to zero. The duration of fall is much shorter than the time required on closing for the current to rise from zero to its full strength.

Since now, the electromotive forces induced in the secondary coil for equal alterations of the primary current are inversely as the durations of change, the current arising in the secondary coil on opening the primary must have a greater electromotive force, or tension, and therefore also a greater intensity than the closing current. On the contrary, the quantities of electricity discharging in both currents (*i.e.* the product of intensity and duration) are equal. This is plain from the fact that the alternating currents of the secondary coil, when led by platinum electrodes through a solution of copper sulphate, produce upon neither electrode a copper precipitate, which would necessarily form if one current transferred a greater quantity of electricity in one direction than the other in the opposite direction. A galvanometer also gives, both for the opening and for the closing currents, the same deflection, though in opposite directions; for, since the duration of both induced currents is far shorter than the time of vibration of the needle, in both cases the entire quantity of electricity discharged by the current acts relatively instantaneously (*ballistic galvanometer*). When the interruptions of the current follow each other in rapid succession, the needle remains at rest, because the opposite impulses neutralize each other.

The distinction, therefore, between the opening and the closing current, consists in the fact that the discharge of the same quantities of electricity during extremely short intervals of time, in both cases, requires relatively a longer time for the latter current than for the former.

**236. Measurement of the Galvanic Resistance in Electrolytes.**—As was just mentioned, the alternating currents of an induction coil on passing through a liquid, do not cause chemical decomposition, and, consequently, no polarization is produced by them. If we should attempt to determine the resistance of a liquid, by the bridge method (230), with the use of the ordinary galvanic current ("constant current"), the current-strength would be found to be weakened not only by the resistance of the liquid itself, but also by the reaction of polarization, and the measured resistance would be found too great. This difficulty, due to polarization, is avoided by replacing the constant current by an induced alternating current (F. Kohlrausch, 1868). But instead of inserting in the bridge a galvanometer, which with alternating currents gives no deflection, an electro-dynamometer (256) must be used. Since in the solenoid and in the multiplier the currents are reversed simultaneously, deflection always takes place in the same direction.

But a new difficulty attends this application of alternating currents. In the coils of the resistance-boxes (225) used for measuring purposes, the electromotive force of the extra current reacts against the original electromotive force. The intensity is thereby diminished and the resistance apparently increased. To prevent this disturbing effect of self induction, the winding is such that the wires of every pair run in opposite directions. This is accomplished by bending the wire at its middle and winding it doubly. The extra currents in neighbouring windings then mutually destroy each other, and the coil is said to be wound *bifilarly*, or *free from induction*. The magnetizing effect of the coil is also destroyed by this mode of winding.

**267. Physiological Effects of Induced Currents.**—A galvanometer is not needed to prove the existence of induced currents. Their effects are directly perceptible to us through their strong effects upon the nerves.

If one grasps with the hands the poles of a galvanic battery whose electromotive force is not too small to send a current through the body, a twitching sensation is felt at the moment of closing the current. A current flowing with constant intensity through the body produces no sensation. A second pricking sensation occurs on letting go of one or both of the poles, thereby interrupting the current. An invariable, or *constant*, current produces no perceptible effect on the nerves; but its beginning, or ceasing, or any alteration of its intensity, evokes a response from the nervous system, which becomes more pronounced the more sudden the change. From this it is seen why the discharge of a Leyden jar is so keenly felt; the very small electrical mass condensed in the flask to a high potential discharges in an extremely short time, generating an electrical current, increasing with great rapidity to its full intensity, and then dropping again just as quickly to zero. Since the induced currents are likewise of short duration, and within this short time they rapidly rise and fall, in spite of the small electrical mass set in motion by them, they produce a very strong excitation of the nerves of animals, or a pronounced "physiological effect," which is heightened by the opening and closing currents set in motion in rapid succession through the body by the incessant play of the interrupter. The more intense and more rapidly discharging opening current produces, then, a far stronger effect than does the closing current. To pass induced currents through the human body, cylindrical brass

handles are ordinarily connected by metallic cords to the ends of the induction coil, the handles being grasped with moistened hands. In case of feeble currents, only a prickling sensation is felt, while with stronger currents cramping muscular contractions are experienced. On account of their effect upon the nerves, induced currents are extensively used in the practice of medicine. In medical parlance, these currents are named for Faraday, the discoverer of induction, "Faraday's currents" and the treatment of the human body by means of them is called "faradizing."

**268. Inducing Apparatus.**—To procure induced currents of high potential (electromotive force), the primary coil is made of thick wire with but few turns, so that its resistance may be small, and consequently the strength of the primary current as great as possible, while, on the contrary, the secondary coil is made of a large number of windings of slender wire, because electromotive force increases with the number of windings. The sliding mechanism of Dubois-Reymond (Fig. 237) is an inducing apparatus especially adapted to medical purposes. The secondary coil, *N*, whose wire terminates in the binding-screws, *a* and *b* (for connecting the wires running to the handles), is fixed to the board, *S*, which slides in the grooves (*gib*) of the baseboard. It can, therefore, be drawn at will wholly, or partially, over the primary coil, *H*, which is held horizontally by the vertical board, *B*. The strength of the induced currents may be altered in this way to suit the needs of each special case. The interruption of the primary current, whose pole wires are screwed into the binding-posts, *c* and *d*, is effected by the magnetic hammer, *M*. The ends of the primary are, furthermore, connected with the binding-screws, *e* and *f*, to which the wires of the handles are attached, when it is desired to use the induced extra-current in the primary wire itself. At each interruption of the primary current, the secondary current discharges through the secondary closing wire between *e* and *f*.

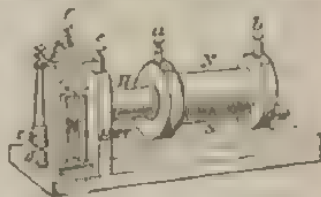


FIG. 237. — Sliding Apparatus.

The inducing effect of the primary coil is very materially

intensified by inserting a bar of soft iron into its central cavity. The original primary current magnetizes the iron core, i.e. it compels the little circular currents, which Ampère supposes to surround the molecules of iron, to assume its own direction of flow. After its cessation, however, these currents return to their former unorganized positions, and the iron core loses its magnetism. These molecular currents, first becoming directed, and then losing their direction again, excite in the secondary coil induced currents which are like-directed with those simultaneously induced by the primary current directly, and, accordingly, strengthen them. This advantage of the iron core is, however, partially neutralized by a detrimental effect, also due to it. As would be the case with any continuous metallic mass inserted within the primary coil, currents are induced in the iron bar on starting and stopping the primary current. These induced currents, passing from molecule to molecule, encircle the circumference of the bar, retard the rise and fall both of the primary current itself, and also of the magnetizing effect, and, accordingly, they prolong the duration of the induced currents arising in the secondary coil. This process leaves the quantity of electricity in motion unaltered, but reduces its potential. The development of these injurious (Foucault's) currents may be avoided by constructing the core of the primary coil of a bundle of slender iron wires, insulated from one another by coatings of varnish, instead of using a single thick bar of iron. The retarding currents circuiting the iron core do not now arise. The induced currents in the secondary coil assume the desired direction, and act much more strongly upon the nerves than would be the case with a single mass of iron.

**269. Sparking Apparatus—Inductors.**—If the ends of the inducing coil are not connected, the electricities set in motion by starting and stopping the primary current collect here in the secondary wire and produce the phenomena characteristic of electrical potential. When tested electroscopically, either end appears to be charged alternately, positively and negatively in rapid succession, according as the collected electricity is at the instant due to the opening or to the closing current. When the tension becomes sufficiently high, sparks pass from either

end of the open secondary coil to an approached conductor. This discharged electricity is, however, always due to the opening current, for this alone attains a sufficient potential to force a spark through the intervening atmospheric layer. On interposing a layer of air, one end of the induction coil, accordingly, appears always positive, and the other end negative, for which reason they are designated opposite electrical poles, or "electrodes." Inducing apparatus exhibiting the phenomena of tension thus strongly are called *sparkling apparatus*, or *inductors*. Ruumkorff's inductor is represented in Fig. 238. To secure sufficiently high potential, the induction coil is made of a great number of windings of fine wire, while at the same time care is taken to remove as far as possible the effect of the opening extra-current. This effect retards the disappearance

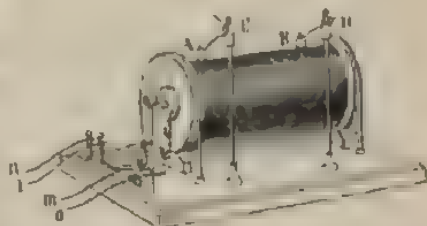


FIG. 238.—Sparkling Apparatus.

of the primary current by the formation of sparks at the break-piece. The reduction of this effect is accomplished by connecting two points of the primary wire, one on either side of the break-piece, to the coatings of a condenser, which constitutes a part of the apparatus. The condenser absorbs the two electricities of the extra-current on opening and permits them to flow back again on closing. The poles, A and B, of the induction coil are connected with the insulated binding-screws, C and D, into which the pole wires may be screwed. If the ends of these wires are approached, a brilliant stream of sparks passes between them, similar to those of the influence machine. When, as with the influence machine, the poles are connected with the coatings of a Leyden jar, a succession of crackling sparks is obtained. A large Leyden battery may also be charged very quickly by the aid of this apparatus. By means of the sparking apparatus, therefore, it is possible, with the help of a galvanic battery of low potential, to reproduce all the

phenomena of frictional electricity, which depend upon high potential.

**270. Geissler's (Pluecker's) Tubes.**—Very remarkable—but as yet insufficiently explained—luminous phenomena are produced by discharging the sparking apparatus through rarefied gases. The gases are ordinarily enclosed in glass tubes, into which platinum, or aluminium, wire electrodes are fused at proper places. These electrodes terminate outward in hooks to receive the conducting wires (Gassiot, 1854; Geissler and Pluecker, 1858). One of the simplest of the manifold forms given to these Geissler tubes is reproduced in Fig. 239. If the tube contains moderately

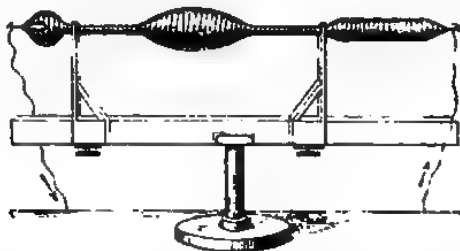


FIG. 239.—Geissler's Tubes.

(e.g. to  $\frac{1}{100}$ ) rarefied air, and the electrodes are connected with the poles of a sparking apparatus (or of an influence machine), the negative electrode (*cathode*) appears surrounded by a mellow, deep-blue, luminous envelope, called the *glow-light* (cf. 178). From the positive electrode (*the anode*), however, issues a peach-red sheaf of light, extending through the entire tube almost to the negative envelope, but remaining separated from the anode by a dark intervening space. Quite frequently, especially when vapours of oil of turpentine, carbon disulphide, or other combustible gases, are present in the tube, this sheaf appears to be decomposed into a succession of bright and dark layers, lying perpendicularly to the axis of the tube, and appearing to advance with a wave-like movement from the positive toward the negative pole. With reference to an electrical current brought near, or to a magnet, the positive sheaf of light comports itself as a movable conductor. It is, for example, deflected by a magnet, according to the same laws as a movable conducting wire, and

rotates continuously about a magnetic pole. This may be conveniently shown by means of the device of Fig. 240. Within an egg-shaped glass vessel containing sufficiently highly-rarefied air, is inserted an iron bar, *E*, covered with a glass envelope. The current flows parallel to the iron bar between the platinum electrodes, the one of which, *a*, is attached to the upper end of the egg, while the other, *b*, encircles the iron bar farther below. If the egg is placed upon the pole of an electro-magnet, *M*, the iron bar becomes magnetic, and the sheaf of light rotates about the bar precisely as does a conducting wire (242), with movable suspension. The direction of rotation reverses also whenever, by means of the commutator, *K*, the poles of the electro-magnet are interchanged.

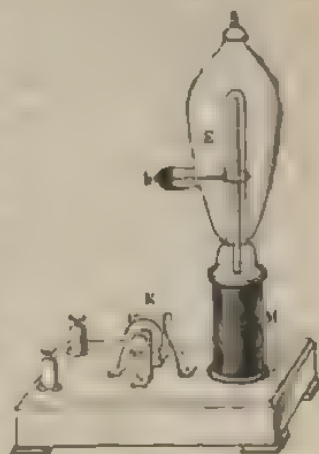


FIG. 240. —Rotation of a Luminous Current about a Magnet.

Although a Geissler's tube appears to emit its mellow light continuously, it is really composed of a rapid succession of sudden, though distinct, discharges whose images, falling upon the same part of the retina of the eye, fuse into a single continuous luminous impression. If, however, the tube is rotated rapidly about one end with the aid of a centrifugal machine, so that the images of the individual discharges may fall upon different parts of the retina, a brilliant star, formed, as it were, of many luminous tubes, may be seen.

The colour of the positive light varies with the constitution of the gas in the tube. With hydrogen it is reddish purple; with carbonic acid it is greenish. The light is, however, always rich in those violet and ultra-violet rays, which are capable of producing that peculiar self-luminosity of glass known as "fluorescence." By constructing portions of the tube of strongly fluorescent glass, *e.g.* of bright green uranium-glass, prepared in ornamental forms, the exquisiteness and

variety of the phenomena may be very considerably augmented.

When the air in a tube is still further rarefied, as in the ordinary Geissler's tube, the blue negative light and the dark space separating it from the positive light, extend much farther toward the positive pole; while, at the same time, the positive light gradually becomes feebler, and finally vanishes completely. While the positive current of light in an ordinary Geissler's tube effects an electrical connection between the electrodes, just as a movable conductor does, and follows all of the accidental windings of the tube, propagating itself even in tubes where the air is rarefied to one-millionth of an atmosphere, the negative, or *cathode light*, passes only in straight lines (cathode rays), radiating perpendicularly from the surface of the cathode, and is wholly uninfluenced in direction by the position of the anode. Crooke (1879) used the following device to prove the existence of this peculiarity

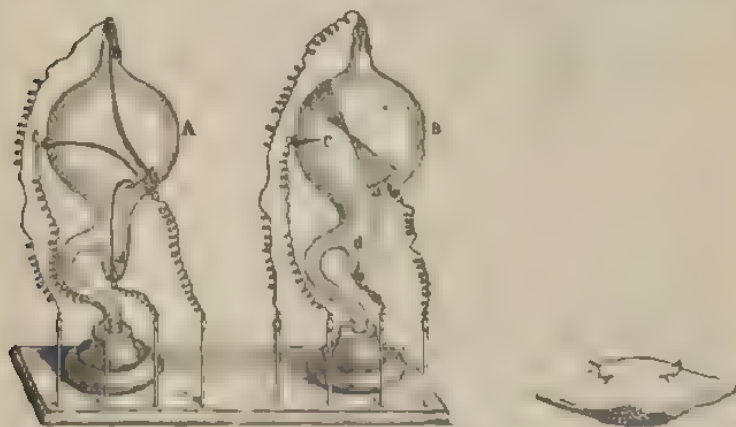


FIG. 241—  
Crooke's Tube

of cathode light, which was discovered by Hittorf (1869). In the V-shaped tube (Fig. 241), three wires, *a*, *b*, *c*, are melted, each of which carries a small circular plate of tin. If *a* is connected with the negative, and *b* with the positive pole of an inducing apparatus, the negative light passes in a straight line only to *c*, where it does not turn the corner, but when *a* is connected with the positive, and *c* with the negative pole, the negative light issues in a direction perpendicular to the cathode plate, *c*, and rectilinearly toward *b*, apparently without any regard whatever to the positive electrical charge at *a*. The essential difference between the electrical discharge in moderately, and in very strongly rarefied air, may be very clearly perceived with the two equal spherical vessels, *A* and *B* (Fig. 242), the former being exhausted to a moderate degree, 2 mm. mercury, and the latter to about one-millionth of an atmosphere.

When the cup-shaped electrode, *a*, is connected with the negative pole and the electrodes, *b*, *c*, *d*, in turn with the positive pole, a red beam of light, passing from the positive electrode towards the negative, is seen in the first vessel, and

at the latter electrode, the blue negative luminous envelope appears, while in the other vessel, the positive luminous sheath is not seen at all. The rays of cathode light, however, emanate from the cup-shaped negative electrode, converge into the centre of the sphere of which the electrode forms a part, whence they proceed in a diverging cone and produce upon the opposite wall a spot of green phosphorescent light, accompanied, however, by the sensation of heat. This path is taken



FIGS. 242, 243.—Crooke's Tubes

by the cathode rays, no matter which of the wires, *b*, *c*, or *d*, is made the anode.

Where the rays of the cathode light impinge upon the walls of the vessel, they excite the glass to vivid self-luminosity (*fluorescence* and *phosphorescence*). Thuringian glass, from which these vessels are usually prepared, shines with a brilliant apple-green colour; uranium glass, with a dark green; and English glass, with a blue colour. To observe the phosphorescence of other bodies under the action of cathode rays, they are enclosed in tubes such as Fig. 243. Ruby and calc spar shine red; diamond, bright green; clime, emerald green, and pectolite, yellow.

The rays of cathode-light are intercepted by solid bodies. In the pear-shaped vessel (Fig. 244), the positive electrode

terminates in a cross, *b*, cut from a plate of aluminium. Since only the rays, *ad*, of the cathode, *a*, arrive at the opposite wall and excite phosphorescence, a dark shadow of the cross is there seen upon a bright green background. If now the cross is turned about the hinge by gently shaking the apparatus, so that the rectilinear cathode rays strike the opposite wall unobstructed, the former dark cross will appear bright upon a



FIG. 244.—Crooke's Tube.

darker ground. The glass has become heated at the places where the rays formerly impinged upon it and has thereby lost, in part, its phosphorescent character. The portion, however, which was previously shaded, has not yet been fatigued, but still possesses susceptibility.

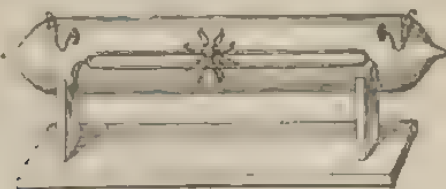


FIG. 245.—Crooke's Tube.

The cathode rays exert an impact upon a body against which they fall and are therefore able, as Crooke discovered, to produce mechanical effects. In the tube of Fig. 245, a glass track is fixed, upon which a small wheel with mica paddles can roll. When the electrodes, situated just above the track, are connected with the poles of an inducing apparatus, the wheel

is driven from the cathode toward the anode, as though a current of air were striking against the upper paddles in the direction from the former to the latter.

The negative stream of light is subject also to the action of a magnet, comporting itself, according to Hittorf, as a rectilinear beam of stiff fibres with one of its ends at the cathode, while the positive stream resembles a flexible conductor with both ends fixed. Parallel beams of cathode rays mutually repel one another.

A body immersed in cathode rays becomes heated. If in the glass globe (Fig. 242, B) a piece of platinum-iridium is placed at the centre of curvature of the cup-shaped negative electrode, *a*, it is raised to white heat, and finally melted by the condensed cathode rays.

The phenomena of the cathode light are most perfectly developed at a degree of rarefaction, corresponding to a pressure of one-millionth of an atmosphere. Beyond this they become weaker, and in a perfect vacuum no electricity whatever passes. At one end of the tube (Fig. 246) a small auxiliary



FIG. 246.—Crooke's Tube.

tube, *k*, is blown, containing fragments of calcium hydroxide. If the tube is filled with carbonic acid gas, and exhausted as far as possible, while *k* is heated, the residue of carbonic acid gas, not removable by the air-pump, will be absorbed by the hydroxide when it again cools. There will then be no substance whatever, to conduct electricity between the electrodes, *n* and *p*. Electricity no longer passes, and the tube remains dark. If the tube be slightly heated, water-vapour is developed, and there appears then, first the cathode light, and later, after considerably further heating, follow the positive rays.

The cathode rays were supposed to be confined to the space within the tube, so that bodies, to be exposed to their effect, had to be enclosed also within the tube, until Lenard (1893) succeeded in releasing them. After Hertz had found that thin plates of metal are transparent to the cathode rays, Lenard fastened in the wall of a Hittorf's tube a thin aluminium plate,

and through this dark "window" in a transparent wall the cathode rays passed outward into the air, and excited it to a diffuse luminosity.

Roentgen (1895) surrounded a Hittorf's tube with blackened opaque cardboard, and found that a fluorescent substance, *e.g.* barium-platinum-cyanide, brought near the apparatus, shines brightly. From those places of the glass wall of the tube, where the cathode rays impinge, an invisible radiation, therefore, takes place which penetrates the envelope of cardboard. These *Roentgen rays* differ from the cathode rays in that *they are not deflected by a magnet*. All bodies are more or less transparent to them. They pass readily through paper, wood, leather, hard rubber, and also through metallic plates, when the plates are not too thick. With the same thickness of plates, the transparency is very essentially dependent upon the density. Lead is practically impermeable to these rays in plates of 1.5 mm. thickness, while ten times as thick a layer of aluminium, though weakening their effect, does not wholly destroy it. Roentgen further showed that these rays are neither regularly reflected nor refracted.

What is especially interesting is that ordinary photographic dry-plates are susceptible to Roentgen's rays, so that their phenomena may be permanently fixed. Since the rays pass through wood and paper, almost without modification, photographs may be made with closed plate cases, or upon plates wrapped in black paper, and *even in a light room*. Metallic objects, such as the brass weights of a balance enclosed in wooden cases, or coins in a closed purse, imprint their images upon the sensitized plate, since the rays penetrate wood and leather in sufficient quantities to blacken the sensitized film. On the contrary, those objects which are partially covered by metals are more or less imperfectly photographed at the places covered. If the hand is placed upon the wooden cover of the case containing the plates, or upon the blackened paper enveloping them, there appears upon the plate a shadowy image of the bones of the hand which shows, in the positive, the dark shadows of the bones within the less darkened image of the hand, because the rays penetrate the softer, fleshy portions of

the hand more readily than they do the bones. A gold ring appears as a gauze-like image floating freely about the finger.

**271. Magneto-Electrical Machines.**—To produce electrical currents by magnetic induction, it is most advantageous to give the steel magnet, as also the iron core, *a*, inserted in the induction coil, the form of a horseshoe (Fig. 247). About the ends of the branches of the core, two rolls of wire are wound and connected with each other. The alternate approach and recession of the magnetic pole is effected, either by turning the magnet (Pixii, 1832), or, still better, by turning the core about the mid-line, *ab*, parallel to its branches (Clarke, 1836). If in this manner the iron core is rotated rapidly, the coils surrounding it are traversed by alternately opposite induced currents, their directions being reversed, whenever the poles of the core stand opposite to those of the magnet. Such a *magneto-electrical machine*, or *motor*, therefore, furnishes alternating currents directly. To give the alternately



FIG. 247.—Magnetic Induction.

opposite currents in the coils the same direction in the outgoing circuit, a commutator is put upon the axis of rotation with the same arrangement as is represented in the model (Fig. 226) of an electro-magnetic motor. The latter may, indeed, be regarded indifferently as a model of a magneto-electrical, or of an electro-magnetic, motor. When, on the passage of a battery-current, the armature of the motor turns, a current directed opposite to this is induced in the coils. This latter current weakens the current from the battery. The work performed by the motor corresponds to this loss of current-energy. When work is expended to turn the armature, *AB*, against the electro-magnetic forces, a current is induced in the coils, whose direction is the same as that of the galvanic current. The latter current is thereby strengthened. The work performed corresponds to this increase of current-energy. When no galvanic current is present, an induced current arises on rotating the armature within the coils if the circuit is closed. This

current is indicated by a galvanometer in circuit with the coils. The induced current also resists the motion of the armature.

Any magneto-electrical motor may be used also as an electromagnetic motor. If a current is sent through it, the mechanism begins to move and to perform mechanical work. If it be set in motion by the application to it of work from without it furnishes a current. All the electromagnetic rotating apparatus hitherto referred to, *e.g.* Barlow's wheel, etc., when moved by an external force, become magneto-electrical apparatus.

To secure a stronger effect by means of magneto-electrical motors, the form and winding of the armature (inductor) are so designed that the magnetic field, in which the armature moves, is used as fully as possible. In the machines of Siemens and Halske, designed for telegraphic purposes, two rows of horizontal magnetic bars,  $MM'$  (Fig. 248), are fastened to a vertical

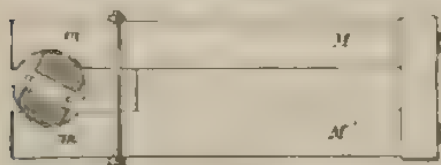


FIG. 248.—Machine of Siemens and Halske.

iron plate, and have their front ends at  $mm'$  so turned out that the cylindrical hollow may receive the "cylindrical inductor." The latter consists of an iron core

whose form is represented by the double T-shaped cross-section,  $aa'$ . The wire is wound longitudinally into its lateral grooves, and the entire cylinder is surrounded by a brass protecting envelope which carries, above and below, the support for the axis of rotation.

A material advance in the construction of magneto-electrical machines was made by the introduction of the *ring armature* of Pacinotti (1860), or Gramme (1871). In Gramme's machine (Fig. 249) a ring of soft iron,  $ABCD$  (Fig. 250), upon which a number of coils of wire are wound, each being connected with the following, rotates between the poles,  $N$  and  $S$ , of a horseshoe magnet, about an axis perpendicular to the plane of its branches. From the connecting points of adjacent coils, metallic continuations,  $RRR, R, R$ , run to the axis of the ring, where they are

turned at right angles, insulated from one another, and fastened upon the axis in directions parallel to it. Two bundles of wire, or brushes (indicated in Fig. 250 by wires running from R and R<sub>1</sub>), slide by gentle friction on both sides against the axis and take up the currents induced in the coils during rotation. Under the influence of the magnet the ring itself becomes magnetized in such way as to make it consist of two semi-circular magnets, ABC and ADC, joined together with their like poles at A and C, and having their neutral sections at B and D. The position of these poles is not altered by the

rotation of the ring, since the soft iron does not retain its magnetism. The effect is the same as if the ring remained stationary with its south pole at A, and its north pole at C, while the coils of wire were being slipped along over the iron core. The coils upon the surface of the ring cut, then, the lines of magnetic force which, proceeding from N, enter the ring and, passing within the core along ABC and ADC, leave the ring at C, toward S, and in opposite senses at C and at A. The currents induced in these lines have, therefore, in the coils of

the lower half of the ring, a direction opposite to that in the coils above. The neutral line BD is the line of alternation. The brushes sliding upon the axis in the neutral line (displaced somewhat in the direction of rotation) take up at R<sub>1</sub> and at R, the opposite currents of the semicircles in the same sense,

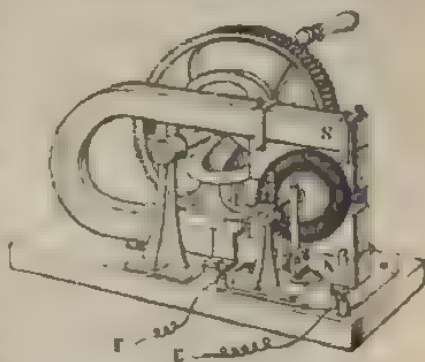


FIG. 249. — Gramme's Machine.

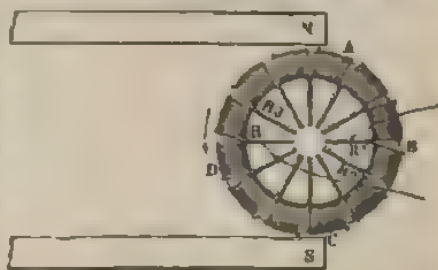


FIG. 250. — Gramme's Machine.

and transmit into the closing circuit a *continuous* current in the same direction. The apparatus furnishes therefore a *constant current* immediately, i.e. without the use of a commutator.

**272. Dynamo-Electrical Machines.**—The steel magnets of a magneto-electrical machine (e.g. of Gramme's machine) may be replaced by electromagnets, about whose cores the current coming from the inductor is made to pass. If the inductor is rotated the weak magnetism of the electro-magnet remaining in the softest iron suffices to excite a weak induced current, which increases the magnetism and thereby again intensifies induction. By continual rotation the current-strength in the closing

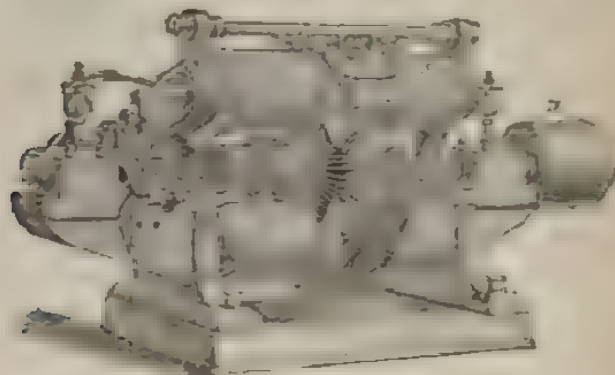


FIG. 251. DYNAMO.

circuit reaches a limit, determined by the saturation point of the magnet and certain other circumstances. By means of this dynamo-electrical principle, discovered by W. Siemens (1867), *dynamo-electrical machines* (*dynamos*), constructed often in colossal dimensions, have made it possible, with comparatively small expense, by aid of the energy of water, steam, or gas motors, to produce the strong electrical currents required for electrical illumination and other purposes of electro-technics.

Figure 251 represents one of the latest and most highly improved forms of the dynamo (the flat-ring-dynamo of Schuckert). MN, SS, are the "field-magnets," between the iron pole-shoes (PP) of which, the Gramme ring rotates about

the axis, A, being driven by a motor (steam-engine, or gas-motor), by means of the belt-wheel, D. The insulated ends of the windings of the ring lead along the axis to the "collector," C, whence the brushes, B (only the front one is visible in the figure), take off the current, conveying it to the binding-screws, KK.

**273. Electrical Transmission of Power.**—If the current of one dynamo is transmitted through the windings of a second, the latter takes up a rotation and may now perform mechanical work as an electromagnetic motor (*electro-motor*). In this way the work of a stationary steam-engine, or of a waterfall, etc., may be conveyed to a remote station, the work being first transformed into current energy by means of a dynamo. As an illustration of this we have the electric street railway car, in the carriage of which is placed the second dynamo which acts as a motor to turn the wheels.

The transmission of strong currents over great distances is attended, nevertheless, with very considerable practical difficulties. If at the remote station a definite current-strength is required, for double the distance, the cross-section of the copper conducting wire must also be doubled to keep the resistance the same, and four times the weight of copper must be used. The expense of power-transmission therefore increases with the square of the distance, and a limit is soon reached at which the process ceases to be economical. In the transmission of force, or more correctly, of work, however, the important question is not how to transmit a definite current-strength, or a fixed potential, but rather how to transmit the energy of the current without too serious a loss. This energy is expressed by the product of tension, or potential, and current-strength (in volt-ampères, or watts), and hence it remains unaltered when the potential is raised, provided the current-strength is diminished in the same ratio. With double the potential, for example, to transmit the same quantity of work requires only half the current-strength and the cross-section, as also the weight of the copper conducting wire, may be diminished to one-fourth their former values.

For greater distances currents of higher potential and of lower current-strength must be applied (Marcel, Desprez,

1882). The construction of constant-current machines for high potential is also attended with serious difficulties. Alternating-current machines of high potential are much more readily built and, consequently, have come to be preferred in recent times for the transmission of power. The great alternating-current motors used in electro-technics are arranged essentially as follows. Along the periphery of a wheel are attached several coils of wire, which, on rotating the wheel, pass between two fixed circles, to whose peripheries electro-magnets are fastened in similar positions. These electro-magnets, energized by a constant-current dynamo, are so connected up that both the adjacent and the opposite magnetic poles have opposite polarities, *i.e.* so that a north pole stands opposite to each south pole, while a south pole stands between two north poles. If the wheel be then rotated uniformly, there arises in the coils an alternating current, that is to say, a to and fro pulsating motion following the same laws as a vibrating pendulum.

**274. Transformers.**—At the place of application of the current, as a rule (*e.g.* for illuminating purposes), a current of great strength and of relatively low potential is required. The alternating current, therefore, transmitted thither from a distance (which, be it noted, on account of its high potential, would be dangerous to life and property) must be converted into a current of low potential but of great current-strength. This is done by means of transformers (Gaulard and Gibbs, 1883), which are a special form of inducing apparatus. By means of the inducing apparatus hitherto discussed, a battery-current of moderate intensity and great strength, which passes through the primary coil, consisting of a comparatively few windings of thick wire, into the secondary coil with many windings of fine wire, is transformed into an induced current of high potential and low strength. If, on the contrary, the primary coil consists of many windings of thin wire, and the secondary of a few windings of a coarse wire, a weak current of high potential in the former produces in the latter an induced current of low potential and of high current-strength. By means of such transformers we have then a means of converting

without alteration of current-energy (disregarding unavoidable losses) high into low potential, and small into great current-strength, and conversely. Since, with the potentials here occurring, the air does not produce sufficient insulation, these transformers are sometimes placed in vessels filled with good insulating oil.

In an inducing apparatus, or transformer, suppose the secondary current to have a potential,  $e$ , and a current strength,  $i = \frac{e}{r}$ , where  $r$  denotes the resistance of the wire. If, now, the secondary wire is stretched to  $n$ -times its initial length, so that the number of its windings is increased  $n$ -fold, and its cross-section becomes the  $\frac{1}{n}$ -th part of the original, its resistance thereby becomes  $n$ -times its former value ( $= n^2r$ ), the potential becomes " $n$ " times ( $ne$ ), and the current-strength,  $\frac{1}{n}$ -th part of the former value (for  $\frac{ne}{n^2r} = \frac{e}{nr} = \frac{i}{n}$ ), while the product of both, i.e., the current energy, remains unaltered. If, conversely, the number of windings be diminished to  $\frac{1}{n}$ -th of the former number, and the cross-section be increased in the same ratio  $n$ -times as great, the potential will then be diminished in the same ratio ( $= \frac{e}{n}$ ), and the current-strength heightened to  $ni$ .

**275. Rotating-Current Motors.**—The currents furnished by an alternating-current dynamo, and thus transformable at will, may be used just as advantageously as a constant current for the direct operation of arc and incandescent lamps. For purposes of transmission, it must be led through a precisely equivalent alternating-current motor, which is thereby brought to exactly the same number of revolutions as that of the first machine; for only then can the direction of the current in the spools reverse simultaneously (synchronously) with the poles of the electromagnet. This sort of reversal is necessary, if the force is to drive the ring of coils always in the same direction. If this exact agreement is disturbed in the least, by loading the motor too heavily, the force reverses its direction, draws the ring of coils backward instead of driving it forward, and the motor comes to rest.

The problem of the transmission of power by means of alternating currents of high potential, was very simply solved

by the invention of *rotating current motors* (Ferraris, 1887; Dolivo-Dobrowolsky, 1890). A fixed iron ring carries two pairs of coils, AA and BB (Fig. 252), placed at right angles to each other. An alternating current passes through each pair. One of these currents remains one quarter of a period of vibration behind the other, so that the current strength in A is greatest when it is zero at B, and conversely. At the instant when this occurs (first Fig.), the coil, A, produces a magnetic pole at B, and, within the ring, a magnetic field is simultaneously developed, whose lines of force have the direction

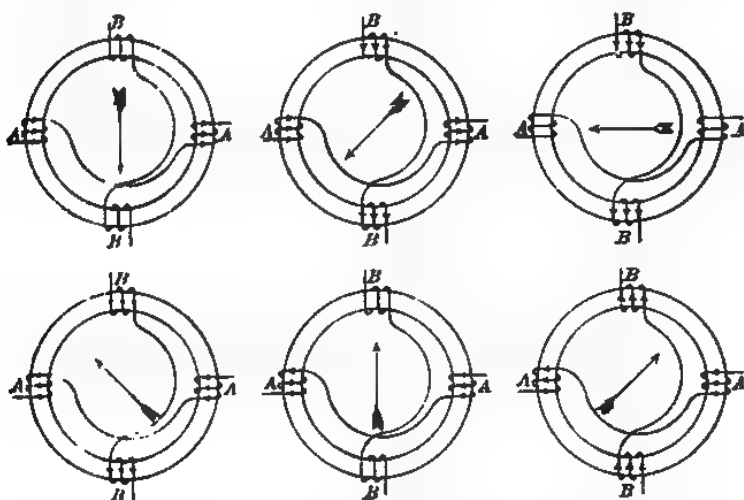


FIG. 252.—Rotating Current.

indicated by the arrow. An eighth of a period later, the currents in both pairs of coils are equally strong and produce a magnetic field, whose position is indicated in the second figure. After one-fourth of a period the current-strength is greatest at BB, and equals zero at AA, where the poles are now formed, and the field assumes the direction indicated in Fig. 3. After three-eighths of a period, the current at AA has reversed its direction, and the field then has the direction shown in the fourth figure. And so the rotation of the magnetic field continues, until, after the lapse of an entire period, it has returned

to the position indicated in the first figure (fifth and sixth figures). If, now, an iron cylinder (with, or without windings), movable about its axis, is brought within the iron ring, it will be carried about in its own direction by the rotating magnetic field. Such a "rotating-current motor" is the simplest conceivable electro-magnetic machine.

By means of apparatus of the sort depicted, the General Electric Company (*Allgemeine Elektrizitäts-Gesellschaft*) of Berlin, in conjunction with the Oerlikon machine factory, in 1891, succeeded in transmitting approximately 300 horse-power of work, from Lauffen, on the Neckar, to the Electro-technical exhibition in Frankfort-a.-M., over a distance of 175 km. A turbine of 300 horse-power operated an alternating-current dynamo at Lauffen which furnished a ("tri-phase") rotating current of 150 volts potential, and 1400 ampères current-strength in all, and, therefore, a current-energy of about 200,000 volt-ampères, or watts. This current was transformed to about 20,000 volts potential, and led away through three pure copper wires of only 3 mm. diameter and well insulated. In Frankfort, the potential was reduced by a transformer to 100 volts, with a corresponding increase in current-strength. A part of the transformed current fed 1000 incandescent lamps, another part operated a pump by means of a rotating-current motor, which supplied the flow for an artificial waterfall of ten meters height. The latter portion of the transmitted energy completed the circle, therefore, for a portion of the energy of the waterfall in Lauffen appeared in Frankfort again as the energy of a waterfall.

**276. Earth Induction.** -Terrestrial magnetism also produces induced currents in a closed conductor, when it is moved in the uniform magnetic field of the earth. A circular frame, MN (Fig. 253), of very large diameter, can be used as an earth inductor, if a number of windings of insulated copper wire be placed about its circumference, and the ring be rotated by means of a crank about a diameter as an axis. When this axis is perpendicular to the magnetic meridian, and the plane of the frame is perpendicular to the inclination of the needle, if the axis be turned quickly through half a revolution, the presence of

an induced current may be detected by a (ballistic) galvanometer. The strength of the induced current will be proportional to the total intensity of the terrestrial magnetism. If the frame

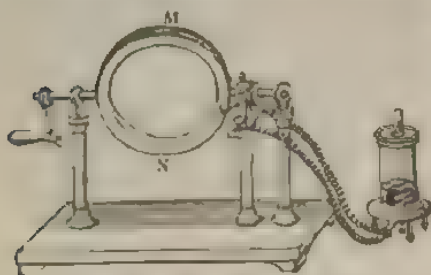


FIG. 258.—Earth Inductor.

be turned half a revolution farther forward (or backward) at the same rate as before, an induced current of equal intensity and of opposite direction will be produced. If the latter rotation is made at the instant when the galvanometer needle, deflected by the first in-

duced current, passes its position of equilibrium on the backward swing, the deflection toward the opposite side will, of course, be greater, because the needle was in motion when it received the second impulse. If this "multiplying process" be continued, after a time a maximum deflection will be reached, which may be taken as a standard for the intensity of terrestrial magnetism. When the frame is turned continuously with uniform velocity, induced currents are obtained, which oscillate, or pulsate, to and fro through the wire, according to the laws of the pendulum. These currents can be given the same direction by means of a commutator attached to the axis, and it is thus possible, without the use of iron, by the mere rotation of a copper wire in the magnetic field of the earth, to produce the same effects as arise from the constant current of a galvanic battery.

If the plane of the windings be placed horizontally, and the axis of rotation is in the magnetic meridian during a half revolution, only the vertical component ( $V$ ) of the earth's magnetism can act inductively. But, if the axis of rotation be placed vertically, and the plane of the frame is at the same time perpendicular to the magnetic meridian, during each rotation through  $180^\circ$  only the horizontal component ( $H$ ) of the earth's magnetism is effective. The galvanometric deflections in both cases are to each other, therefore, in the ratio of these

components ( $V : H$ ), equal velocities of rotation being assumed. But the tangent of the inclination of the needle is also expressed by this same ratio ( $\tan i = V : H$ ). By this mode of procedure, first given by W. Weber (1838), the magnetic inclination may be determined with an earth-inductor.

If the apparatus be placed so that the axis of rotation coincides with the terrestrial magnetic inclination, no deflection of the galvanometer is obtained. In the halves of the circular wire, into which the axis of rotation divides it, arise, according to Faraday's rule (261), opposite induced currents, which neutralize each other, because the lines of force of the field are cut in opposite directions.

**277. Determination of the Absolute Unit of Resistance by means of an Earth-Inductor.**—As was shown above (245), the magnetic moment of a circular current equals the product of the current-strength ( $i$ ) by the area ( $F$ ) of the surface circulted by the current. If the current is turned from its initial position at right angles to the magnetic meridian, about a vertical axis, through  $90^\circ$  of angle, the work expended is expressed by the product,  $iFH$ , of its moment, into the horizontal intensity,  $H$ , of the earth's magnetism. If at the beginning, no current were passing in the circular conductor, with this amount of rotation performed in the time  $\tau$ , a current will be induced in it of (mean) intensity,  $i$ , and of (mean) electromotive force,  $e$ , whose energy of flow equals the work expended, i.e. we should have  $e i \tau = i F H$ , and, consequently, the mean electromotive force of the induced current,  $e = \frac{F H}{\tau}$ . Hence, when the horizontal intensity is known, and the surface enclosed within the windings has been measured, the electromotive force,  $e$ , of the current may be computed in absolute units from the latter equation. If, now, the intensity,  $i$ , is also determined galvanometrically in absolute units (246), with the aid of Ohm's law, the resistance of the entire conductor is found from  $r = \frac{e}{i}$ , likewise in absolute units. Ten times the unit of resistance thus determined, equals exactly one ohm (W. Weber).

**278. Induction in Extended Conductors—Foucault's Currents—Rotatory Magnetism.**—Currents, known as Foucault's currents, directed always so as to retard the relative motion of conductor and magnet, are induced not only in closed coils of wire, but also in any extended conducting body with respect to which a magnet in the immediate neighbourhood changes its position. When, for example, a brass plate is inserted between the poles of a strong electro-magnet, a resistance is perceived similar to that experienced on cutting a tough substance, such

as cheese. A copper disk placed between the poles, when rotated rapidly, is suddenly brought to rest when the electromagnet is energized. The energy of motion, lost by the conductor on account of this "magnetic frictional resistance," is transformed into heat (Joule's heat) just as is the case with ordinary friction. The temperature of the moving conductor therefore rises. A hollow copper cylinder covered with an easily fusible metallic coating, when rotated rapidly between the poles, is heated to such an extent that the metallic coating melts.

The reaction of currents induced in a movable conductor by magnets is also capable of setting these magnets in rotation. For example, let a magnetic needle, movable in a horizontal plane, be placed over a horizontal copper disk, which may be set in rapid rotation by means of a centrifugal machine. When the disk is turned, the needle will also be seen to turn *in the same sense* as the disk. Conversely, a horizontal magnet, rotating rapidly about a vertical axis, will carry a copper disk, movable in a horizontal plane above it, along in its own direction. Arago (1825) called these phenomena, which were discovered by him and explained later by Faraday, to be due to induced currents, the effects of *rotatory magnetism*.

**279. Damping.**—If a horizontal magnetic bar swings freely within a fixed copper envelope, the Foucault's currents induced in the envelope by the magnet, exert a retarding influence upon its motion, and the bar comes to rest far sooner than when allowed to vibrate freely. This means of *damping* the vibrations is used in galvanometers to bring the magnet quickly into its position of equilibrium. If the damping is so strong as to bring the magnet immediately into equilibrium without vibration, the galvanometer is described as "*aperiodic*." The device for damping is called a *damper*.

**280. The Telephone** transmits by the agency of induced currents, spoken words, or other sounds, distinctly audibly to a distant station. Bell's telephone (1875) consists of a steel magnet, A (Fig. 254), upon the soft iron prolongation of one of whose poles a coil of thin wire is wound. A thin circular plate, EE, of soft iron is held about its circumference just in front of this pole. When sounds are uttered against the plate through the

front mouthpiece, called the *transmitter*, the plate vibrates like the head of a drum. The periodic changes of potential caused by the trembling of the plate in the field of the magnet, induce currents in the coil of wire, which are transmitted through the wires, CC, passing through the wooden envelope and thence over a conducting wire connected to the binding-screws, DD, to the remote station. These currents there pass through the coils of a similar instrument and alter the field of its magnet rhythmically with the alterations of field of the first magnet. The iron plate of the second telephone is set into exactly corresponding vibrations, which are audible in an ear-piece, called the *receiver*, and performing the office of an ear-trumpet. The sound produced at the sending station, *e.g.* that of the human voice, is thus propagated to the receiving station with all its characteristic peculiarities of tone.



FIG. 254.—Telephone.

Nothing is heard in the telephone when a constant current passes through its coils, excepting a crackling noise at the instant of opening or closing the circuit. A sound is heard only when an intermittent, or an *alternating current*, or generally, any current subject to periodic vibrations, is transmitted through the instrument. Since the apparatus responds to alternating currents only, it may be used with Wheatstone's bridge, to determine the galvanic resistance in electrolytes (266), instead of an electro-dynamometer (F. Kohlrausch, 1880). The resistance of the *rheostat*, or of the measuring wire, must merely be altered until the telephone remains silent.



FIG. 255.—Microphone.

**281. The Microphone** (Luettlge, Hughes, 1878) in its simpler form consists of a small bar, *k* (Fig. 255), of coke, pointed at both ends, in gentle contact with two pieces of carbon, *K* and *K'*,

fixed to a small wooden plate. The carbons, K and K', are connected with the terminals of a battery wire to a telephone, T, also in circuit. If a word is spoken near the carbon pieces, it is heard in the telephone. The cause of this effect is that the alteration of the intimacy of contact between the carbons due to the sound waves and the consequent variations of the galvanic resistance produces periodic fluctuations of intensity, which in their turn cause corresponding vibrations of the plate of the telephone.

In actual practice, a microphone (*transmitter*) is used as speaking apparatus, and the telephone proper is used only for hearing. Of the numerous forms of microphones now in use, only the universal transmitter of Berliner will here be explained. In a vial-shaped rubber capsule, coarsely-powdered carbon is placed between two horizontal carbon plates. When the lower plate is spoken against through a funnel-shaped mouthpiece, the powdered carbon readily imitates the vibrations of the plate, and produces corresponding fluctuations of battery current. This current, instead of passing directly through the wire coils of the telephone, is sent through the primary coil of a small inductor placed within the case of the transmitter, so that, at each station, the battery is closed by means of the carbon contact and this primary coil, while the hearing apparatus at both stations are connected by the conducting wire with each other and with the secondary coil of the inductor. The fluctuations of current-strength in the primary coil awaken in the secondary coil induced currents which reproduce distinctly audible speech in the distant telephone. It is evident that the microphone, used as speaking apparatus, acts as a relay (247) permitting the strong current of a galvanic battery to be used instead of the very weak current of the telephone itself. The little inductor acts as a transformer, converting the strong current of the battery into a feeble current of high potential, which may be easily transmitted through a small wire to a great distance. As with the telegraph, the return circuit is made here also through the earth.

**282. Diamagnetism.**—A bar of bismuth suspended horizontally by means of a silk fibre between the poles of a powerful

electromagnet (Fig. 256, seen from above) is repelled by both poles and assumes a position at right angles to the connecting line, NS, of the poles, while, as is well known, an iron bar places itself in the line, NS, of the poles. As has been stated above, the connecting line of the poles of a magnet is its *axis*, and the plane perpendicular to its axis at the middle point is called its *equator*. The position, *ab*, is therefore designated the *equatorial*, while the position, NS, is the *axial* position.



FIG. 256.—Diamagnetism.

All bodies may be separated into two groups according to the positions they assume between the poles of a magnet. *Magnetic* bodies are attracted by the magnet, and assume *axial* positions; other bodies are repelled and assume *equatorial* positions. Faraday, who discovered this phenomenon (1845), called bodies of the latter class, *diamagnetic*. Besides iron, nickel, and cobalt, whose magnetic properties have been long familiar, manganese, chromium, cerium, titanium, palladium, platinum, osmium, and almost all iron compounds are magnetic, while the diamagnetic substances are bismuth pre-eminently, then follow antimony, zinc, tin, lead, silver, copper, gold, and most other bodies.

Weber explains diamagnetism on the hypothesis of electrical currents circuiting the molecules of diamagnetic bodies. But he assumes that these currents are not already formed, as are the molecular currents of magnetic bodies which need to be merely directed by an approached magnet. Weber's assumption is that the molecular currents of diamagnetic bodies must be first induced, or called into existence by means of the approached magnet, or other source of magnetic induction. When a conductor is brought near to a magnetic pole, currents are induced in it, which are directed *against* the molecular current of the magnet. These currents (Ponceault's currents, 278) are, however, of very short duration, for while passing through the mass of the copper from molecule to molecule, they have a resistance to overcome, by which their energy is soon exhausted, i.e. transformed into

heat. Besides these ordinary induced currents passing through the mass of the copper, the magnet excites also small circular currents around the molecules of the copper, which are likewise directed against the molecular currents of the magnet; but, since they meet no resistance on circuiting the molecules, they continue to flow, until in consequence of a subsequent action of induction, new and opposite molecular currents arise, which destroy the older currents. Since the former currents are opposite to those of the magnet, it is clear that, according to the laws of the mutual action of electrical currents, repulsion must ensue. These induced molecular currents may form also in non-conductors, though these bodies do not permit the passage of electricity from one molecule to another. Glass, carbon disulphide, and other non-conductors, in fact, exhibit strong diamagnetic properties.

It may also be conceived that diamagnetic bodies in a magnetic field become magnetically weaker than the surrounding medium, e.g. the air, or a vacuum, and, therefore (following the analogy of Archimedes' principle), assume equatorial positions, just as a dilute solution of iron chloride enclosed within a glass tube, when surrounded by air becomes magnetic, while, when surrounded by a stronger solution of iron chloride, it appears diamagnetic. Diamagnetic bodies have negative magnetization numbers (149): bismuth,  $-0.0000015$ ; water,  $-0.0000001$ .

**283. Absolute System of Units.**—The *absolute system of measures*, established by Gauss and Weber, is so called because it reduces all physical magnitudes to the three fundamental ideas of *length*, *mass*, and *time*. Gauss and Weber selected the millimeter and the milligram as units of length and of mass, respectively. But to avoid inconveniently large numerical values, the centimeter and gram were later selected, after the example of the British Association. The fundamental units of the absolute system of measures are, accordingly, for length, the *centimeter* (cm.); for mass, the quantity of matter contained in 1 cm. of water at  $4^{\circ}$  C., or the *gram* (g.), and for time, the *second* (sec.), i.e. the  $\frac{1}{86400}$  part of the mean solar day.

Before deriving from these magnitudes the units of electrical and magnetic quantities, let us call to mind the absolute units of the most important motions of mechanics. By the *velocity* of a moving body at any instant of time, we mean the ratio of the distance traversed by the body in the next succeeding infinitesimal interval of time, to the length of this interval, i.e. the ratio of a *length* to a *time*. The unit of velocity is accordingly a magnitude obtained by dividing the linear unit, cm., by the time unit, sec., and is expressed by cm.; sec., or cm.-sec.<sup>-1</sup>. This expression cm.-sec.<sup>-1</sup>, which represents the mode of composition of the derived unit "velocity," from the fundamental units, is called the "dimension" of the derived unit, and to every numerical value in the absolute system, the corresponding dimension-symbol must be either appended, or understood, to denote without ambiguity the nature of the derived unit to which the numerical value refers. *Acceleration* is the

ratio of the increment of velocity of the moving body to the short period of time within which this increment is acquired, or, since a velocity-increment is itself a velocity, acceleration is the ratio of a velocity to a time. The dimension of a unit of acceleration is accordingly,  $\text{cm.} \cdot \text{sec.}^{-1} : \text{sec.}$ , or  $\text{cm.} \cdot \text{sec.}^{-2}$ . Since, now, *force* is understood to be the product of the mass moved, into the acceleration of its motion, the dimension of the force-unit, or *dyne*, which, acting on a mass of 1 g. during 1 sec. imparts to it the acceleration 1, is  $\text{cm.} \cdot \text{g.} \cdot \text{sec.}^{-2}$ . The product of a force by the distance through which it moves a mass represents the *work* of the force. The work-unit, or the *erg*, has then the dimension  $\text{cm.}^2 \cdot \text{g.} \cdot \text{sec.}^{-2}$ , and is the work performed by 1 dyne, in displacing a body through 1 cm. in the direction in which the force acts. The *energy of motion*, or *kinetic energy* (the half-product of the mass moved into the square of its velocity), has the same dimension as work. The same is also true of a *quantity of heat*, since it is equivalent to a definite amount of work. The work performed by a force in one second is called its *performance*, or *effect*. The unit of effect is the "erg per second," and has the dimension  $\text{cm.}^2 \cdot \text{g.} \cdot \text{sec.}^{-2}$ .

Since the nature of electricity and of magnetism are unknown to us, to express electrical and magnetic ideas in absolute measures, it is necessary to return to their known effects and so to specify these ideas in such way that the forces exerted and the work performed may agree with the already-defined ideas of force and work in mechanics. This will be the case when the electrical and magnetic notions are of the same dimensions in  $\text{cm.} \cdot \text{g.}$ , and  $\text{sec.}$ , as are the mechanical notions. By proceeding, then, from the effects of electrical charges upon one another (from electrostatic effects), or from the magnetic effect of the electrical current (from electromagnetic effects), two different absolute systems of measures are arrived at, viz. the *electrostatic* and the *electromagnetic* systems, both of which are equally justifiable on a scientific basis, though the latter, being of greater practical significance, is entitled to first consideration.

According to Coulomb's law, two magnetic poles act upon each other with a force directly proportional to the product of their intensities and inversely to the squares of their distance. If we select as the *unit of pole-strength* ( $m$ ), that exerted by the force 1 (dyne) upon an equally strong pole at a distance of 1 cm. (144), the force,  $m^2 : \text{cm.}^2$ , must equal the force-unit,  $\text{cm.} \cdot \text{g.} \cdot \text{sec.}^{-2}$  (dyne). To make this possible, we must ascribe to the  $m$ , the dimension  $\text{cm.}^{\frac{1}{2}} \cdot \text{g.} \cdot \text{sec.}^{-\frac{1}{2}}$ , and, accordingly, to the  $m$ , the unit of pole-strength, the dimension  $\sqrt{\text{cm.}^3 \cdot \text{g.} \cdot \text{sec.}^{-1}}$ , or,  $\text{cm.}^{\frac{1}{2}} \cdot \text{g.}^{\frac{1}{2}} \cdot \text{sec.}^{-\frac{1}{2}}$ . *Magnetic moment*, or *bar magnetism*, is the product of the distance of the poles of a magnet into its pole-strength. The unit of moment is then obtained by multiplying the unit of pole-strength by the unit of length,  $\text{cm.}$ , obtaining thus the dimension  $\text{cm.}^{\frac{3}{2}} \cdot \text{g.}^{\frac{1}{2}} \cdot \text{sec.}^{-\frac{1}{2}}$ . A definite *magnetic intensity*, or *strength of field*, corresponds to every point in a magnetic field. By virtue of this intensity, a force is exerted upon a magnetic pole situated at the point which equals the product of field-strength by pole-strength. That this product may have the dimension of a force ( $\text{cm.} \cdot \text{g.} \cdot \text{sec.}^{-2}$ ), since  $\text{cm.}^{\frac{1}{2}} \cdot \text{g.}^{\frac{1}{2}} \cdot \text{sec.}^{-\frac{1}{2}}$  is the unit of pole-strength, the unit of field-strength must have the dimension  $\text{cm.}^{-\frac{1}{2}} \cdot \text{g.}^{\frac{1}{2}} \cdot \text{sec.}^{-\frac{1}{2}}$ . The intensity of the earth's magnetism at any point upon its surface is of this dimension.

The effect of a circular conductor, traversed by an electrical current upon a magnetic pole, may be replaced by the effect of magnetic bar, whose moment equals the product of current-strength into the area of the surface enclosed by the circuit. The current-strength is, therefore, a magnitude,

which, multiplied by a surface, i.e. by the square of a length, has the same dimension as a magnetic moment, viz.  $\text{cm.}^2 \cdot \text{g.}^{\frac{1}{2}} \cdot \text{sec.}^{-1}$ . Its dimension is, therefore,  $\text{cm.}^{\frac{1}{2}} \cdot \text{g.}^{\frac{1}{2}} \cdot \text{sec.}^{-1} : \text{cm.}^2$ , or  $\text{cm.}^{\frac{1}{2}} \cdot \text{g.}^{\frac{1}{2}} \cdot \text{sec.}^{-1}$ . The unit of current-strength,  $\text{cm.}^{\frac{1}{2}} \cdot \text{g.}^{\frac{1}{2}} \cdot \text{sec.}^{-1}$ , is the strength possessed by an electrical current, which, flowing through a circular arc of radius 1 cm. and of length 1 cm., exerts upon a magnetic pole of strength 1, situated at the centre of the circle, the force 1 dyne. The electrical mass, flowing through a conductor in a certain time, equals the product of this time into the current strength. The unit of electrical mass is, therefore, that mass which furnishes a current of strength 1 in 1 sec., and its dimension is  $\text{cm.}^{\frac{1}{2}} \cdot \text{g.}^{\frac{1}{2}} \cdot \text{sec.}^{-1} \times \text{sec.}$ , or  $\text{cm.}^{\frac{1}{2}} \cdot \text{g.}^{\frac{1}{2}}$ . The effect of heat produced by an electrical current, is, according to Joule's law, proportional to the current-strength and to the resistance of the conductor. The unit of resistance must, therefore, possess the dimension  $\text{cm.} \cdot \text{sec.}$ , i.e. that of a velocity, in order that the product of this unit by the square of current-strength ( $\text{cm.}^2 \cdot \text{g.} \cdot \text{sec.}^{-2}$ ) may be equal to the dimension of thermal effect ( $\text{cm.}^2 \cdot \text{g.} \cdot \text{sec.}^{-2}$ ). The unit of resistance is, then, that of a conductor, in which a current of strength 1 develops in 1 sec. a quantity of heat equivalent to the unit of work (erg). According to Ohm's law, which requires that the electromotive force (difference of potential, tension) shall equal the product of current-strength and resistance of conductor, the unit of electromotive force, or of electrical potential, must be of the dimension  $\text{cm.}^{\frac{1}{2}} \cdot \text{g.}^{\frac{1}{2}} \cdot \text{sec.}^{-1}$ . It is thus so defined as to produce in a conductor of resistance 1, an electrical current of intensity 1, and a quantity of heat equivalent to the unit of work. By the capacity of a conductor is meant the ratio of the electrical mass upon it, to the electrical tension attained by this mass. The unit of capacity has, then, the dimension  $\text{cm.}^{\frac{1}{2}} \cdot \text{g.}^{\frac{1}{2}} : \text{cm.}^{\frac{1}{2}} \cdot \text{g.}^{\frac{1}{2}} \cdot \text{sec.}^{-1}$ , or  $\text{cm.}^{-1} \cdot \text{sec.}^2$ . It belongs, consequently, to a conductor (or to a condenser), which is charged by the unit of electrical mass to the unit of potential.

The dimensions of the units defined above are collected in the following table:—

Units of the Absolute Electro-magnetic System.				Dimension.
m	pole-strength	...	...	$\text{cm.}^{\frac{1}{2}} \cdot \text{g.}^{\frac{1}{2}} \cdot \text{sec.}^{-1}$
M	magnetic moment	...	...	$\text{cm.}^2 \cdot \text{g.}^{\frac{1}{2}} \cdot \text{sec.}^{-1}$
T	field-strength	...	...	$\text{cm.}^{-\frac{1}{2}} \cdot \text{g.}^{\frac{1}{2}} \cdot \text{sec.}^{-1}$
E	electrical mass	...	...	$\text{cm.}^{\frac{1}{2}} \cdot \text{g.}^{\frac{1}{2}}$
J	current-strength or intensity	...	...	$\text{cm.}^{\frac{1}{2}} \cdot \text{g.}^{\frac{1}{2}} \cdot \text{sec.}^{-1}$
R	resistance	...	...	$\text{cm.} \cdot \text{sec.}$
V	electromotive force (potential)	...	...	$\text{cm.}^{\frac{1}{2}} \cdot \text{g.}^{\frac{1}{2}} \cdot \text{sec.}^{-1}$
C	capacity	...	...	$\text{cm.}^{-1} \cdot \text{sec.}^2$

These absolute electromagnetic units, based upon the cm., g., and sec., lead in practical work with electrical and magnetic magnitudes to inconveniently large numerical values, although this is not so much the case in the absolute system as in that of Gauss and Weber. The resistance of a Siemens' unit (324 for example, amounts to 943,400,000 absolute units of resistance or  $9434 \times 10^5 \text{ cm.} \cdot \text{sec.}^{-1}$ ; the electromotive force of a Daniell's element to 110,400,000, or  $1104 \times 10^5 \text{ cm.}^{\frac{1}{2}} \cdot \text{g.}^{\frac{1}{2}} \cdot \text{sec.}^{-1}$ . The British Association and, after it, the Paris Electrical Congress (1881), have, therefore, established suitable decimal multiples and sub-multiples of the absolute units as *practical units*, and to these are ascribed the names of celebrated physicists, whose special study in the science of electricity has made them worthy of the

honour. A resistance of 1000 million absolute units, i.e. of the magnitude  $10^9 \text{ cm.}^2 \cdot \text{sec.}^{-1}$ , which differs only by a few per cent. from the Siemens' unit, was chosen as a higher unit of resistance, and is called the *ohm*, 100 million-fold the value of the absolute unit of electromotive force, differing considerably from 1 daniell, forms, under the name *volt*, the practical unit of electromotive force, so that 1 volt is  $10^8 \text{ cm.}^2 \cdot \text{g.}^{\frac{1}{2}} \cdot \text{sec.}^{-1}$ . The strength of the current, which produces the electromotive force of 1 volt in a circuit of 1 ohm's resistance, is called the *ampère*. One ampère is one-tenth part of the absolute unit of current-strength, or  $10^{-1} \text{ cm.}^{\frac{1}{2}} \cdot \text{g.}^{\frac{1}{2}} \cdot \text{sec.}^{-1}$ , and is a little more than ten times as great as the oxy-hydrogen unit of Jacobi. The quantity of electricity flowing in 1 second through the cross-section of a wire with current-strength 1 ampère is called the *coulomb* (Fig. 212). It equals one-tenth of the absolute electrical unit, or  $10^{-1} \text{ cm.}^{\frac{1}{2}} \cdot \text{g.}^{\frac{1}{2}}$ . Finally, the capacity of a conductor, which, under the influence of the electromotive force of 1 volt, takes up the quantity of electrification of 1 coulomb, is termed the *farad*. One farad is the 1000 millionth part of the absolute unit of capacity, or  $10^{-9} \text{ cm.}^{-1} \cdot \text{sec.}^2$ . These practical units of the electromagnetic system are collected in the following table, together with their absolute values:—

Unit of—	Name.	Value in absolute units.
Electrical mass	coulomb	$10^{-1} \text{ cm.}^{\frac{1}{2}} \cdot \text{g.}^{\frac{1}{2}}$
Current-strength	ampère	$10^{-1} \text{ cm.}^{\frac{1}{2}} \cdot \text{g.}^{\frac{1}{2}} \cdot \text{sec.}^{-1}$
Resistance	ohm	$10^9 \text{ cm.}^2 \cdot \text{sec.}^{-1}$
Electromotive force	volt	$10^8 \text{ cm.}^2 \cdot \text{g.}^{\frac{1}{2}} \cdot \text{sec.}^{-1}$
Capacity	farad	$10^{-9} \text{ cm.}^{-1} \cdot \text{sec.}^2$

In accordance with the decision of the first Conference of Electricians (1882), the ohm was to have been defined by the resistance of a mercurial column of 1 sq. mm. cross-section at 0° C. The measurements (277) then at disposal did not, however, show the necessary degree of accordance to warrant proceeding upon them to the establishment of a normal system of measures. The Conference, which considered an accuracy to 1 mm. to be required, recommended the continuation of investigation, and reserved the establishment of the ohm to an international commission. Pursuant to this conclusion, by the time of the second Conference of Electricians in 1884, an extensive series of careful observations were at hand, but the degree of accuracy required had not yet been attained. The Conference, however, considered the degree of correspondence attained by this time, to suffice for the needs of practice, and decided the "*legal ohm*" to be the resistance of a column of mercury 106 cm. long and of 1 sq. mm. cross-section at 0° C. (224). The *legal volt* was taken as the electromotive force, which, in a circuit of 1 (legal) ohm resistance, produces the unit of current-strength, or 1 ampère. To establish this latter unit, it was referred to the foregoing scientific definition, because the direct determination of its theoretical value appeared easier than its derivation from the ohm and volt.

In the *absolute electrostatic system*, the definition of electrical mass is based upon the mutual effect of electrical charges. If we take as the electrostatic unit of electrical mass,  $E$ , that mass, which exerts upon another mass equal to it, at a distance of 1 cm., the force of 1 (dyne) (162, according to Coulomb's law,  $E^2 : \text{cm.}$  must equal this unit of force, giving the dimension  $\text{cm.} \cdot \text{g.} \cdot \text{sec.}^{-2}$ , or  $E^2 = \text{cm.}^2 \cdot \text{g.} \cdot \text{sec.}^{-2}$ . The *unit of electrical mass* has, therefore, in this system the dimension  $\text{cm.}^{\frac{1}{2}} \cdot \text{g.}^{\frac{1}{2}} \cdot \text{sec.}^{-1}$ . Since the current-strength ( $I$ ) here also indicates the quantity of electricity flowing through a conductor in the unit of

time, the *unit of current-strength* is  $E' : \text{sec.}$ , or  $\text{cm.}^{\frac{1}{2}}\text{-g.}^{\frac{1}{2}}\text{-sec.}^{-2}$ . The *unit of electrical capacity*,  $C'$ , equals the linear unit,  $\text{cm.}$ , for we know that the capacity of a sphere equals its radius; and since the electrical mass upon a conductor equals the product of its capacity and its potential ( $E' = C'V'$ ), the *unit of electrostatic potential* must be of the dimension,  $V' = E' : C'$ , or  $\text{cm.}^{\frac{1}{2}}\text{-g.}^{\frac{1}{2}}\text{-sec.}^{-1}$ . According to Ohm's law, the resistance  $R' = V' : J'$ . Hence the *unit of resistance* has the dimension  $\text{cm.}^{\frac{1}{2}}\text{-g.}^{\frac{1}{2}}\text{-sec.}^{-1} : \text{cm.}^{\frac{1}{2}}\text{-g.}^{\frac{1}{2}}\text{-sec.}^{-2}$ , or  $\text{cm.}^{-1}\text{-sec.}$ . We have then the following summary of units of the electrostatic system :—

$E'$	electrical mass	...	...	...	$\text{cm.}^{\frac{1}{2}}\text{-g.}^{\frac{1}{2}}\text{-sec.}^{-1}$
$J'$	current-strength	...	...	...	$\text{cm.}^{\frac{1}{2}}\text{-g.}^{\frac{1}{2}}\text{-sec.}^{-2}$
$R'$	resistance	...	...	...	$\text{cm.}^{-1}\text{-sec.}$
$V'$	potential	...	...	...	$\text{cm.}^{\frac{1}{2}}\text{-g.}^{\frac{1}{2}}\text{-sec.}^{-1}$
$C'$	capacity	...	...	...	$\text{cm.}$

These magnitudes are thus seen to furnish measures in the electrostatic system, in terms of units of different dimensions from those in the electromagnetic system. The ratios of the numerical measures of like-named magnitudes in the two systems are, therefore, not abstract numbers, but have the following dimensions :—

$$\begin{aligned} \frac{E'}{E} &= \text{cm.-sec.}^{-1}, & \frac{J'}{J} &= \text{cm.-sec.}^{-1}, & \frac{R'}{R} &= \text{cm.}^{-2}\text{-sec.}^2, \\ \frac{V'}{V} &= \text{cm.}^{-1}\text{-sec.}, & \frac{C'}{C} &= \text{cm.}^2\text{-sec.}^{-2}. \end{aligned}$$

It will be observed that all these ratios are expressible by a number of the dimension  $\text{cm.-sec.}^{-1}$ , i.e. by a velocity. This magnitude was determined by a comparison of electrical masses (W. Weber and R. Kohlrausch), measured in terms of electrostatic and electromagnetic units; of potentials (W. Thomson, Maxwell), and of capacities (Ayrton, Perry, J. Thomson), and it was found equal to  $3 \cdot 10^{10} \text{ cm.-sec.}^{-1}$ , or 300,000 km.-sec. It is, therefore, almost equal to the velocity of propagation of light. The electromagnetic unit of electrical mass contains, then, 30,000 millions of electrostatic units, and a current of 1 ampère intensity carries, in 1 sec., 3000 million such units through each cross-section of the conductor.

## IX. WAVES AND SOUND.

## A. WAVE MOTION.

**284. Wave Motion.**—A wave motion arises whenever a swinging motion is propagated with uniform velocity along a succession of points, or in a continuous medium of any kind, provided that the motion is of such character that each succeeding particle in the direction of propagation begins its vibration later than the foregoing by an interval of time corresponding to its distance from the particle first set in motion. A field of waving grain furnishes a graphic picture of the progress of events with wave motion. Each head of wheat is bent downward by the wind, but is again raised by virtue of the elasticity of the stalk, bends again downward, etc., executing thus regularly its repeated vibrations. The following heads are set in vibration by the impulse of the wind which started the first, by an interval of time after the first, which increases the farther down the line of heads it stands. This regular alternation of rise and fall of the heads of grain gives to the surface of the field at any instant the form of alternate elevations and depressions. A wave is seen to pass across the field with the velocity of the wind, while each particular head, rooted to its place, vibrates only within a very limited region.

Wave motion may also be well imitated by means of so-called wave machines. In Mach's machine a row of spherical pendulums is suspended along a graduated scale, or bar, each by two threads, so that each may swing only in a direction perpendicular to the line of centres of suspension. If all of the spheres be drawn aside equally by means of a bar, and the bar be then drawn in the direction of the line of suspension (AB, Fig. 257) with uniform velocity, one after the other is

released, and, since each vibrates perpendicularly to AB, so that seen from above at any moment in their totality the appearance is as that in Fig. 257, with curves first toward the one side, and then toward the other, the curves seeming to glide smoothly along the line of balls.

In case of the field of waving grain, and also in the row of pendulums, the question as to what transmits the motion from each particle to the next does not arise. In a medium whose



FIG. 257.—Series of Waves.

particles mutually act and react upon each other by virtue of the forces acting between them, this transmission is mediated by these forces themselves. Liquid waves furnish an exemplification of this fact.

If a stone is thrown upon the smooth surface of standing water, the liquid at the point where the stone strikes is depressed, but it is immediately compelled to rise by the pressure of the surrounding water. Having reached its original level, instead of remaining at rest, it continues to move upward, until gravity acting against it, compels it again to sink. The particle of water first displaced from its position of rest by the stone, executes a succession of such upward and downward vibrations. But the equilibrium of the surface of the water cannot be disturbed at one place without a disturbance being transmitted to neighbouring particles, because of the equal transmission of the pressure of the water in all possible directions. This transmitted disturbance compels the surrounding particles to vibrate in unison with the particle first disturbed, each remoter particle beginning its vibration a little later than the one immediately before it. Every rise of the first particle occasions a sinking of those surrounding it, and, since the effect advances in all directions, a circular depression is produced about the centre of disturbance. This depression causes, likewise, a corresponding elevation of the particles around it, which follows the preceding *trough* in the form of a *crest*. While, therefore, the original particle is completing an entire vibration consisting

of an elevation and a depression, it produces a complete wave formed of a *crest* and a *trough* and, continuing to vibrate, new circular waves continue to emanate from it. These waves expanding outward with uniform velocity *advance* over the surface of the water. It is, however, only the *form* of the liquid surface that advances, and *not the water itself*. The particles of water leave their places no more than do the stalks of a waving grain field. They merely vibrate upward and downward, as may be readily seen by means of a small piece of floating wood, which imitates the vibratory motion of the water particles. The complete series of rings, proceeding from the same centre of disturbance, forms what is called a *wave-system*. A straight line, drawn from the centre of the system upon the surface of the water, is called a *wave-ray*. All particles which, in a state of rest, lie on this line are during vibration partly above and partly below it, according as they constitute a portion of a wave crest, or of a wave trough at the instant considered. They form, therefore, in their continuous succession a *wave-line* curved upward and downward. The portion of the ray intercepted by a complete wave, i.e. by a crest and a trough, is called a *wave-length*.

To observe the motion of the particles of water during the propagation of a wave, the brothers E. H. and W. Weber (1825) used a long narrow trough with side walls made of glass plates. A powder of the same specific gravity as water (amber) was mixed with the water. The particles of the powder made visible the motion of the particles of water within which they floated, and whose motions they shared. It was found that the water particles describe curvilinear paths in the vertical plane through the direction of propagation. These paths were found to be circles at the surface, but below the surface they were ellipses of smaller and smaller vertical diameters with increasing depth below the surface.

In Fig. 258, I., let the circles 0 to 12 represent the paths of 13 particles of water, which, in a state of rest, lie at equal distances apart, upon the horizontal surface of the water. Let us consider the positions of all the particles when the one at 0, after having completed an entire circuit and returned again

into the plane of the water, is just on the point of starting upon its second circuit. If, during the first circuit, the motion has been propagated to the particle 12, the latter is just on the point of starting upon its first circuit, i.e. it is an entire circuit behind the motion of the particle at 0. The particle 1, being only  $\frac{1}{12}$  as far from 0 as 12, is only  $\frac{1}{12}$  of a circuit behind the particle 0, and has, therefore, completed  $\frac{1}{12}$  of a revolution. Similarly, the particles 2, 3, 4, . . . have at the instant considered, performed respectively only  $\frac{2}{12}$ ,  $\frac{3}{12}$ ,  $\frac{4}{12}$ , . . . of a revolution, and are situated at the places indicated in the drawing by black dots. We should find the positions of all intermediate particles not shown in the drawing, if the points designated should be connected by a continuous curve giving the line of the wave. After  $\frac{1}{12}$  of the period of revolution, each of the points will have advanced by  $\frac{1}{12}$  of the circumference, and the points will now lie upon the dotted line of the wave, which differs from the former only in that it is displaced forward in the direction of propagation.

We recognize immediately that during a period of revolution a complete wave consisting of a crest and a trough is produced. The wave length is then the distance through which the motion advances through the series of particles, while any particle is completing a whole revolution, or an entire vibration. If  $\lambda$  denote the wave length,  $V$  the velocity of propagation, and  $T$  the period, or time of vibration, we have  $\lambda = VT$ . If  $n = \frac{1}{T}$ , is the number of vibrations, i.e. the number of revolutions per second, since each vibration produces a wave; within the distance,  $V$ , by which the motion is propagated in 1 sec., as many wave lengths must be contained as there are units in the number of vibrations, or  $V = n\lambda$ .

While a particle of water is describing its circle, it is driven simultaneously vertically upward and downward, and horizontally forward and backward, in the direction of propagation. We may, if we choose, regard the uniform motion in a circle, as was done (40) with the pendulum, as compounded of two rectilinear vibrations perpendicular to each other, and both of the same duration. The former vibrations perpendicular to the direction

of propagation, and hence, also in the direction perpendicular to the wave-ray, are called *transverse*; while the latter, which take place in the direction of propagation, and hence parallel to the ray itself, are called *longitudinal*. Waves composed of these vibrations are similarly designated. A wave of water is compounded of a longitudinal and a transverse wave.

In other modes of vibration, these simple waves may exist separately. If the vertical displacement upward and downward from the positions of equilibrium of the particles be plotted alone, as in Fig. 258, II., we obtain the geometrical representation of a transverse wave. If only the horizontal displacement be plotted, we obtain a picture of the horizontal wave. Since in the latter case no particles move out of the direction of the ray, longitudinal waves do not exhibit wave-like forms; they do not have crests and troughs; but it is readily observed that the particles indicated in Fig. 258, II., by small circles, from 0 to 3 and from 9 to 12 are separated farther from one another, while those from 3 to 9 are crowded together more closely than when they are in a state of rest. In the medium through which it advances, therefore, a longitudinal wave produces

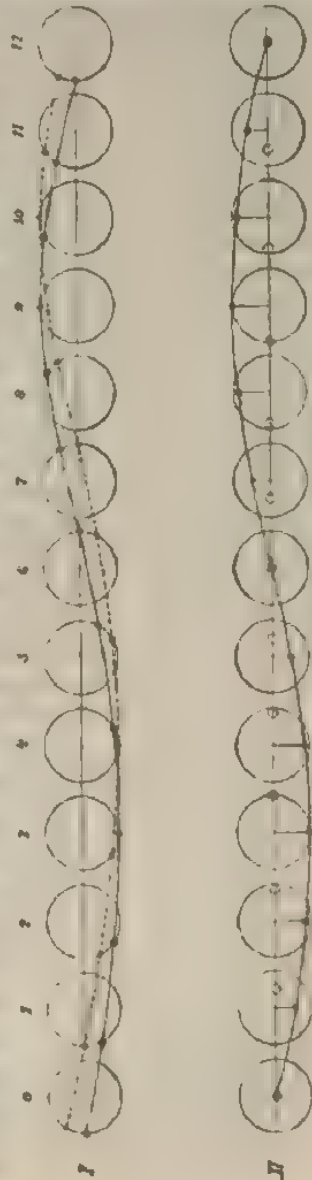


FIG. 258.—Origin of Progressive Waves.

alternate condensations and rarefactions, which have their greatest values at the points O and B, removed from each other by half a wave-length.

Mach's wave machine, in the above experiment, furnished a representation of a transverse wave, for all the balls vibrated perpendicularly to the line, in which they lay when at rest. The waves of a cord which are obtained, for example, by vibrating with the hand, in a direction perpendicularly to its length, a long rubber tube fastened at its ends, are transverse waves. Mach's apparatus is so arranged that, after the vibrations are started in the wave indicated above, all the planes of vibration may turn simultaneously through  $90^\circ$ . The balls then swing along the line of the supports, and the former transverse wave is transformed into a longitudinal with alternate condensations and rarefactions. In Fig. 258, II., this rotation through  $90^\circ$  is indicated by a dotted circular arc. A real longitudinal wave, propagated not by external means, but by the elastic forces of substances, is obtained when an impulse is imparted at one end of a long spiral wire, in the direction of the axis of a spiral, which is suspended horizontally by threads. The swinging to and fro of the separate windings gives rise to the appearance of a condensation and rarefaction of the spiral while the wave passes along in the direction of its length.

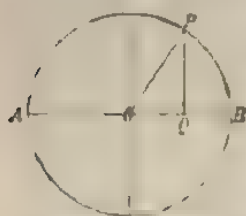


FIG. 259. - Law of Vibration.

If a particle, P, moves uniformly in a circle (Fig. 259) of radius,  $a$ , and centre at O, and if after the time,  $t$ , the arc, AC, corresponding to the angle,  $\alpha$ , has been traversed from A, its displacement perpendicular to the direction of propagation, AB, is indicated by the perpendicular, PQ =  $y = a \sin \alpha$ , dropped from P upon AOB. But if T denote the period of revolution, or the time of

vibration, we have  $\alpha : 360^\circ :: t : T$ , whence  $\alpha = \frac{360^\circ t}{T}$ . The displacement

after the time,  $t$ , is therefore  $y = a \sin \frac{360^\circ t}{T}$ , or, if the angle is expressed in arc,  $y = a \sin \frac{2\pi \cdot t}{T}$ .

The maximum deviation,  $a$ , which occurs when  $t = \frac{1}{4}T, \frac{3}{4}T, \frac{5}{4}T, \dots$  is called the amplitude; the instantaneous condition of motion expressed by  $\frac{2\pi t}{T}$  is called the phase, and  $t$  is the corresponding time of phase. If a particle

in the direction of the ray is situated at the distance  $x$  from the particle just considered, its time of phase is  $t - \tau$ , if  $\tau$  is the time required for the swinging motion to be propagated through the distance  $x$ . Since the motion advances through the succession of particles during the interval of one vibration,  $T$ , by the wave-length,  $\lambda$ , we must have  $\tau : T = x : \lambda$ , or  $\frac{\tau}{T} = \frac{x}{\lambda}$ , and the deviation

at the place of the ray given by  $x$  is  $y = a \sin 2\pi \left( \frac{t}{T} - \frac{x}{\lambda} \right)$ .

This equation gives for each particular value of  $x$  the displacement during the time of the particle located by  $x$ , and for each particular value of  $t$  the positions for this same instant of all the successive particles along the ray, whence the form of the transverse wave is a *sinusoid*. This equation is called simply the equation of a wave-ray. It holds, moreover, just as well for a longitudinal wave, since the latter arises from a transverse wave by a mere rotation of the direction of vibration through  $90^\circ$  without any alteration of the amount of the deviation (cf. Fig. 258, II.). The displacement,  $y$ , is periodic, both in respect to  $t$  as also in respect to  $x$ . If, with a constant value of  $x$ , the time,  $t$ , is increased by  $T$ , or with a constant value of  $t$ , the distance  $x$  is increased by  $\lambda$ ,  $y$  passes through the entire period of its values lying between  $-a$  and  $+a$ .

If we consider the two particles which at the instant in question lie at the tops of two adjacent crests, both particles will be found to be just on the point of departing from this highest position to move downward. These particles, which are obviously at a distance of one wave-length apart, are in the same phase (*condition of vibration*). This holds generally of every pair of particles separated by one, or more, entire wave-lengths from each other. Their motions take place in complete unison. If, on the contrary, we select two particles separated by a distance of one-half a wave-length; one, for example, at the top of a crest and the other at the bottom of the adjacent trough, we shall find them to be in exactly opposite conditions of vibration. While the former is starting downward from its highest position, the latter is just on the point of starting upward from its lowest position. In general, it is apparent that the motions of two particles separated by a distance equal to half a wave-length, or to any odd multiple of half a wave-length, are in all respects in complete opposition to one another.

**285. Interference.**—If two stones are thrown into standing water at some distance from each other, two systems of waves arise, which cross each other as they spread outward. At the places of intersection the surface is seen to be covered with a beautiful network of small elevations and depressions due to

the combined action, or to what is termed the *interference*, of the waves of the two systems. Wherever two crests meet, water rises to twice the height of a single crest, and wherever two troughs cross one another it sinks to double the depth of a single trough. On the other hand, where a crest meets a trough, the water is brought to its original position of equilibrium, *i.e.* the two undulations here mutually destroy each other. In general, every particle in a medium disturbed by two or more equal, or unequal, systems of waves of small deviations, suffers a displacement equal to the *sum* of all the displacements imparted to it at the instant considered by all the individual wave systems. To form this sum, all elevations are added and all depressions subtracted. The motion of the particle actually occurring is, so to speak, the *balance* of all the partial motions to which it is subject. This proposition is called the *principle of superposition* of vibrations, because it merely expresses the fact that every wave-system lies upon a surface already disturbed by undulations precisely as it would if it alone were present upon the quiet surface. Each system of waves is formed as though no other system were present, maintains its particular existence during interference with other systems, and advances, after crossing other systems and acting together with them (interfering), over the quiet surface of the water as though it had suffered no disturbance whatever. For example, we observe the wavelets caused by falling drops of rain upon the waves produced by a boat just as distinctly as upon the quiet surface. We have also observed how these large waves, when passing across a portion of the surface disturbed by the wind, take the smaller ones upon their backs, so to speak, pass apparently beneath them, leaving the surface disturbed by the wind behind them apparently untouched, advancing beyond in their original form.

**286. Stationary Waves.**—The interference of two waves of equal length and duration, when moving in opposite directions, is particularly noteworthy. If, at the instant considered, two waves have the position of Fig. 260, I., such that the crests of the one and the troughs of the other exactly coincide, the displacements are then everywhere equal and opposite, and all particles

are, for the instant, in the position of equilibrium. If now the waves indicated by the fainter full line and by the dotted line, respectively, advance in opposite senses, *e.g.* in the position of Fig. 260, II., from their interference there will arise such a wave as is indicated by the heavy full line of equal length and duration. At the points of intersection,  $B_1, B_2, B_3, \dots$  with respect to which both waves in their opposite courses remain symmetrically situated, the displacements are equal and in the same direction, and, consequently, add to each other. At these places, separated by half a wave-length from one another, the particles are driven alternately upward and

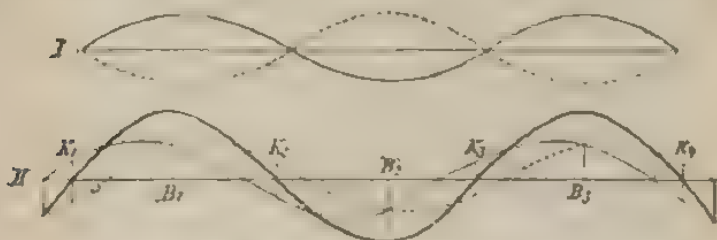


FIG. 260.—Interference of Waves which meet.

downward by double the former displacement. At the points  $K_1, K_2, K_3, \dots$  on the contrary, lying midway between the points  $B_1, B_2, B_3, \dots$  and hence also separated from one another by half a wave-length, the displacements are equal and opposite, and neutralize each other. If the waves move farther across each other, it will be seen that at the points  $K_1, K_2, K_3, \dots$  the displacements are always equal and opposite, while at the points  $B_1, B_2, B_3, \dots$  they remain always equal and in the same sense. The particles,  $K_1, K_2, K_3, \dots$  remain, therefore, always at rest in their positions of equilibrium, while the particles,  $B_1, B_2, B_3, \dots$  vibrate with increased emphasis upward and downward, reaching their maximum deviation when the waves have advanced from the position, I, each by a quarter of a wave length, so that now everywhere crest coincides with crest and trough with trough. The forms assumed by the resultant waves in the successive

stages of an entire vibration are given in Fig. 261, each after  $\frac{1}{2}$  of a vibration. It is evident that all particles pass simultaneously through their positions of equilibrium (at 0, 6, and 12), reach at the same instant their maximum deviation (at 3 and 9), and are always, at a given moment, in the same phase of vibration, the amplitude alone changing periodically from place to place. The form of the wave, therefore, does not advance,

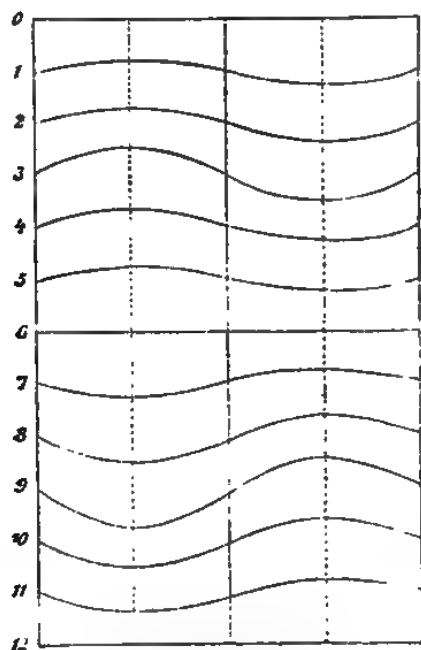


FIG. 261.—Stationary Waves.

for which reason such waves are called *stationary* in contradistinction to the *progressive waves* considered above, where each successive particle in the direction of propagation passes its position of equilibrium later than the preceding. The points  $K_1, K_2, K_3, \dots$  which remain always at rest, are called *nodes*; the points  $B_1, B_2, B_3, \dots$  where the most vigorous vibration takes place, are called *antinodes*.

Stationary transverse waves are easily produced by means of

a rope, or of a long, slack rubber tube. If the tube is fixed at one end, and a sudden upward displacement is imparted to the other end by the hand, this displacement may be seen to pass as a crest along the tube and return again as a crest, or a trough. The wave is, therefore, *reflected* at the fixed end. If the second end is movable, as is the case with a freely hanging flexible tube, or with such a tube attached to a long slender thread, the crest returns again as a crest. Reflection, therefore, takes place at the free end with the same direction of vibration.

If now the end of the tube held in the hand is moved rhythmically upward and downward, the train of waves produced at the hand interferes with that reflected from the other end, and stationary waves with nodes and antinodes are formed. Since, at the fixed end, there is necessarily a node, and at the free end an antinode, and since two adjacent nodes, or antinodes, are always at half a wave length apart, the rhythm of the motion must be so regulated that the wave-length, and accordingly also the number of vibrations, bear a definite ratio to the length of the tube. The experiment is more easily performed by producing circular vibrations with the hand, which may be regarded as composed of two rectilinear vibrations perpendicular to each other. All points of the tube will then describe circles, whose planes are perpendicular to the length of the tube (transverse circular vibrations). Stationary waves may be very beautifully illustrated by a linen thread attached to the prong of a tuning-fork, or to the vibrating spring of a magnetic hammer (248). The more the tension of the string, and accordingly also the velocity of propagation are diminished, the greater will be found to be the number of nodes and antinodes (Meldi, 1859).

Stationary waves may be exemplified by Mach's wave machine by bringing the balls into their position of greatest departure by means of a properly bent wire, and then releasing them simultaneously by turning the wire quickly to one side. If, then, the planes of vibration be rotated by 90°, the *transverse* are transformed into *longitudinal stationary waves*. It is then seen that at the nodes, where the adjacent particles on both sides swing simultaneously toward the

stationary particle and then from it, alternate condensations and rarefactions occur, while at the antinodes, where the greatest motion occurs, condensation and rarefaction never take place. Stationary longitudinal waves may also be produced by means of the spirally wound wire mentioned above (vide 284). This is done by fastening one end of the spiral and exerting a pull at the other in the direction of the length. The coils of the wire, now left to themselves, vibrate vigorously to and fro, while at the node on the fixed end the coils are alternately crowded together and drawn apart. If the free ends of a spiral wire are drawn apart, a node is formed at the middle. The motion of the particles in a stationary longitudinal wave may be illustrated also by means of the drawing (Fig. 261), by carrying it along beneath a slit parallel to the line 0 - 12, from right to left. Such a drawing may also be placed upon a cylinder, so that the points 0 - 12 lie upon the surface in the direction of its length. If the cylinder is turned behind a slit placed parallel to its axis, the points seen through the slit imitate the curves of motion of the particle of a stationary wave, with nodes at 0, 6, and 12, and with antinodes at 3 and 9.

The displacement produced by a (transverse, or longitudinal) wave at the distance  $x$  from its origin, is

$$y_1 = a \sin 2\pi \left( \frac{t}{T} - \frac{x}{\lambda} \right) = a \sin 2\pi \left( \frac{t}{T} - \frac{t}{\lambda} + \frac{l - x}{\lambda} \right).$$

At the same point ( $x$ ) a wave coming from a distance  $2l$ , to meet the former and having the same amplitude and length as the former, causes the deviation, or elongation:

$$y_2 = a \sin 2\pi \left( \frac{t}{T} - \frac{2l - x}{\lambda} \right) = a \sin 2\pi \left( \frac{t}{T} - \frac{l}{\lambda} + \frac{l - x}{\lambda} \right).$$

The displacement produced by the interference of the two motions at the point  $x$  is accordingly

$$Y = y_1 + y_2 = 2a \cos 2\pi \frac{l - x}{\lambda} \sin 2\pi \left( \frac{t}{T} - \frac{l}{\lambda} \right).$$

In this equation of a stationary wave,  $2a \cos 2\pi \frac{l - x}{\lambda}$  represents the amplitude which changes periodically from place to place, and  $\sin 2\pi \left( \frac{t}{T} - \frac{l}{\lambda} \right)$  represents the phase of vibration common to all points.

## B. SOUND (ACOUSTICS).

**287. Sound** is the sensation mediated from without by the hearing. Whenever the sensation of sound is produced, the body whence the sound proceeds may be readily identified, and we easily convince ourselves by means of the sense of sight, or of touch, that this body, called the "source of sound," is in a state of trembling, or vibratory, motion.

If an alarm clock, which consists of a little hammer striking against a metallic bell, is brought under the receiver of an air-pump, the strokes of the bell cease to be distinctly audible when the receiver is exhausted of air as far as possible. But when the air is allowed to enter the receiver gradually, the strokes gradually become more easily perceptible and are soon heard as distinctly as at the beginning. *Sound is not propagated in a vacuum.* The report of the most violent explosion cannot pass beyond the limits of our atmosphere, and sounds originating outside of the atmosphere are also wholly beyond the reach of our perception. In rarefied air, e.g. upon high mountains, the intensity of sound is much feebler than in air of ordinary density.

Sound is propagated, not only in air and in other gases, but also in liquids and solids as well. A diver hears what is spoken on the shore, and the lightest taps against the end of a long beam are distinctly perceptible to an ear at the other end. The toy telephone (lover's telegraph) is made of two hollow cylinders, with membranes stretched tense over one end of each, the centres of the membranes being connected by means of a long taut string. Words spoken against one of the membranes are distinctly heard at the other.

**288. Mode of Propagation.**—That no material particles of the sound-producing body itself, nor of the surrounding air are transmitted to the ear is evident from the fact that the strokes of an alarm clock, when placed under the receiver, though slightly deadened, are still distinctly perceptible. Glass is known to be impenetrable to either air or other material. The idea is much more plausible that the sound-producing body transmits vibrations through the atmospheric particles within

the receiver: these then impart their motions to the particles of glass, which latter pass them on to the molecules of the outside air. A row of equal, elastic balls (percussion machine, 55) furnishes a picture of this mode of propagation. If the first ball is allowed to strike against the second it gives up its velocity to the ball it strikes and comes to rest. The second ball transmits likewise its motion to the third, and so on to the last. A *progressive longitudinal wave* passes meanwhile along the entire row of balls. If the balls are not all equal—if, for example, a row of larger balls, all equal to each other, follows a row of smaller ones, also equal to each other, the ball at the limit of the two rows does not come to rest, and a portion of the wave returns toward the place whence it started, or it is *reflected*; while another portion passes on into the other row. If the impinging ball is the larger, it maintains the direction of its motion and imparts to the smaller ball a higher velocity than its own; if, on the contrary, it is the smaller, it reverses its motion, while the larger moves forward with reduced velocity.

Similarly, the vibrations of the bell in the experiment with the alarm clock are propagated through a series of atmospheric particles as longitudinal waves, to the glass wall, where, on passing to the larger molecules of the glass, the vibrations suffer partial reflection. The waves in the glass on passing outward into the air, are again partially reflected and the sound heard from without is thus considerably deadened.

**269. Enfeeblement of Sound due to Transmission.**—Sound waves radiate spherically from a vibrating point in air of homogeneous constitution, *i.e.* in the form of spherical envelopes, which are alternately in a condition of condensation and rarefaction. Any radius of such a spherical wave is called a *sound ray*, and the vibrations of the atmospheric particles take place in the direction of the ray.

Since the surfaces of these spherical shells, and, accordingly, also the masses contained within such shells of equal thickness, increase as the square of their radii, the energy of motion of the source being continually disseminated throughout a greater atmospheric mass, the intensity of the sound per

surface-unit must continually diminish with increasing distance and in the precise ratio of the inverse square of the distance from the source. Or, in other words, at twice, three times, four times, . . . , the distance from the source, the intensity with which the sound affects the ear is only  $\frac{1}{4}$ ,  $\frac{1}{9}$ ,  $\frac{1}{16}$ , . . . , of that perceived at the distance 1. If the free propagation of sound rays in all directions is in any way obstructed, by compelling the waves, for instance, to enter a tube of uniform bore, this loss of intensity does not occur. Upon this principle depends the utility of *communicating tubes* (*speaking tubes and trumpets*) in public houses, upon steamboats, etc.

**290. Velocity of Propagation.**—To obtain the velocity of propagation of sound in air, cannons were fired at night and at prearranged instants, from two stations, the distance between which was accurately measured; while at each station, the time which elapsed between the flash of light and the report was observed (Committee of the Parisian Academy under A. von Humboldt and Arago, 1822). When the measured distance was divided by one-half the time required by the sound to traverse it in both directions, the distance traversed per second was obtained independently of the direction of the wind. The velocity of sound was thus found to be equal to 340 m. per second at 16° C. It increases with the temperature, but is independent of the pressure. Liquids and solids are traversed by sound with unequal velocities: in water, for example, it moves at the rate of 1435 m. per second (Colladon and Sturm, 1827).

If  $V$  denotes the velocity of propagation of a longitudinal wave in any medium whatever, a vibration of  $T$  seconds' duration produces a wave of length  $\lambda = VT$ . If in another medium, whose velocity of propagation is  $V'$ , a wave of the same length  $\lambda$  is produced by a vibration of  $T'$  seconds' duration, we have also  $\lambda = V'T'$ . Consequently,

$$\lambda T = \lambda T', \text{ or } V : V' :: T' : T :: \frac{1}{T} : \frac{1}{T'}.$$

Imagine now the distance,  $\lambda$ , to be cut up by sections perpendicular to the direction of propagation, into a number of equal thin segments. The masses of these segments will be as the densities of the media, and the forces, under whose influence the segments vibrate in the direction of the length, are as the elasticities  $c$  and  $c'$ . In the expression  $T = 2\pi\sqrt{\frac{m}{p}}$ , which gives the time

of vibration (53), the mass,  $m$ , may be assumed proportional to the density,  $d$ , and the force,  $p$ , to the elasticity,  $e$ . We have, therefore,

$$T : T' = \sqrt{\frac{d}{e}} : \sqrt{\frac{d'}{e'}} \text{ or } \frac{1}{T} : \frac{1}{T'} :: \sqrt{\frac{e}{d}} : \sqrt{\frac{e'}{d'}}$$

consequently,

$$V : V' = \sqrt{\frac{e}{d}} : \sqrt{\frac{e'}{d'}}$$

i.e. the velocities of propagation are directly as the square roots of the elastic forces and inversely as the square roots of the densities of the respective media. By properly choosing our units, we may write, with Newton (1667),

$$V = \sqrt{\frac{e}{d}}$$

The elastic force of gases is merely their expansive force, or their internal pressure, and, instead of  $e$ , the pressure per superficial unit (1 sq. cm.) must be substituted, and instead of  $d$ , the mass of the volumetric unit (1 ccm.). 1 ccm. of air at 0° and 76 cm. pressure weighs  $\frac{1}{773}$  g. If furthermore,  $s_0$  denote the specific gravity of the gas at 0° and 76 cm. pressure, the specific gravity  $s$ , at a pressure  $b$  and at a temperature  $\theta$ , is given by

$$s = \frac{bs_0}{76(1 + \beta\theta)}$$

where  $\beta = \frac{1}{273}$  signifies the coefficient of expansion of gases. The weight of the volumetric unit (1 ccm.) is accordingly  $\frac{s}{773}$  g., or

$$\frac{bs_0}{773 \times 76(1 + \beta\theta)}$$

whence the density, or the mass of the volumetric unit (in the terrestrial system of units) is found by dividing the latter expression by  $g = 981$  (cm. sec.<sup>-2</sup>):—

$$d = \frac{bs_0}{981 \times 773 \times 76(1 + \beta\theta)}$$

On the other hand, with a barometric height,  $b$ , the pressure upon 1 sq. cm. amounts to  $b \times 13.595$  g., where the latter number is the specific gravity of mercury referred to water. We have then  $e = b \times 13.595$ . If this value be substituted in Newton's formula, we obtain for the velocity of propagation in gases—

$$V = \sqrt{\frac{13.595 \times 981 \times 773 \times 76(1 + \beta\theta)}{s_0}}$$

This velocity of propagation is independent of the pressure, since, with constant temperature, pressure and density change proportionally according to Mariotte's law, and therefore  $b$  disappears.

From the preceding formula, the velocity of sound in air at 0° ( $\theta = 0$ ,  $s_0 = 1$ ) is found to be 279.91 m., which is considerably smaller than the value obtained experimentally. As Laplace showed later (1816), Newton's formula does not consider the circumstance that in the condensed portion of the wave the temperature is increased and in the rarefied portion decreased. Since, in consequence of the small conducting and radiating power of air, this difference of temperature cannot be uniformly distributed within the short

duration of a vibration, the differences of pressure, and hence the elastic forces, are increased in the ratio  $\frac{c}{c'} = 1.41$  of the specific heats  $c$  and  $c'$  of air under constant pressure and volume respectively. To make the formula complete, therefore, the expression under the radical sign must be multiplied by  $\frac{c}{c'}$ , and we then obtain for dry air (expressed in  $m$ )  $V = 332.4\sqrt{1 + \beta\theta}$ , which is in close accord with experience. The velocities of propagation in other gases are in the inverse ratio of the square roots of their specific gravities.

With solids,  $c$  must be replaced by the modulus of elasticity,  $E$  (52); with liquids,  $c$  must be derived from their compressibility.

**231. Reflection of Sound.**—Sound rays are reflected and refracted (the latter on passing into layers of air of different densities, or from air into other bodies) according to laws analogous to those for light rays. Sound rays are reflected from a plane surface as though they came from a point lying in a perpendicular from the source of sound upon the surface and situated as far behind the surface as the source is in front of it. The explanation of the *echo* comes from this fact. When a loud cry is uttered at some distance from a wall, a cliff, the edge of a wood, etc., after a lapse of time sufficient for the sound to pass to the wall and back again, the cry is again heard from the direction of the wall. The wall reflects the sound precisely as a mirror reflects light, so that the reflected cry is heard precisely as though a second person, playing the part of the image of the crier, were situated just as far behind the reflecting surface as the latter is before it, and were sending back precisely the tone which reached him. To speak a syllable requires at least one-fifth of a second. If, then, the experimenter stand so far from the wall that sound-waves consume one-fifth of a second in passing to it and back again, the reflected sound will return exactly at the instant of completing the syllable. Since, in one second, sound traverses 340 m., the experimenter must be at least 34 m. from the wall to perceive a monosyllabic echo. If he be 2, 3, 4, . . . times as far from the reflecting surface, he may speak 2, 3, 4, . . . syllables before the first returns, and may accordingly hear an echo of 2, 3, 4, . . . syllables. If the surface is less than 34 m. distant, the reflected sound returns before the utterance of the syllable is complete, and blends in part with it. In churches

and large halls this echo is often unpleasantly noticeable. When there are several reflecting surfaces at different distances, several independent echoes are produced simultaneously. Between the cliffs of the Lurlei, for example, a pistol shot is repeated 17 to 20 times with varying intensity resembling a roll of thunder.

If two concave mirrors are placed facing each other and a watch is at the focus of one, when the ear of the observer is at the focus of the other, even at a considerable distance from the first mirror, the ticking of the watch is distinctly heard. The sound-waves proceeding from the first mirror, are reflected from its surface in parallel directions and, striking upon the surface of the second, they are brought together at its focus. If the parallel rays reflected from the first mirror fall upon a glass plate, they are reflected by the latter at equal angles and, by properly turning the plate, they may be sent in any desired direction.

The speaking tube and the ear trumpet are based upon this principle of the reflection of sound. The *speaking trumpet* is a funnel-shaped tin, paper, or gutta-percha tube, provided with a mouthpiece, by means of which the sounds of words spoken into it are prevented from spreading and are thereby rendered audible at a long distance. The rays of sound issuing from the mouthpiece are so reflected by the walls of the tube as to leave these walls in almost the direction toward which the tube is turned. They proceed then without loss of intensity, and with sufficient power to reach the distant ear. For example, a speaking tube of 15 to 20 m. in length and 17 cm. of opening, such as are used in ships, can be understood at a distance of 1000 to 1500 m. The *ear trumpet* has to solve the reverse problem of collecting the sound rays entering from its funnel and reflecting them into the narrow opening to the ear passage, thereby assisting the partially deaf to a more distinct perception of sounds. The *stethoscope* is an ear trumpet used by physicians to intensify the sounds accompanying respiration and circulation.

A sound produced at the centre of a hollow sphere is reflected from all directions again to this centre. Sound waves

emerging from one focus of an ellipse are gathered together by reflection into the other. In a room whose walls are curved elliptically, words spoken softly at one focus are distinctly perceptible at the other, while elsewhere in the room the words are wholly inaudible. Buildings constructed thus either purposely, or accidentally, enable words spoken in a low tone at one point on the interior to be heard at another point, and are called *whispering galleries*. Halls constructed for parliamentary and concert purposes must be *acoustic*, that is, they must be so constructed that the waves of sound proceeding from the stage, or orchestra, are reflected toward the auditorium without the disturbing influence of intermediate reflecting surfaces.

Sound waves are not only reflected from a solid wall, but wherever they pass into a medium of different constitution, *e.g.* from a denser into a rarer atmosphere, or conversely, reflection also takes place. Sound is heard much farther by night than by day, because, in the latter case, it is weakened by numerous reflections which it suffers at the unequally heated and consequently unequally dense ascending and descending currents of air, while it passes unhindered through the uniformly heated layers of the night air. Tyndall observed that the fog signals which are given by steam whistles, and by large sirens along the coasts, as a warning to seafarers, are heard much farther during cloudy than during clear weather, and this is true because on a clear day the air is unequally heated by the rays of the sun and is thereby rendered less transparent to sound, or it is, so to speak, disturbed by an "acoustic cloud."

**292. Sounds of Various Kinds—Sirens.**—Sound sensations are quite varied in kind, and our speech is correspondingly rich in symbols to express their peculiar nature. By means of a single violent concussion, a crack is produced; by an irregular succession of vibrations *noises* are produced (rushing, roaring, rustling, rattling, rippling, crackling, clattering, crunching, etc.). A musical sound, or a tone, on the contrary, is produced by the *regularly repeated* (*periodic*) or "vibratory" motion of a sonorous body. A musical sound may be produced by means of a card held against the periphery of a uniformly rotating

toothed wheel (Savart), or by means of puffs of air repeated regularly and in a similar way. The latter method is illustrated in the siren, the simplest form of which, as given by Seebeck (1843), consists of a round disk of paper, or metal, perforated with several circular rows of equally distant holes. When a puff of air is driven through a glass tube toward the innermost row of holes, the disk being rotated rapidly and uniformly by means of a centrifugal machine, a passage is opened to the current of air escaping from the tube, whenever a hole passes above its mouth, and closed again as soon as an unperforated portion of the disk takes its place. The puffs of air following in regular succession, produce in the ear the sensation of a tone of definite *pitch*. If, with the same velocity of rotation, one of the outer rows of holes be blown against, a

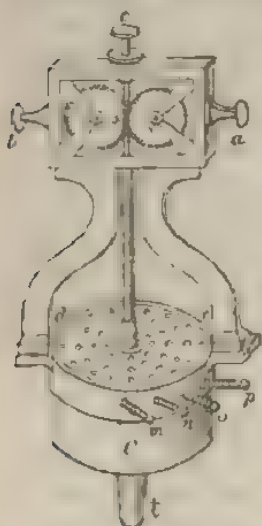


FIG. 262. Siren.

greater number of holes passes the mouth of the tube in the same time, and, therefore, also a greater number of impulses result. A musical sound, or tone, is again heard, which differs from the former in being of a higher pitch. It is furthermore noticed that the tone is pitched higher, the greater the number of periods of vibration occurring in a given time, or the greater the number of vibrations per second. Cagnard-Latour has constructed a more highly perfected siren in which the disk is rotated by the atmospheric current itself. Fig. 262 represents this apparatus in the still more highly perfected form given to it by Dove. A horizontal metallic disk, *ds*, perforated by four

rows of holes, rotates readily about a vertical axis. The disk is situated above a cylindrical chamber, *C*, whose cover is perforated by a corresponding system of holes. The holes of both cover and disk are bored obliquely and with opposite inclinations, so that the current, issuing obliquely from a hole of the cover, strikes almost at right angles against the sides of the

holes of the disk, thereby setting the disk in rotation. Beneath each row of holes is placed a movable metallic ring with the same number of holes disposed in a similar way. These rings may be placed by means of the pegs, *m, n, o, p*, so that any one, or more, of them closes the corresponding row of holes of the chamber cover, or they may be so placed that the holes of a ring fit exactly upon the holes of the corresponding row of the cover. By pressing on one, or more, of the pegs, any number of rows may be blown upon at the same time. The chamber is placed upon the mouth of a pipe leading to an organ bellows, by means of the tube, *t*. The axis of the disk carries at its upper end an endless screw, *s*, which engages in a toothed pinion, whose motion permits the number of revolutions in a given time to be read from a dial plate (not shown in the figure), from which the number of vibrations per second may be determined. By pressing the knob, *a*, the toothed wheel mechanism may be connected and set in motion, and by pressing upon *b* it may again be thrown out.

**293. The Scale (Gamut).—**The first row in the siren contains 8, the second 10, the third 12, and the fourth 16 holes. If the first and then the fourth row of holes be blown upon, two sounds are heard, which in music are distinguished as the *fundamental tone*, or root, and its *octave* respectively. The octave vibrates, therefore, twice as rapidly as the fundamental tone. When both tones are sounded simultaneously, they blend together perfectly into a single composite tone, producing a pleasing sensation. They form what is termed a *consonance*. A consonance is more nearly perfect the simpler the ratio of the number of vibrations of the tones sounded together. Next to unison (1:1), the octave and the fundamental tones form the most perfect consonances, for their ratio is the simplest conceivable, namely 2:1. The next perfect consonance is obtained by means of the first and third rows of holes. The tone produced by the latter bears to the fundamental tone the ratio 12:8, or 3:2, and is called the *fifth* of the fundamental tone. The first and second rows give the somewhat harsher ratio 10:8, or 5:4. The higher tone is called the *major third* of the fundamental. The fundamental

tone is designated by the letter C, its major third, by E, the fifth, by G, and the eighth, or octave, by C. The simultaneous production of three, or more, tones in agreeable relations to each other is called a *chord*. The fundamental tone, major third, and fifth (CEG) form together the C-major chord. By sounding together other pairs of rows of the siren, still other consonances are produced. The fourth and third rows give the ratio 16 : 12, or 4 : 3, called the ratio of the *fourth*. The fifth of C is designated F. The third and second rows furnish the ratio 12 : 10, or 6 : 5. The higher tone is here called the minor third of the lower, and is designated, with respect to the fundamental C, by E $\flat$ . Recapitulating this succession of sounds, which always maintains its musical characteristics even with a varied velocity of rotation of the siren, we obtain, if we leave aside the minor third, the following group, where under the designation of the sound, its vibration ratio to the fundamental tone is also given:—

C	E	F	G	c
1	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{5}{4}$	2

To satisfy the requirements of music each sound must again be the fundamental of a C-major chord, i.e. it must be possible to ascend from each tone again in thirds and fifths. But the fifth of G must make  $\frac{3}{2}$  as many vibrations as G, hence  $\frac{3}{2} \times \frac{5}{4} = \frac{15}{8} = 2\frac{1}{8}$ . The tone thus found is higher than the octave c. To remain within the octave, therefore, we take the next lower octave of the tone  $\frac{15}{8}$ , whose number is  $\frac{15}{16}$ . The corresponding tone is designated by D and is called the *second* of C. The major third of G has the vibration-ratio  $\frac{5}{4} \times \frac{3}{2} = \frac{15}{8}$ . It is called the seventh of the fundamental tone and is designated by H. The fifth of the tone F has the number  $\frac{3}{2} \times \frac{4}{3} = 2$ . The octave of C is therefore at the same time the fifth of F. The major third of F has the ratio  $\frac{4}{3} \times \frac{3}{2} = 2$ , and is designated by A and called the *sixth*. We thus obtain the *diatonic* (major) scale, which, within a single octave, consists of the following tones: prime, or fundamental, tone C, second D, major third E, fourth F, fifth G, sixth A, seventh H, and the octave c, with the corresponding

vibration-ratios placed below the symbols in the following group:—

C	D	E	F	G	A	H	c
1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2

If the vibration-ratio of each of these tones is divided by that of the preceding, the *interval* of the tones is obtained, i.e. the number indicating how many times greater the vibration-ratio is than that of the next lower. In the following series the values of these intervals are placed in the second line between the tone-symbols standing in the first:—

C	D	E	F	G	A	H	c
$\frac{9}{8}$	$\frac{10}{9}$	$\frac{8}{9}$	$\frac{7}{8}$	$\frac{6}{5}$	$\frac{5}{4}$	$\frac{16}{15}$	

It is apparent that the intervals in the diatonic scale are very unequal. The intervals between third and fourth, and between seventh and eighth ( $\frac{16}{15}$ ) are considerably smaller than the rest. It is therefore said that the intervals from E to F, and from H to c, are *half tones*, or *semi-tones*, while the remaining intervals are regarded as *whole tones*. To advance by more nearly uniform intervals, half tones must be interpolated between the whole tones, and the entire series of one octave, consisting of twelve semi-tones (chromatic scale) is then—

C	C $\sharp$	D	D $\sharp$	E	F	F $\sharp$	G	G $\sharp$	A	A $\sharp$	B	H	c
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Since, however, the whole tones do not possess equal intervals, but from C to D, from F to G, from A to H, the interval is one whole major tone ( $\frac{9}{8}$ ), from D to E and from G to A it is a whole minor tone ( $\frac{10}{9}$ ), the intervals of the chromatic scale are also unequal, which makes it impossible to ascend from any desired tone as a fundamental always in the same way. If, for example, we advance in pure thirds, we obtain an impure octave, and the same is true on advancing according to pure fifths. But since the octave forms the most perfect consonance, the impurity of which is disagreeably noticeable, it is the custom rather to sacrifice the purity of the remaining tones by allowing them to "flow" ("Schweben") a little above, or below, the pitch required by the diatonic scale, and to maintain rigorously the purity of the octave. Such an

adjustment is called the *temperament*. The uniformly flowing *equal temperament*, which is the simplest of all in extended use, and underlies the construction of all musical instruments which have a fixed key (*e.g.* the piano), has all its intervals equal. Since, in the chromatic scale, twelve steps or chromatic semi-tones are present, the interval of a half tone (called *semi-tone*) must be so chosen that when repeated twelve times it leads to a pure octave, *i.e.* to a vibration-ratio double that of the fundamental tone, or if  $x$  denote the desired interval,  $x^{12}$  must equal 2. This interval is, therefore, expressed by the number  $\sqrt[12]{2} = 1.05946$ . We thus obtain a uniformly flowing scale with the following vibration-ratios:—

C ...	...	1.00000	G ...	1.49831
C# ...	..	1.05946	G# ...	1.58740
D ...	...	1.12246	A ...	1.68179
D# ...	...	1.18921	B ...	1.78180
E ...	...	1.25992	H ...	1.88775
F ...	...	1.33484	c ...	2.00000
F# ...	...	1.41421		

in which each number is obtained from that of the preceding semitone by multiplication with the number 1.05946.

**284. Absolute Vibration-numbers, or Frequencies.**—Hitherto we have considered only the vibration-ratios of the tones within an octave, but have said nothing about the *absolute vibration numbers*, or *frequencies*. When the vibration number is known for one of these tones it is known for all, because the vibration-ratios are all known.

The siren may be used to determine the absolute vibration numbers. Suppose the vibration number of a tuningfork were desired, for example. The siren is rotated with such velocity that one of its rows of holes gives the same tone as the fork. From the number of revolutions per second furnished by the toothed wheel mechanism above and the number of holes, the number of vibrations of the tuningfork per second is obtained.

As a fundamental tone in the tuning of musical instruments the so-called *concert pitch* is usually selected. This tone is given by means of a normal tuningfork. A *pure a* of 440 vibrations constitutes the basis of the *German method of keying*, proposed by Scheibler. The *Parisian key* introduced into

France, and also used in many orchestras outside of France, takes for the *tempered a* the vibration number 435. For purposes of computation the *physical key* is very convenient. This takes the *one-lined C* at 256, and the *tempered a*, consequently, at 480·5 vibrations. For the fundamental tones designated in the following table, the appended absolute vibration numbers, called also *frequencies*, are thus derived:—

Position of Octave.		German Key.	Parisian Key.	Physical Key.
Suboctave-C	C <sub>-2</sub>	16·5	16·2	16
Contra C	C <sub>-1</sub>	33	32·3	32
Great C	C <sub>0</sub>	66	64·7	64
Small C	C <sub>1</sub>	132	129·3	128
One-lined-C	C <sub>2</sub>	264	258·7	256
Two-lined-C	C <sub>3</sub>	528	517·3	512
Three-lined-C	C <sub>4</sub>	1056	1034·6	1024

The subcontra-C of 16 vibrations per second constitutes the lower limit of perceptibility for the human ear. The upper limit is assumed at about *c*, of 16896 vibrations per second. The human ear embraces, therefore, ten octaves. The tones used in music lie between 30 and 4000 vibrations, which corresponds to an interval of about seven octaves.

**295. Wave-Lengths.** — When the vibration number, or frequency, of a tone is known its wave-length in air may be readily obtained. *All tones, high and low, are propagated in the air with the same velocity of 340 m. per second.* That this is true is apparent from the fact that if high tones moved more or less rapidly than lower tones, a piece of music played at a distance would be heard as a disagreeable confusion of sounds, because the high and low tones would not reach the ear on the same beat. Since, however, each complete vibration produces also a complete wave within the distance of 340 m., as many waves must be comprised as there are vibrations per second. The length of a wave is then found by dividing the *velocity of propagation of sound by the frequency*. For the tone *a*, of 440 vibrations, for instance, the wave-length is  $\frac{340}{440} = 0·772 \text{ m.} = 772 \text{ mm.}$

**296. Pipes.**—A vibrating tuningfork, when held in the open air, gives out a faintly audible tone. The sound is, however, rendered more distinct by placing the tuningfork before the

mouth of a tube of proper length, *e.g.* over a cylindrical glass vessel, within which the atmospheric column may be shortened by pouring water into the vessel until the tone becomes sufficiently strongly re-enforced. For the *a*-fork, experiment shows that to produce this effect the column of air must be 193 mm. long, i.e. equal to the fourth part of the wave-length, 772 mm. In general, the length of the shortest column of air capable of being excited into sympathetic vibrations by a sounding body, equals one-fourth of the length of the sound wave emitted by the body. The entering wave of air is reflected at the closed end of the tube. By means of the combined effect (interference) of the incident and reflected waves, the peculiar condition of vibration is produced in the tube, which we have heretofore called stationary longitudinal waves. At the closed end of the tube the wave, in which the layers of air perpendicular to the axis of the tube vibrate longitudinally, is reflected (cf. 286) with the opposite phase of vibration, and hence the motion of the incident wave is destroyed by that of the reflected wave. The layer of air next to the closed end of the tube remains at rest, forming a node. Similar nodes are also formed at sections of the tube, which are at  $\frac{1}{2}$ ,  $\frac{3}{2}$ ,  $\frac{5}{2}$ ,  $\frac{7}{2}$ , . . . of a wave-length from the bottom of the tube. On the other hand, at the points  $\frac{1}{4}$ ,  $\frac{3}{4}$ ,  $\frac{5}{4}$ ,  $\frac{7}{4}$ , . . . of a wave-length from the bottom of the tube, the incident and reflected waves meet always in the same phase of vibration. At these places, called *antinodes*, a vigorous vibration of the atmospheric layers always occurs. The layers of air at the nodes are alternately condensed and rarefied, by reason of the neighbouring layers crowding toward them, or withdrawing from them, simultaneously. These *condensations* and *rarefactions* occur in such a way that neighbouring nodes are always in opposite states. At the antinodes condensation and rarefaction never occur, but always the most vigorous vibration of the air takes place here. During these vibrations the particles of air pass simultaneously through their positions of equilibrium and attain at the same time their greatest *elongation*, which is greatest at the antinodes, and diminishes continually toward the adjacent nodes, where it is zero. The air thus set in vibration is thereby converted into a sonorous body.

or into a source of sound. Since the open end of the tube communicates with the outer air, neither condensation nor rarefaction can occur here. An antinode must necessarily exist at the mouth of the tube. When the air contained in the tube is required to vibrate sympathetically with a sonorous body, i.e. to be put into a state of stationary vibration, its length must be  $\frac{1}{2}$ , or  $\frac{3}{4}$ , or  $\frac{5}{4}$ , etc., of the wave-length of the tone. It may also be readily seen that the same tuningfork will set three, or five, times as long a column of air into sympathetic vibration, but that it will not produce this effect with a column which is two, or four times as long. Consequently, a tube will respond to those tones,  $\frac{1}{3}$  of whose wave-length is contained once, or three times, or five times, etc., in its length. The frequencies of the tones must then be as the odd numbers 1, 3, 5, . . . during the response, the column of air is separated by 1, 2, 3, . . . nodes into the same number of vibrating subdivisions. This is represented in Fig. 263, where the arrows indicate the alternating phases of vibration at the antinodes. The lowest of these tones is called the *fundamental tone* of the tube, and the others are the *overtones*.



FIG. 263 - Vibration in a Tube closed at one end.

Air may also be set in stationary vibration in a tube open at both ends: for at these open ends a sort of reflection of the wave entering at the other end takes place, since the external air whose particles are freely movable in all directions, acts as a medium rarer than the enclosed air, whose mobility is restricted to the direction of the length of the tube. Since this reflection takes place at a rarer medium, the vibrations of the incident and reflected waves always meet in the same phases, thereby intensifying each other. Antinodes must then form at the open ends of the tube, and the length of the tube is therefore  $\frac{1}{2}$ , or  $\frac{3}{4}$ , or  $\frac{5}{4}$ , etc., of the wave-length of the tone, while the frequencies of the series of tones of which it is capable are as the numbers 1, 2, 3, 4, 5, . . . With the first of these tones, which is the fundamental, the column vibrates with a node at its middle, and its length is then half the wave-length of this tone. For the overtones, the column is divided by 2,

3, 4, . . . nodes, as is shown in Fig. 264. The fundamental tone of an open tube is an octave higher than the fundamental of an equally long closed tube. To make an open tube furnish the same fundamental as a closed one, it must then be made twice as long (Daniel Bernoulli, 1762).

As a source of stationary waves, the sounding body may be dispensed with, since the same effect may be produced by blowing. A tube arranged to be used with a blast of air is called a



FIG. 264.—Vibration in a Tube open at both ends.



FIG. 265. Organ Pipe

pipe (mouth-pipes). Fig. 265 represents the cross-section of an open, wooden organ-pipe. A puff of air is forced through the aperture at the foot, into the chamber, *K*, through the slit, *cd*, against the sharp lip, *ab*, of the mouth, *aled*. The flat current of air thus produced has, by virtue of its velocity, a certain rigidity, which renders it capable of vibrating, like the prong of a tuningfork, at the mouth of the pipe. But while the rigid tuningfork possesses a fixed and invariable duration of vibration, the yielding current of air regulates its movement so as to conform to the time of vibration peculiar to the pipe. The pipe becomes sonorous when blown upon, and gives forth a definite

fundamental tone, conditioned by its length alone. When an open pipe sounds its fundamental, a node forms at its middle. The existence of this node may be very easily proved by means of the *manometric flame* of Koenig. In the wall of an open pipe (Fig. 266) three holes are bored, the one at the middle, and the others at one-fourth the length of the pipe from its ends. Three "manometric capsules," *a, b, c*, are screwed into these holes, the arrangement being shown in Fig. 267. The hole, *o*, in the wall, *w*, is separated from the inner space of the capsule, *bb*, by means of a thin rubber membrane. Illuminating gas is admitted into the chamber of the capsule through the rubber tube, *d*, from the chest, *ee* (Fig. 266), which is filled through the tube, *f*. The gas flows from the capsule, *bb*, through the tube, *s*, where it burns with a small pointed flame. When the pipe sounds its fundamental, a node forms at its middle, at which alternate condensations and rarefactions of the air take place. At each condensation the membrane is pressed outward, expels the gas from the capsule into the



FIG. 266. — Pipe with Manometric Flame.



FIG. 267. — Manometric Capsules.



FIG. 268. — Rotating Mirror.

burner, and the flame burns high. On rarefaction, the membrane curves inward, the gas follows it, the flame withdraws

into the burner and becomes quite small. The alternate flashing up and sinking of the flame takes place so rapidly that when observed directly, by reason of the persistence of the image on the retina, only a trembling of the flame is perceived. A rotating mirror (Fig. 268) is used to observe the flame. The mirror consists of a four-sided prism whose faces are covered with plane mirrors, the prism being easily and rapidly movable about its vertical axis. A steady flame appears to stretch out into a continuous band of light, when the mirror is rotated. The alternate lengthening and shortening of the flame on sounding the pipe, appears in the form of separate flame-images alternating with dark spaces (Fig. 269a). When the pipe sounds its fundamental, the manometric flame placed at its



FIG. 269.—Flame Images in Rotating Mirror.

middle indicates the existence of a node, while the other flame remains relatively steady. If the puff of air is strengthened, the pipe will sound the octave of its fundamental, its first overtone. An antinode will now form at its middle, while at the places *b* and *c* (Fig. 266) nodes will appear. The middle flame now burns quietly, while the other two are broken up into flame-like images, which, with the same rate of rotation of the mirror as before, appear to stand only half as far apart as did the former (Fig. 269, *b*).

A tube, open at both ends, may also be made sonorous by means of a gas flame (Fig. 270) burning inside of it and near its lower end (*singing flames, gas harmonicas*). The illuminating gas vibrates alternately into and out of the burner, while the flame extinguishes and rekindles with a light puffing noise, keeping time with the stationary vibrations of air in the tube, which control the rate of burning. If the length of the tube be increased, by raising the adjustable stopper, *s*, the

tone grows deeper. A tube, on the point of emitting a tone, resounds whenever its characteristic tone is produced at a distance from it (*sympathetic flames*). When seen in the rotating mirror, the singing flame also shows a series of separate images.

From the laws of vibration of atmospheric columns, the velocity of propagation of sound may be determined by simple experiment. When the vibration frequency of the fundamental tone produced by a closed pipe has been obtained with the siren, the velocity of sound may be found by multiplying four times the length of the pipe (i.e. the wave-length of its fundamental) by the frequency. If the pipe is filled with any other gas, it will give out a different tone, and in precisely the same way as before the velocity of propagation in this gas also may be found. It may be readily verified in this way that the velocities of propagation in different gases are as the square roots of their specific gravities. The velocity of sound in liquids may also be determined by pipes filled with the liquid and blown upon by a jet of liquid.

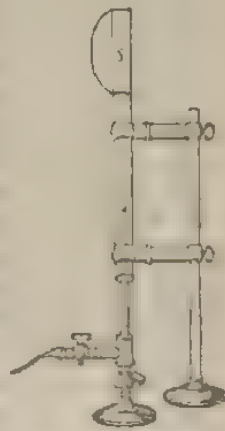


FIG. 270.—Singing Flame.

Assuming the velocity of sound in air to be known, the converse problem of finding the vibration frequency of a tuningfork, for example, may be solved, by varying the length of the column of air within a tube by pouring in water, until it is made to vibrate as powerfully as possible under the stimulus of the vibrating fork. The velocity of sound, 340 m., divided by fourfold the length of the air-column, gives the desired vibration-frequency.

**297. Longitudinal Vibration of Bars.**—Liquid columns and solid bars may be set into stationary longitudinal vibration, the vibrations following the same laws as atmospheric columns. A metal bar, for example, is made to emit sound waves of this sort by grasping it firmly at the middle, or at one end, and stroking it

at the other end, in the direction of the length, with resin-covered fingers. When held at the middle, the bar behaves like an open pipe, and when held at the end, like a closed one, the individual cross-sections of the bar vibrating in the direction of its length, alternate condensation and rarefaction occurring at the points grasped. The longitudinal vibration of the bar at its free end may be shown by a suspended ivory ball, which hangs so as to touch lightly against this end. The ball will be forcibly hurled away, when the bar is rendered sonorous. In the same way as with pipes, the velocity of sound in the substance of the bar may be computed from the vibration frequency of the tone and the length of the bar. It has been found that sound is propagated in silver, 9 times, in copper, 12 times, in iron, 16½ times, and in fir wood, 18 times as rapidly as in atmospheric air.

**298. Kundt's Tubes.**—The nodes and antinodes in a column of air are made visible by the following process which is due to Kundt (1866). In a horizontal glass tube a small quantity of light powder (cork filings) is sprinkled. A smaller glass tube, held at the middle of the horizontal tube, by a cork closing the end of the latter, projects into the larger tube and carries at its inner end a cork which does not quite fill the larger tube, and, consequently, remains free to move. The other end of the wide tube is closed by a cork, by displacing which the distance between it and the inner cork may be varied. If, now, the glass tube be set in longitudinal vibration by rubbing it with a scrap of moist cloth, stationary waves form within it, which are rendered apparent by the collecting of the powder at the antinodes in fine transverse lines, and at the nodes in round heaps. Since the distance between two nodes, or between two antinodes, is half a wave-length, by dividing the velocity of sound in air, 340 m., by the wave-length thus found, we obtain directly the vibration frequency of the glass tube, and hence, as before, the velocity of propagation in glass, or other solid, if the glass tube be replaced by a bar of the solid material. From the velocity of propagation,  $V$ , and the density,  $d$ , of a solid, the modulus of elasticity,  $E = V^2 d$  (290), also results.

**299. Vibration of Cords.**—Strings (or cords), in the acoustic sense, are tense, flexible threads of solid matter, which, when

drawn aside from the rectilinear positions of equilibrium by picking them, or by stroking them as with the bow of a violin, pass into a condition of transverse vibration, their particles swinging back and forth in paths perpendicular to the direction of their length (Fig. 271). To investigate the laws of vibration of cords, the so-called *monochord* (372) may be used.

This apparatus consists of a resonance box, or *resonator*, upon which the cords are stretched, between the bridges, *a* and *b*, either by means of a key, *s*, or by the weight, *P*. The frequency is found to be

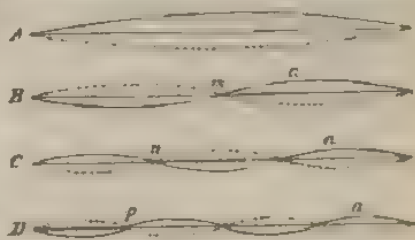


FIG. 271.—Vibrations in a Stretched Cord.

greater, the shorter and thinner the cord. When stretched with four times the weight, it gives the octave of its fundamental tone, and, consequently, the frequency is doubled, *i.e.* the frequency is proportional to the square root of the tension. When made of heavier material, the cord emits a lower tone, and the frequency is found to be inversely proportional to the square root of its specific gravity. When the cord vibrates as a whole (Fig. 271. A), it gives out its fundamental tone. But it may also be divided by stationary points (nodes) into 2, 3, 4, . . . vibrating parts (antinodes), whereupon the harmonic overtones, whose vibration-numbers are 2, 3, 4, . . . times as great as that of the fundamental, are successively produced. To produce the forms of vibration, B, C, D (Fig. 271), the cords should be touched with a pencil point at *m*, *n*, and *p*, at the same time that they are stroked, or picked, at *a*. The nodes may be made visible by placing light paper riders at the nodes and antinodes. The riders will be instantly thrown off at the antinodes, but will remain on the string at the nodes.

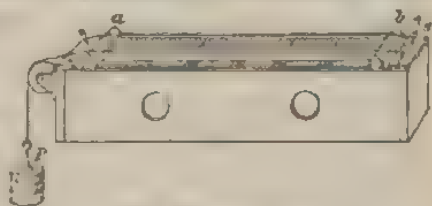


FIG. 272. Monochord.

The frequency,  $N$ , of the fundamental of a cord is given by the expression—

$$N = \frac{1}{l} \sqrt{\frac{gS}{\pi s}},$$

where  $l$  denotes its length,  $d$  the thickness,  $S$  the tension,  $s$  the specific gravity, and  $g = 9.81$ , and  $\pi = 3.14159$  (Taylor's formula, 1716).

**300. Transverse Vibrations of Bars.**—While the tendency of a cord to return to its position of equilibrium after being drawn aside from it, must be due to an external force, tension bars must possess within themselves the elasticity necessary to vibration. When a bar is fastened at one end, it assumes the various conditions of vibration represented in Fig. 273, vibrating either as a whole, or with 1, 2, 3, . . . nodes. With a glass fibre of the proper length, fastened to a prong of a tuningfork, the nodes may be studied observationally. If both ends are free, the bar possesses two nodes (Fig. 274) in its simplest mode of vibration.

FIG. 273.—Vibrating Forms of a Bar fixed at one end.

These nodes are about one-fifth of the cord's length from its ends, and, to permit the bar to vibrate freely, it must be supported only at these points. The frequency of vibration of a bar is directly as its thickness,

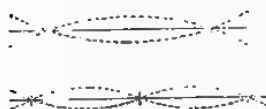


FIG. 274.—Vibrating Forms of a Bar free at both ends.

inversely as the square of its length and independent of its breadth. The overtones corresponding to the higher vibrating forms, are not harmonic with the fundamental, but rise much more rapidly than do the harmonics. The lengths of bars of equal thickness, whose fundamental tones are the notes of the scale, must be made inversely as the square roots of the frequencies.

The vibration-number,  $N$ , of a bar is expressed by—

$$N = C \frac{d}{l^2} \sqrt{\frac{gE}{s}},$$

where  $E$  denotes the modulus of elasticity,  $s$  the specific gravity, and  $C$  a constant factor depending upon the mode of fixing, or supporting, and upon the number of nodes.

With a bent bar, the nodes lie nearer its middle than with a straight one. A *tuningfork* is a bar bent in the form of a horseshoe. In it the two nodes lie very near to the bend of the bar (Fig. 275, *cc*).

**301. Vibrating Plates.**—When stroked along their edges with a violin bow, *plates* may be subdivided by *node lines* in manifold ways. If certain of their points are prevented from vibrating by being clamped, or touched with the finger, sand sprinkled over a vibrating plate withdraws from the vibrating regions, and collects along the node lines, thus rendering them apparent. The acoustic figures of Fig. 276, first produced by Chladni, and named for him, arise in this way. Each corresponds to a different tone in the plate, the tone being of higher pitch the more numerous the vibrating subdivisions of the plate. In the drawing the points, which must be held, to produce the respective figures, are designated by *a*, and the point where the violin bow is to be applied, by *b*. Circular plates, fastened at the centre and stroked at the edge, subdivide by diametral lines into 4, 6, 8, . . . equal sectors. Two adjoining subdivisions of a plate vibrate always oppositely. Bells may be considered as thin curved plates. While ringing, they subdivide into vibrating portions separated by stationary node lines.

**302. Reed Pipes.**—A reed is an elastic metal strip, fixed at one end, vibrating according to the same law as bars and, at the same time, interrupting a current of air at regular intervals by its vibrations. This current passes from the tube, *pp*, of the reed-pipe (Fig. 277), whose base is set over the mouth of a bellows-tube into the cylindrical brass canal, *rr*, whose slit is alternately opened and closed by the vibrating tongue, *l*. The air escapes thence through the orifice, *v*. The tuning



FIG. 275.—  
Tuningfork.

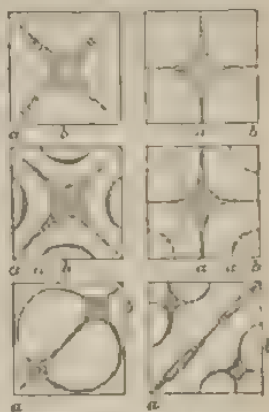


FIG. 276.—Chladni Acoustic  
Figures.

wire, *d*, is inserted through the wooden cap, *ss*, by which the reed is held in place in the pipe. By pressing *d* downward, or raising it upward, the tongue may be tuned to a higher,



FIG. 277 —  
Reed Pipe.

or lower pitch. A conical funnel may be placed upon the mouth of the opening, *v*, to intensify and vary the tone. When the funnel is short, it exerts no appreciable influence upon the fundamental tone of the reed, but, when sufficiently long, it alters the frequency very materially. The reed is neither as stiff as a tuningfork, nor as yielding as the current of air, of an ordinary pipe. The stationary wave formed in the funnel, when sufficiently long, forces the tongue to accommodate itself to the issuing waves. Still, another form of reed is that

of the *membranous tongue*. This is formed by two elastic plates, or bands (of rubber), which by their vibrations alternately open and close a slender

slit between them, and thus rhythmically interrupt

the air current flowing through the slit. The pitch may be heightened by increasing the tension of the bands. The organ of human speech is merely a membranous reed-pipe, within which the vocal cords, or bands, stretched at the sides of the glottis, act as reeds.



FIG. 278. — Compound  
Vibrations of a Bar.

**303. Composition of Rectangular Vibrations.**—A bar of rectangular cross-section fixed at one end (Fig. 278), can be made to vibrate in the directions, *ab* and *cd*, perpendicularly to each other, the frequencies in the two directions being related to each other as the thickness of the bar in the respective directions. By means of an oblique impulse, both modes of vibration are produced simultaneously, and the free end of the bar describes a curved line (Fig. 279), whose form depends upon the ratio of the frequencies. If these frequencies are equal, or in the ratio 1 : 1, the vibrating figure is a circle, or an ellipse.

If the ratio is 1 : 2 (fundamental and its octave), the figure has the form of an 8, and so forth. These ornamental figures may be very prettily exhibited with bars carrying brightly shining knobs at their upper ends (Wheatstone's *kaleidophone*, 1827). By a process given by Lissajous (1857), these vibrating figures may be thrown upon a screen by the agency of a beam, or ray, of light. Two tuningforks, R and S (Fig. 280), the one placed vertically and the other horizontally, carry small mirrors at C and B. The ray, AB, from the lamp, A, is reflected from B to C, from C upon a screen at D, forming here a luminous point, when the forks are at rest. When the fork, R, vibrates alone, the luminous point is replaced by a band of light standing in a vertical position, and, when S alone vibrates, a horizontal band is seen. When both forks vibrate simultaneously, a curvilinear figure, similar to those of Fig. 279, is observed, from whose form the vibration ratio of both forks may be obtained.

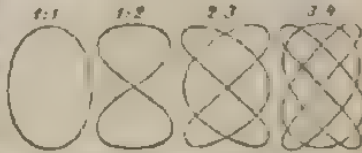


FIG. 279.—Figures of Vibration.

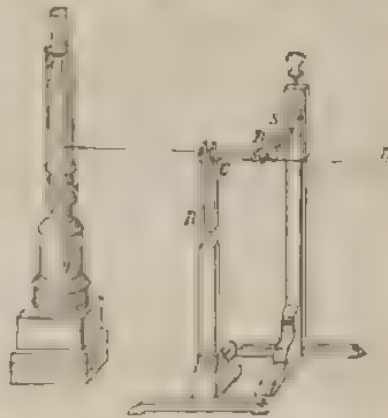


FIG. 280.—Lissajous's Optical Method of comparing two Tuningforks.

**304. Vibrography.**—The vibrations of a tuningfork may be permanently drawn upon a surface by providing its prongs with points (Fig. 281, *v*) made of thin sheet brass, and allowing these points to stand just above a glass plate coated with soot, while the fork is in vibration. Or, the glass plate may be replaced by a cylinder coated with lampblack (Fig. 281, *TT*), which, during the rotation, is carried by means of the threads, *Ab*, along in the direction of the axis, the cylinder at the same

time revolving at a uniform rate, just in front of the tuning-fork. In this case the point draws a waved line (Fig. 282) in the coating, which is a true graphic representation of the law of

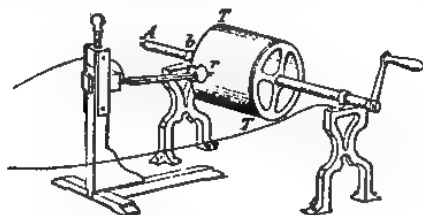


FIG. 281.—Phonautograph.

motion of the fork. Being a sinusoid, it shows that the vibrations of a tuningfork follow the same laws as do those of the pendulum. This apparatus, known as the *phonautograph*, makes possible the accurate determination of the frequency of a tuning-fork. This is done by leading wires from the stand supporting the cylinder and the base of the fork to an *inductor*, and inserting into the circuit a seconds pendulum, so that, at each vibration, the current is closed for an instant. At this instant a spark passes from the point to the cylinder, leaving upon the waved line a distinct mark (Fig. 282, *abc*). The number of vibrations



FIG. 282.—Waved line of a Tuningfork.

of the fork in one second may now be easily counted. If the frequency of the fork is known, the number of waves contained between two such marks furnishes an exact measure of the small interval of time, which elapses between the production of the marks (*tuningfork-chronoscope*, *vibration chronoscope*). To draw waves of air by means of the phonautograph, a funnel-shaped condenser is placed before the coated cylinder, with its narrow end covered by an elastic membrane, carrying a light steel point, sliding gently against the coated surface (phonautograph of Scott and Koenig, 1859).

**305. Interference of Sound Waves.**—Two sound waves of equal pitch and intensity will mutually destroy each other

on combining (*interference*), thereby producing silence, provided they meet in phases differing by half a wave-length. This may be observed when two pipes tuned to the same pitch are placed upon the same organ bellows. The motion of the air must be so regulated that when at the node of one condensation occurs, rarefaction simultaneously takes place at the other. An ear at a small distance from the pipes receives then simultaneously a wave of condensation and of rarefaction, and, as a consequence, the fundamental tone of the pipes is not perceptible, while the overtones for which such opposition of motion does not exist are distinctly audible. Fig. 283 represents an apparatus (Quinke) designed to quench the tone of a tuning-

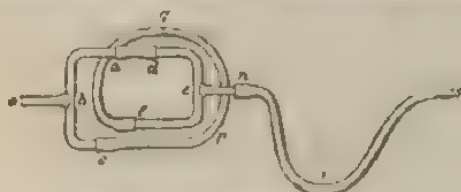


FIG. 283. —Interference of Sound.

fork by interference. Two fork-shaped glass tubes, *obac* and *nedf*, are connected on one side by the short tube (*ad*), and on the other by a longer rubber tube (*fjpc*). If the end, *o*, of the apparatus is placed at the ear, a tuningfork placed before the open end of the rubber tube, *nrs*, is wholly inaudible when the tube, *fjpc*, equals half a wave-length of the tuningfork. A tone is, however, heard at once when the tube is pressed together with the fingers.

**306. Beats.**—When two tones are sounded together, whose frequencies differ but little, alternate increase and decrease of intensity, known to musicians as *beats*, are perceived. If two tuningforks vibrating respectively 512 and 508 times in a second are sounded together, and if, at a given instant, their motions are in such agreement that both send waves of condensation to the ear simultaneously, an intensified sound is heard. This impression is repeated every  $\frac{1}{4}$  of a second, since during this time the first fork vibrates 128, and the second 127 times. After  $\frac{1}{2}$  of a second, on the contrary, the former has

vibrated 64 times, and the latter only 63·5. The second fork is therefore behind the first by half a vibration, and sends a wave of rarefaction to the ear. This exactly neutralizes the wave of condensation simultaneously emitted from the first fork. Four beats are therefore heard each second, which number is merely the difference between the frequencies of the two forks. Generally, the number of beats in a second is equal to the difference of the rates of vibration. If there are more than thirty beats per second, they cannot be perceived separately. They produce a resultant effect upon the ear which is disagreeably rough, and which is the chief cause of what is called *dissonance*. With the aid of these beats, even the unpractised ear may very easily attune two cords, pipes, etc., to perfect unison, because the approach to this condition is indicated by the beats becoming slower and slower. A series of tuningforks, or of reeds, each of which gives with the next following, a definite number of (four) beats per second, may be used as a *tone-meter* (*tonometer*) to determine the frequencies of tones in the neighbourhood of the series.

**307. Difference-tones.**—On sounding two powerful tones together whose pitches are not so nearly equal as to render their beats indistinguishable, a third and deeper tone is heard, whose vibration frequency equals the difference of the frequencies of the two tones. This is called a *combinational tone*, *Tartini's tone*, or, according to Helmholtz, a *difference tone*. It is also known as the *grave harmonic* of Tartini (Sorge, 1745). The next lower octave of a tone, for example, is heard when the tone is sounded simultaneously with its fifth. Helmholtz added to this another tone, called a *summational tone*, whose frequency is the sum of the frequencies of the constituent tones.

**308. Resonance** is the sympathetic sonorous vibration of a body produced by sounding in its vicinity its characteristic tone. We have already had an illustration of this in the sympathetic vibration of a column of air in a tube with a tuningfork, giving the same tone as would be produced by sounding the tube.

If one of two cords stretched beside each other be struck, the other also resounds, if the cords are tuned together, i.e. if

their characteristic tones have been given the same pitch. The second remains silent, however, if they differ in their pitch by never so little. The cord which is struck emits waves of sound, which, striking against the other cord, are able to set it also in motion. If the wave impulse be properly timed with the vibration of which the cord is capable, i.e. if both cords are tuned together, the cord at rest receives a forward impulse just on the instant when it is on the point of moving forward, and a backward impulse during its return. The successive impulses act incessantly in this way to intensify the motion, which, at the first impulse, was extremely weak, so that the cord at length vibrates with sufficient vigour to hurl off paper riders placed upon it. But if the vibration-frequency of the on-coming wave differs from that of the cord, the latter impulses act against the feeble vibration produced by the former, and the effect is destroyed completely. The tones of the cord are distinctly audible, as is well known, only when they are stretched above a wooden *resonance box*, or *resonator* (Fig. 272).<sup>\*</sup> The elastic fibres of the wood, together with the air contained in the resonator, re-enforce, by their sympathetic vibrations, the feeble tone of the cord. The value of a stringed instrument depends very materially upon the quality of its resonator. Tuningforks, which of themselves emit but feeble sounds when fastened upon a box closed at one side, and of length equal to one-quarter of the wave-length of the tone of the fork, are powerfully re-enforced by the air in the box which acts after the manner of a closed pipe. If one of two forks standing beside each other and tuned in unison is made to emit sound, and then, by touching it with the hand, is again silenced, the other fork continues to resound, as may be shown by allowing it to repel an ivory ball suspended so as to touch one of its prongs.

**309. Quality.**—Sounds differ from each other not only as regards *pitch* and *intensity*, but also as regards their *quality* (*timbre*). It is to this latter property of sound that the peculiar character of one and the same note is due, when heard from the violin, clarinet, the piano, or from the human voice. While the intensity of a sound depends only upon the amplitude of

its vibrations, being proportional to the squares of them (53), and the pitch depends only upon the vibration-number, or frequency, the quality depends upon the *vibration-form*. By the vibration-form is meant the form of the wave-line, which represents the law of the condensations and rarefactions of the sounding body (as shown by the phonautograph, for example). In Fig. 284, A and B, the heavy full wave-lines represent two wave-motions of equal pitch, but of different vibration-forms. The former corresponds to the simple motion of a tuningfork according to the laws of the pendulum. The

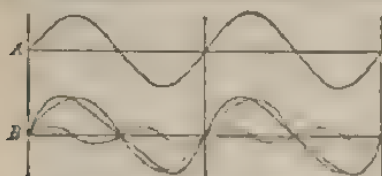


FIG. 284.—Vibration Forms.

latter is compounded of two pendulum-like motions, indicated by the fainter wave-lines, one produced by the fundamental, and the other by its octave. The displacements produced by each

wave singly, along any vertical line, add to each other, so that the longer wave carries the shorter, as it were, upon its back, thereby producing a new wave-form indicated by the heavy line. This compound form, though formed by the fusing together of two pendulum-like vibrations, does not itself conform to the law of the pendulum. In a similar way, any desired motion of vibration whatever, which does not itself follow the laws of the pendulum, may be compounded of simple pendulum-like vibrations, or it may be decomposed into them (Fourier). The frequencies of the component vibrations will bear to each other the ratios of the natural numbers, 1, 2, 3, 4, . . . This decomposition, however, is not merely an imaginary one. It is, in point of fact, unconsciously perceived by the ear. For, according to a proposition due to G. S. Ohm, the ear perceives only a pendulum-like, or simple harmonic motion of the air as a simple tone, while it decomposes every other vibratory motion into simpler constituent harmonic motions, each of which is perceptible in the composite sound as a series of simple tones. The lowest tone heard in a sound is called its fundamental, and the higher are called overtones (partial tones). The great varieties of quality are due to the association of the

fundamental tone with various overtones of greater, or less intensity. To assist the ear, which readily accustoms itself to the perception of every sound as a complete whole, in hearing the separate partial tones, Helmholtz's *resonators* (Fig. 285) are used. These are hollow spheres of glass, or brass, having one mouth, *a*, turned to the source of sound, while the other cone-shaped one, *b*, is placed at the ear. Any given resonator intensifies only those simple tones, to which the air contained in it is attuned, and it is therefore capable of assisting the ear to disentangle only this particular tone from a mixture of sounds. By means of a series of resonators based upon a fundamental tone and the corresponding overtones, one may

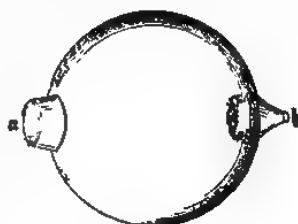


FIG. 285.—Resonator.

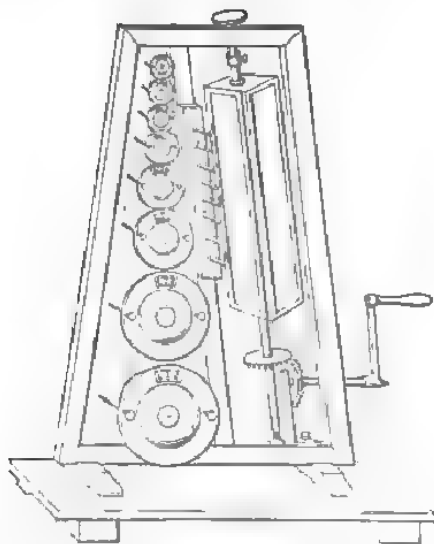


FIG. 286.—Sound Analyzer.

test the composition of any sound with the same fundamental tone by decomposing it into its simple partial tones. This analysis of sound may even be rendered visible by means of

Koenig's *sound analyzer* (Fig. 286). In this, eight resonators are fastened one above the other to a vertical standard, the back opening of each being connected by means of a rubber tube with a manometric capsule (v. Fig. 267). The gas-flames of these capsules are placed above one another along an inclined line, and are observed in a rotating mirror. Those flames whose resonators are affected by the sound give a series of separate flame-images in the mirror, while those whose resonators are not so affected appear in the form of bright bands.

When sounding bodies, such as strings, pipes, etc., are caused to subdivide by nodes, they give, simultaneously with the fundamental tone, all overtones not prevented from forming by the special mode of excitation. To illustrate: if a cord is picked at a point  $\frac{1}{2}$  of the length from the end, and then touched at its middle with a pencil, the fundamental ceases to sound and its octave is heard. If touched at the points,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{6}$  of the length from the other end, there will be heard, in turn, the fifth of this octave (the *duodecimo*), the double octave, and then the third and the fifth of the latter. The partial tones of the seventh order, not occurring in the scales in most extended use, cannot form, because the cord was picked at the precise point where it would require a node. It is the simultaneous sounding of the harmonic overtones with the fundamental and their blending together, which produces the rich and melodious quality that makes chords of such great value in music. If, as is the case with stroked strings, still higher partial tones intermix, the sound becomes rougher and sharper, but gains greatly in expressiveness. Open pipes give the same series of harmonic overtones, but especially the lower ones. If, as with closed pipes, only the overtones of odd frequency are present, the sound seems muffled and hollow. Simple tones, such as those given by tuningforks, are agreeable and soft, but empty and expressionless, and are, therefore, but little used for musical purposes. The ringing sounds of bars and plates whose fundamental is accompanied by high and unharmonic overtones (Helmholtz, 1865) are still less musical. The consonance of two sounds is always more complete, the more partial tones they have in common.

**§10. Production of Vocal Sounds.** — The human voice is produced by the vibration of two elastic membranes, or bands, called vocal cords, which are stretched in the larynx from the front backwards so as to form the edges of a narrow space called the *vocal slit*. By the vibration of the cords, the slit is alternately opened and closed in such way as to interrupt the current of air passing from the *trachea*, or windpipe, at regular intervals. This produces a sound which is higher, or lower, according as the cords are more, or less, tensely stretched by the action of certain voluntary muscles controlling them. This sound is quite rich in overtones. By varying the size and form of the cavity of the mouth one, or more, of these overtones may be especially emphasized so that the quality of the voice may be varied in a multitude of ways. The cavity of the mouth acts as a resonator to increase, by the simultaneous vibration of the air contained within it, the intensity of those overtones to which it is momentarily attuned. The differences of vowel sounds depend upon the variations of quality produced in this way. While with *u* almost the pure fundamental tone is heard, with *o* the octave is associated with its fundamental, and with *a*, *e*, and *i*, still other overtones are intermixed. Besides these overtones, which refer to the fundamental and are pitched in accordance with it, each vowel is distinguished by one, or more, other overtones of definite pitch. These overtones are merely the instantaneous tones of the cavity of the mouth. If this cavity is brought alternately into the positions corresponding to the vowels *a* and *o*, and these sounds be successively uttered, while a vibrating tuningfork, whose tone is  $b$ , is held before the mouth, the air in the mouth will vibrate vigorously with the *o*, while with the *a* no vibration will occur. But if the fork is tuned to  $b_2$ , the response is heard with *a*, but not with *o*;  $b_2$  is, therefore, the characteristic tone for *o*, and  $b_3$  for *a*.

If the vowels are spoken with the same position of the vocal organs against the membrane of a manometric flame, the zigzag row of flames seen in a rotating mirror shows the difference in the composition of the vowel sounds.

The consonants are brief and impure sounds produced by the lips, tongue, teeth, and gums.

**311. Phonograph — Gramophone.**—By means of the phonograph of Edison (1877) the sounds of the human voice, and, in fact, any sound may be permanently recorded and reproduced at will after any length of time. A brass cylinder (C, Fig. 287) is borne by an axis, AA', the threaded end of which,



FIG. 287.—Phonograph.

A', works in a nut, cut in the support. Upon the surface of the cylinder is cut a spiral groove of the same pitch as that of the screw, A'. The cylinder is coated with a thin sheet of tinfoil, or in the later improved apparatus with a removable layer of wax. The coating is used for the reception of characters. The writing apparatus consists of a mouthpiece, D (Fig. 288), back of which a thin plate, E, stretched like a drum-head, which

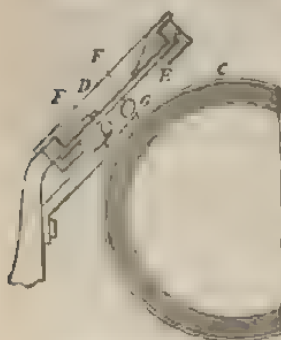


FIG. 288.—For the Phonograph.

presses a style, G, borne by a metallic spring, with the help of the dampers, FF (pieces of rubber), against the cylinder. When the crank, B, is turned, the style describes a spiral line following a groove of the cylinder. If the mouthpiece is spoken against while the cylinder is rotated uniformly, the plate vibrates and the style produces impressions in the tinfoil, or the wax. The profile of these impressions imitates the form of vibration of the spoken sounds. To reproduce these

sounds, the writing apparatus is raised from the surface, and the cylinder is turned backward until the style and mouthpiece are again in the initial position. On rotating the cylinder as at the outset, the style, gliding over the impressions in the foil, sets the plate into precisely such vibrations as previously made the impressions upon the cylinder. The apparatus thus

reproduces spoken words with more or less distinctness, and with a quality similar to that of the original sound.

In the gramophone (Berliner, 1888) the waves of sound collected by a guide, are written upon an etching-ground, by means of a vertical membrane carrying a writing style at its centre, as spiral wave-lines upon a horizontal metallic disk, which lines are then etched in. The plates so obtained may be duplicated any number of times. To reproduce the sounds (speech, cries of animals, pieces of music, etc.), the style of a vertical membrane is drawn along in the depressed wave-line by rotating the disk, and the membrane is thereby set into corresponding vibration which reproduces the original sounds and may repeat them indefinitely.

312. *Hearing*.—The external ear is closed at its inner end by a drum, or tympanum, from which the train of small bones (hammer, anvil, and stirrup) transmits the vibrations through the *tympanum cavity* to the liquid contained in the *labyrinth*. The latter consists of the *vestibule*, the *semicircular canals*, and the *cochlea*, or *snail shell*. In its bony walls are two openings which may be closed by membranous covers. The openings are termed the *round* and the *oval windows*. The stirrup is attached by means of its foot-plate to the membrane of the oval window. The auditory nerve branches in the labyrinth. Its slender terminals are connected in the *snail shell* with a row of vibrating fibres, *Corti's fibres*, each of which, like the cord of a harp, has its own definite tone consisting of between 30 and 16,000 vibrations. Whenever, by stimulating the corresponding nerve, one of these fibres is set in vibration, the simple tone peculiar to it is perceived.

## X. LIGHT.

## (OPTICS.)

**313. Light—Sources of Light.**—Any stimulation of the optic nerve, which spreads out as the retina over the inner surface of the eye, awakens in consciousness the sensation of *brightness*. This is moreover true, no matter what the nature of the stimulus may be. A stroke, or pressure, upon the eyeball, any alterations of a galvanic current passed through the eye, and even the circulation of the blood in the vessels of the retina, are all perceived by the eye as brightness.

Whenever an external object is perceived a certain something must proceed from it, penetrate to the retina, and stimulate it. This something—this cause of the visibility of objects—is called *light*.

Bodies which emit light, such as the sun, the stars, flames, incandescent solids, are called *self-luminous bodies*, or *sources of light*.

All artificial sources are based upon the development of light by the *glowing* of solids. An ordinary gas flame (as also the flame of a candle, or of a lamp) owes its luminosity to finely divided particles of carbon floating in an incandescent condition within the glowing gaseous mass, which, coming to the edge of the flame, combine with the oxygen of the air, and burn to carbonic acid. The existence of these carbon particles may be easily shown by holding a cool body in the flame. The film-like particles of carbon are deposited upon its surface in the form of *soot*. In *Bunsen's flame* the illuminating gas admitted into the tube of the burner through a rubber pipe, mixes with the air flowing in behind it, through a lateral opening. This flame contains, therefore, within it, the oxygen needed to consume the

carbon. The carbon burns then to gaseous carbonic acid, before it can be deposited from the flame. The flame itself consists wholly of glowing gases, which possess a far lower luminosity than do glowing solid particles. It emits, therefore, a feeble bluish-green light, but, in consequence of more complete combustion, it develops considerably more heat than an ordinary gas flame and deposits no soot.

A piece of calcium, heated to white glowing by an ordinary gas flame saturated with oxygen, furnishes a bright white light (*Drummond's calcium light*, 1836). To produce this light conveniently, the *calcium lamp* is used. With this lamp the flame plays obliquely upward against a bar of calcium placed on an adjustable support, and held in front of the curved tube of the burner. The latter consists of two concentric tubes, the inner of which conducts the oxygen from a gasometer into the flame of illuminating gas, which flows from the circular passage between the tubes.

The *magnesium light* is produced by burning magnesium. This lustrous metal is drawn along in the form of a ribbon between a pair of small cylinders by clockwork into an appropriately constructed lamp, while an alcohol flame is allowed to play against it. The magnesium burns to solid magnesium oxide, at the same time emitting a blinding white light, accompanied by a cloud of dense white smoke.

Electrical sources of light also depend upon the glowing of solids. A carbon filament, heated to glowing within a partially exhausted glass globe by means of an electric current, furnishes the *incandescent electric light*, while the glowing carbon points, between which the current passes in the form of an arc, furnish the *electric arc light*. The latter is the most brilliant of all artificial sources of light.

**314. Non-luminous Bodies—Diffuse Reflection.**—Non-luminous (dark) bodies can be seen only because they send back the light, which they receive from self-luminous bodies, from their rough surfaces in all directions by the process known as *diffuse reflection* (diffusion of light). A body thus illuminated acts in turn as a source of light. It shines with borrowed light. The moon and the planets are the heavenly bodies known to be in

this condition. They, as also the terrestrial objects about us, are illuminated by the sun. The diffused sunlight, given off by clouds, atmospheric particles, and objects on the surface of the earth, produces not only the general brightness of the day, but it also converts the earth and the other planets into shining bodies. The tender shimmer of light visible over the feebly illuminated portion of the moon's disk near the time of new moon, is merely the counter-glow of the earth illuminated by the sun (*earth-shine*).

**315. Transparency.**—Bodies, such as water, air, glass, etc., which transmit light, are called *transparent*; when they transmit it only imperfectly, as is the case with horn, ground-glass, etc., they are called *translucent*, and when they transmit no light at all, they are *non-transparent*, or *opaque*. This distinction is, however, not based upon an opposite behaviour of substances. For it is possible on the one hand to prepare sheets of the most densely opaque bodies, *i.e.* the metals, so thin that light will shine through them, while, on the other hand, transparent bodies become less and less permeable to light the thicker they become. At great depths in the sea almost absolute darkness prevails, because the thick layer of water above can be penetrated by only the palest shimmer of light.

**316. Rectilinear Propagation of Light—Shadows.**—An opaque body is illuminated by a luminous point only upon the side turned toward the point. The other side, as also a definite portion of space behind the body, called the *shadow*, remains in darkness. A sharply outlined, circular, dark spot, called the *shadow spot*, is formed upon a plane surface placed within the shadow. It may be readily shown that any straight line drawn from the luminous point toward a point of the shadow spot, passes through the opaque body, and that only those points of the screen receive light which are so situated that the straight lines, drawn from the luminous point toward them, pass without the shadow-casting body.

We conclude from these facts that light (in a homogeneous medium) is transmitted from a luminous point in straight lines, called *light-rays*.

If, as heretofore assumed, the source be a point, the shadow (Fig. 289) extends behind the body in the form of a diverging cone. This cone is limited by the rays, which pass along tangent lines to the shadow-casting body. The points of tangency form a line about the body, separating the illuminated from the unilluminated side.

If the light proceed from a luminous body (A, Fig. 290), which contains a countless number of luminous points, to understand the constitution of the shadow, it is necessary to consider the shadow cone of every individual luminous point,



FIG. 289.—Shadow.

as has been indicated in Fig. 290, for two points. The space behind the opaque body (B), which is common to all the cones, receives no light, and is termed the *umbra* (BS). This space is bounded by another extending backward in the form of a divergent cone, within which all points receive a portion of the luminous rays and are accordingly partially illuminated. This latter region is designated the *penumbra*. Upon a plane held



FIG. 290.—Umbra and Penumbra.

at *mn*, perpendicularly to the axis of the cone, is formed the shadow-spot, shown at the side of the figure. This consists of a perfectly dark spot corresponding to the umbra, surrounded by a circular court less intensely dark, and whose darkness diminishes continuously outward, passing at the edge gradually into full illumination. This court is the *penumbra*. The shadow-spot is sharper the nearer the screen is placed to the shadow-casting body, because the breadth of the diffuse penumbra becomes less the nearer the body is approached. If the source of light, A, is greater than the obstacle, B, the penumbra forms a cone contracting backward and coming to a

point at *S*. This condition of things prevails in the illumination of the planets by the sun. The umbra behind the moon almost exactly equals the radius of the moon's orbit, and may, therefore, when the moon passes between the sun and the earth, as sometimes occurs at new moon, extend to, and, under favourable conditions, even beyond the surface of the earth. For all places within the umbra a complete obscuration of the sun is produced by the moon and a *total solar eclipse* occurs. At those places lying within the penumbra, a crescent-shaped portion of the sun's disk remains visible, and the eclipse is only *partial*. The umbral shadow of the earth extends to a distance of 216 radii of the earth, reaching, therefore, far beyond the orbit of the moon, whose mean distance is only 60 radii of the earth. At the time of full moon, it may happen that the moon dips wholly, or partially, into the earth's shadow, and give what is known as a *lunar eclipse*.

317. *Camera Obscura* (Porta, 1589).—If a small aperture (from 1 – 3 mm. diameter) is made in the shutter of a dark

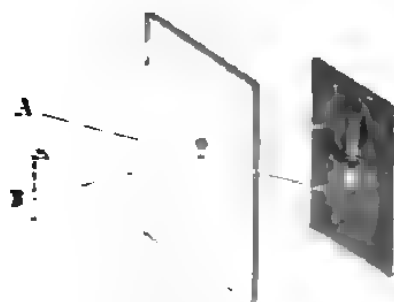


FIG. 291.—Formation of Image by a small Aperture.

room, an inverted image of external objects, with all their forms and colours, is seen upon a paper screen placed upon the opposite side of the room. The origin of this image is explained by the following experiment. A burning candle (or an incandescent electric light) is placed in front of a screen provided with a small aperture, (O, Fig. 291),

while a white paper screen stands behind it. Of the countless light-rays emitted in all directions by the point *A*, for example, of the flame, a slender conical bundle, *Aa*, passes through the orifice and produces upon the screen a small bright spot, *a*, which by virtue of the rectilinear propagation of light is illuminated only by the light from *A*. For this same reason, no other portion of the screen can receive light from the point *A*.

Similarly, the point, *b*, at the top of the screen is illuminated only by the lower point, *B*, of the object. And thus each point of the object sends its light separately to some other point of the screen, and by the continuous aggregation of the numberless bright spots, an image, *ab*, is formed, which, as appears from the drawing, is inverted and similar to the object, *AB*. The image grows larger the further the receiving screen is removed from the aperture; but at the same time it grows more indistinct, because the same quantity of light must then be distributed over a greater area. It is apparent that only small apertures can produce such images; for they alone are able to separate the rays of light, and this separation is the fundamental condition to the formation of an image. Wide apertures fail to accomplish this separation because, at every point of the screen, rays from many, or all, points of the object are superposed. The smaller the aperture the sharper, but, at the same time, the less brilliant is the image.

Since the numberless individual spots of light, of which the image is composed, overlap at the edges and thus obscure their outlines, the form of the aperture has no influence upon the image of the whole. The irregularly shaped openings between the leaves of trees act as so many small apertures, and cast countless round images of the sun upon the shaded ground. During a partial solar eclipse, these spots show a distinctly crescent form.

**318. Visual Angle.**—The angle formed between the lines drawn from the extreme points of the object, *AB*, to the corresponding points of the image (Fig. 292, *ab*) formed upon the retina of the eye by the object, is called the *visual angle*. These lines intersect within the eye at the so-called *intersecting point*. An object appears larger to the eye, the greater the area of the retina covered by its image. The *apparent size* of an object, therefore, depends upon the visual angle under which it is seen. A body is seen under a smaller visual angle.



FIG. 292.—Visual Angle.

and its apparent size is, therefore, correspondingly smaller, the farther it is removed from the eye, and two bodies of unequal size ( $AB$  and  $A'B'$ , Fig. 292) appear under the same visual angle, when their distances from the eye are proportional to their diameters. If the true size of an object is known, its distance from the eye may be inferred from the visual angle, and conversely, when the distance and the apparent magnitude are known, the real magnitude may be estimated. Astronomers use these simple considerations to obtain the distance and size of heavenly bodies. From suitably made observations, for instance, it has been found that the radius of the earth seen from the sun, would subtend a visual angle of only  $8.85$  seconds of arc (this magnitude is called the parallax of the sun), and from this value the distance of the earth from the sun is computed to be  $23,500$  times the earth's radius. After this distance has become known, it is found from the visual angle of  $32$  minutes under which the sun is seen from the earth, that the solar diameter is  $110$  times as great as that of the earth. Our judgment from earliest youth has been unconsciously performing the operations which have led the astronomer to the above results, whenever we have estimated by the eye, the distance and size of terrestrial objects. The visual angle, under which a human form, or other object, of known size appears, furnishes a criterion for judging of its distance, and the known distance enables us to infer the real size of the object. Since the apparent diameter of the sun is only  $32'$ , solar rays can diverge in their directions by no greater angle than this (approximately  $\frac{1}{2}^\circ$ ), and, for this reason, they may be regarded as practically parallel.

**319. Photometry.**—If, about a luminous point, a number of spherical surfaces be described, whose radii are related as  $1 : 2 : 3 : 4$ , etc., each surface, if alone, would intercept the entire quantity of light emitted from the point, and be thereby illuminated. Since the surfaces of these spheres are as the squares of their radii, the same number of rays is spread out over surfaces which are twice, three times, four times, etc., as far from the luminous point, and whose areas are accordingly  $4, 9, 16, \dots$  times as great as that of the unit sphere. Equally

large portions of the surfaces at 2, 3, 4, . . . times the distance, must therefore be illuminated with  $\frac{1}{4}$ ,  $\frac{1}{9}$ ,  $\frac{1}{16}$ , . . . the intensity at the distance unity. *The intensity of illumination of a surface is, therefore, inversely as the square of its distance from the luminous point.*

This law is used to compare the illuminating powers, or luminous intensities, of two sources of light as well as to measure them in any arbitrarily chosen unit. Apparatus designed to be used for this purpose are called *photometers*. All these instruments depend upon the equalizing of the luminous intensities of two surfaces lying beside each other by the variation of the distance of the luminous sources to be compared. This may be done with great accuracy, since the eye is capable of appreciating a difference of from  $\frac{1}{16}$  to  $\frac{1}{128}$  of the luminous intensity of either. According to the above law, then, the intensities of the two sources are to each other as the squares of the distances from the equally illuminated surfaces.

The photometer of Rumford (Fig. 293) is extremely simple.



FIG. 293.—Rumford's Photometer.

An opaque bar, *c*, of about the thickness of a lead-pencil, stands just in front of a surface of white paper, *ab*. Each of the sources to be compared casts upon the screen a shadow (*d* and *e*) of the post, and each shadow is illuminated by the source that produces the other. By displacing the source (*s*) the two shadows, which for better comparison are brought near each other, may be readily made equally bright. The paper surface is now illuminated equally by both sources. By the principle enunciated above, the luminous intensities of the flames must then be in the ratio of the squares of their distances from the surface. Ritchie's

process was to illuminate, with the intensities to be compared, the two perpendicular faces of a wooden wedge, *p* (Fig. 294), covered with white paper, the wedge being enclosed within a blackened chamber, whose two sides facing the surfaces of the wedge were provided with openings, *oo*. By means of a tube, *r*, opening through the upper wall of the chamber, both sides of the wedge may be observed simultaneously, and by displacing



FIG. 294.—Ritchie's Photometer.

the sources of light, the surfaces may be brought to equal brightness. The photometer of Bunsen (Fig. 295) is more accurate than either of the two just mentioned, and for technical purposes is the instrument now most extensively used. It consists essentially of a paper membrane, in the centre of which is placed a spot of stearine. This spot appears bright against

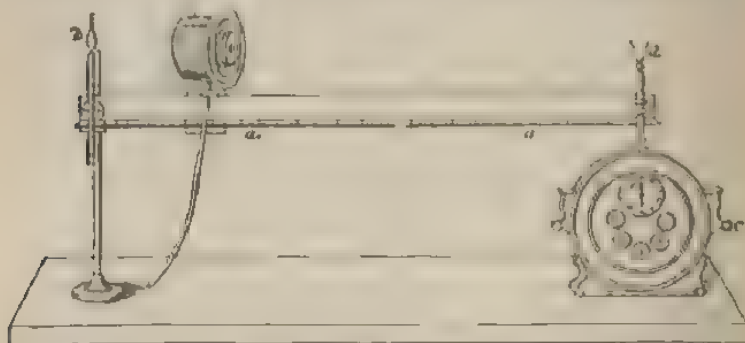


FIG. 295.—Bunsen's Photometer.

a dark ground, whenever the membrane is more intensely illuminated from behind, and dark upon a bright ground, whenever the front surface is the more intensely illuminated. When two sources of light are to be compared, the one is placed in front of, and the other behind the membrane, whereupon, by shifting one of the flames, or the membrane, a position is soon

found, where the spot of stearine can no longer be distinguished from the general surface. When this occurs, the illumination is the same on both sides, and the ratio of the intensities is again found by the same simple computations as above. To avoid the necessity of computation, the bar along which the paper screen slides, and at the ends of which the luminous sources are placed, may be so graduated that the intensity can be immediately read off from the position of the membrane. Such a graduation is shown in the arrangement of the Bunsen's photometer represented in Fig. 295. At one end of the graduated bar, *aa*, the flame, *b*, is placed, and is used as the unit of comparison (the normal flame). At the other end the source to be tested, for example, a gas flame, *d*, is placed. A gas-meter, *c*, indicates the hourly consumption of gas. A cylindrical box, whose rear wall is opaque, slides freely along the graduated bar, while a paper screen with its spot of stearine is placed in the front wall. A small gas flame burns within the box. The box is now brought to a point 20 cm. from the normal flame, and the small flame is then so regulated that the spot turned toward the normal flame disappears. The box, together with its membrane, is turned toward the flame, *d*, to be tested and moved along the bar until the spot again vanishes. A pointer rigidly connected with the box indicates, then, upon the bar the desired intensity.

If the flames are unequally coloured, these measures will be unreliable, because the eye is not certain in its judgment of the equality of two differently coloured illuminations.

The normal flame used in Germany is that of a paraffin candle of 2 cm. diameter, burning with a flame 50 mm. high, or the amylacetate lamp of Hefner (Hefner's lamp), whose height of flame is 40 mm. The ratio of the light of Hefner's lamp to that of the normal candle is 1 : 1.2. The International Conference of Electricians (1884) established as unit of intensity that of 1 sq. cm. of molten platinum at the temperature of solidification.

Any one of the photometers just described may serve the converse purpose of confirming the above fundamental law experimentally, by proving that four candle flames standing

close beside one another produce the same illumination at double the distance, as one flame does at the single distance.

Thus far it has been assumed that the plane surface, MN (Fig. 296), to be illuminated, is struck by the rays perpendicularly, or almost so. If, however,



FIG. 296:—Oblique Illumination.

the surface be turned through the angle,  $i$ , into the position, NO, it will be met by a beam of rays whose cross-section, MP, is to the entire surface, MN, or MO, as  $\cos i : 1$ . The angle,  $i$ , which is equal to that between the directions of the incident ray and of the perpendicular, MQ to MO, is called the *incident angle*, or *angle of incidence*. The intensity of illumination of obliquely incident rays is therefore proportional to the cosine of the *angle of incidence*.

**320. The Velocity of Propagation of Light** is so very great that the greatest terrestrial distances by which light signals can be separated are traversed almost instantly. The Danish astronomer, Olaf Roemer, was (1676) the first to obtain its value. He made use of celestial light signals in his determination. The largest planet of the solar system, Jupiter, is encircled by five moons, which at each revolution suffer eclipse by passing into the shadow cast by the planet. With the second moon (i.e. the one next to the nearest to Jupiter) the time between two successive eclipses is 42 hours, 28 minutes, 36 seconds. Roemer found that when the earth has reached its greatest distance from Jupiter, the eclipse is seen later by 16 minutes, 36 seconds than it should have occurred according to computation, if the earth had remained at its least distance from Jupiter. This retardation is merely the time consumed by the light emitted by Jupiter's satellite at the instant of eclipse, in traversing the distance equal to the difference between the least and the greatest distances of the earth from Jupiter. Since this distance equals the diameter of the earth's orbit, i.e. about 299 million km., and is traversed in 996 seconds, light must traverse in one second about 300,000 km.

Bradley derived the same number fifty years later from the *aberration of light*. Suppose the axis,  $mo$  (Fig. 297), of a telescope, AB, to be directed toward a heavenly body, e.g. a star. The rays of light coming from the star will be condensed at the point,  $m$ , into an image of the star. If now the axis of the

telescope move parallel to itself in the direction,  $m'm$ , perpendicular to the incident rays, and at such rate as to traverse the distance,  $m'm$ , during the time consumed by the light in passing from  $o$  to  $m$ , the rays of light entering at any instant through  $o$ , unaltered by the motion of the telescope, are always condensed at the same point,  $m$ , into a focus. But this point, which was occupied at the



FIG. 297.—Aberration.

instant in question by the centre of the field of view, will, at the instant of focussing the light, be occupied by the point,  $m'$ , lying at the side of the field. The image of the star is then, in consequence of the motion of the telescope, seen at a point of the field, where, with a stationary instrument, incident rays coming from the direction,  $s'om'$ , would be focussed. By virtue of this so-called "aberration of light," instead of seeming to occupy its true place, the object is seen in the direction,  $mos$ , and, to bring its image to the centre of the field, the axis of the telescope must be pointed in this direction by turning it through the angle,  $mom'$ . A telescope is, in fact, always in motion, being carried about by the earth in its annual journey about the sun. Every star, therefore, whose rays strike the path of the earth perpendicularly, appear displaced in the direction of the instantaneous motion of the earth by an angle,  $mom'$ , whose magnitude depends upon the ratio of the distance,  $m'm$  to  $om$ . The former distance being that traversed by the earth, and the latter that traversed by light during the same interval of time, the size of the angle,  $mom'$ , may be said to be determined by the ratio of the velocity of the earth to that of light. This aberration angle, as it is called, which is the same for all the heavenly bodies, may be measured. It is quite small, being only about 20"·25 seconds of arc. But in a right-angled triangle,  $mom'$ , whose angle at  $o$  is 20"·25, the side,  $om$ , must be 10,000 times as great as the side,  $mm'$ . Consequently, the velocity of light must be 10,000 times as great as that of the earth in its orbit. But the earth traverses 30 km. every second, and hence, during the same time, light must traverse 300,000 km.

By a very ingenious method, Fizeau (1849), and later Cornu, measured the velocity of light from terrestrial sources. If, through one of the spaces on the circumference of a toothed wheel, a ray of light be let fall exactly perpendicularly upon a mirror situated at some distance, this ray will return along the same path, and, if the wheel is at rest, it will pass through the same space, to the eye of the observer. But, if the wheel be rotated with gradually increasing rapidity, a rate will be reached at which, during the time consumed by light in passing from the wheel to the mirror and back again, the wheel will have advanced by the breadth of a tooth, so that the returning ray strikes the tooth, which now occupies the position formerly occupied by the space, and the observer does not see it. Finally, Foucault (1850), by a method to be explained later, succeeded in measuring the velocity of light, even within the contracted limits of a single room.

From these experiments with terrestrial light, its velocity was again found to be 300,000 km. In one second, therefore, a ray of light traverses a distance 7.5 times as great as the circumference of the earth (40,000 km.). The fixed stars are so inconceivably remote that, notwithstanding its tremendous velocity, light consumes years in passing from them to us. If Sirius were extinguished at this moment, we should see him twinkling on for fourteen years, for it would require this interval for the last rays, which have already been emitted by this luminary, to reach the eye.

**321. Law of Reflection.**—If a ray of light, *am* (Fig. 298), falls



FIG. 298.—Reflection of Light

upon a mirror, *as* (this term is applied to any smooth surface), a portion of it is reflected in a definite direction, *mb*, from the surface into the space in front of the surface. For convenience of reference to the directions of the incident (*am*) and of the reflected (*mb*) rays, a perpendicular is imagined to be drawn

to the reflecting (plane, or curved, surface at the point, *m*, where

the incident ray impinges against it, and this perpendicular is called the *normal*. The plane, passing through the incident ray and the normal (the plane of the drawing), and hence perpendicular to the reflecting surface, is called the *plane of incidence*. Since it always contains the reflected ray, it is also called the *plane of reflection*. The directions of the incident and reflected rays are determined by the angle of incidence,  $i$ , and the angle of reflection,  $r$ , which the incident ray on the one side and the reflected ray on the other, make with the normal. *The angle of reflection is always equal to the angle of incidence.* A ray ( $pm$ ) striking the mirror perpendicularly, is reflected into itself (toward  $mp$ ).

From this law it follows immediately that all rays (Fig. 299,  $am$ ,  $am'$ , ...) passing from a luminous point,  $a$ , and striking against a plane mirror, are reflected along straight lines ( $mb$ ,  $mb'$ , ...) as though they came from a point,  $a'$ , lying upon the perpendicular,  $aa'$ , dropped from the luminous point upon the surface, and situated just as far behind the reflecting surface as the luminous point,  $a$ , lies in front of it. An eye, placed in front of the mirror (e.g. at  $b''$ ), receives the reflected ray as though the point,



FIG. 299. - Formation of a Punctual Image by a Plane Mirror.

$a'$ , situated symmetrically to  $a$ , with respect to the reflecting surface, were itself a luminous point. Or, in other words, the eye sees in (i.e. behind) the mirror, in the direction  $b'a'$ , the point,  $a'$ , as the image of the point,  $a$ , situated in front of the mirror. To every point of either a luminous or of an illuminated object, there corresponds, in a similar way, an image behind the mirror, and the sum total of all these *punctual images* constitutes the *image of the object*. To draw this image, imagine a perpendicular to be dropped from each point of the object upon the mirror and prolonged to a point just as far below as the point in question lies above it. The image is, therefore, not wholly identical, or *congruent*, with its

original. It is said to be situated *symmetrically* with respect to it. The right hand shows in the mirror as the left, and the letters in the image of a written page proceed from right to left instead of from left to right, as in the book itself.

One may readily convince himself of the symmetry of positions of image and object by selecting as mirror, a transparent plate of glass, through which the object itself may be seen in the position formerly occupied by its image. From this symmetry of object and its image we may reason conversely to the correctness of the law of reflection. Let a lighted candle be placed in front of the glass plate just mentioned, and behind the plate, at the position of the image found as above, let a flask of water be placed. An observer, standing in front of the mirror, then gets the impression of a candle burning under the water within the flask. This simple experiment explains the appearances known under the name of "ghosts."

**322. Applications of Plane Mirrors.**—The *helio-stat* is an apparatus for turning the rays of the sun to a definite (*e.g.* horizontal) direction into a darkened room. It consists of a plane mirror, which may be either set by hand, or driven by clock-work, so as to follow the sun in such way as to reflect its rays always in the desired direction.

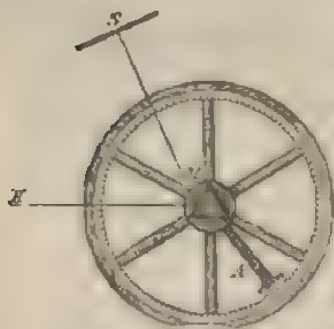


FIG. 300.—Principle of the Reflecting Goniometer.

The small plate, *M* (Fig. 300), with which the pointer, *A* (alidade), provided with a vernier, is rigidly fixed, moves about the centre of a circle which is graduated to degrees. Let a slender beam of parallel rays from the sun fall upon a glass prism, which stands upon the plate, the rays being sent by means of a helio-stat through a vertical slit into the room. The rays reflected

from the front surface of the prism will produce, upon a screen, *S*, a bright vertical streak, whose position may be

designated by a mark. If now the alidade, and with it the prism, be turned until a second surface of the prism reflects the rays in the same direction, MS, *i.e.* until the bright streak is again exactly at the mark, the second surface must then have precisely the same position as the first had formerly. If the second surface was parallel to the first, the alidade would obviously have to be turned through  $180^\circ$  to bring the streak again to the mark. But, if the second surface forms the angle,  $\alpha$ , with the first, the coincidence of streak and mark is reached by a rotation of  $(180 - \alpha)$  degrees. To determine the angle,  $\alpha$ , between the two surfaces, it is necessary only to subtract the angular rotation of the alidade from  $180^\circ$ . Instruments constructed on this principle are used for the accurate measurement of the surface-angles of prisms and crystals, and are called *reflecting goniometers* (Wollaston, 1809).

When the plane of the mirror is turned through any arbitrary angle, the reflected ray is at the same time turned through twice the angle (*conf.* Figs. 136, 302).

The application of plane mirrors to the measurement of small angles of rotation (Poggendorf, 1827), as also that of Jolly's reflected scale, have been mentioned above (100, 104, and 143).

**323. Inclined Mirrors.**—Since the rays reflected from the image behind the mirror proceed from it precisely as though a real object were situated there, the image, when placed before a second mirror, may play the *role* of an object. By using two mirrors with reflecting surfaces turned toward each other, there will be formed, in addition to the two direct images (of the first order), others of the second, third, and higher orders, which, from loss of light due to repeated reflections, become gradually less brilliant. If, for illustration, a lighted candle,  $a$ , is placed between two parallel reflecting surfaces, there will be seen in each mirror an indefinite series of candle flames, which appear to fade out gradually with increasing distance from the candle. The number of these images is limited when the mirrors form an angle with each other (*inclined mirrors*, Fig. 301). The mirrors MN and RN, furnish the images, B and  $B_1$ , of the first order of the object, A, situated between

them. The image, B, behind the first mirror sends its rays to the second mirror, which forms an image of the second order, C, and in the same way the first mirror forms an image, C', of the image B. But with the angle of 72 represented in the drawing, no further images are possible. An eye at O, between

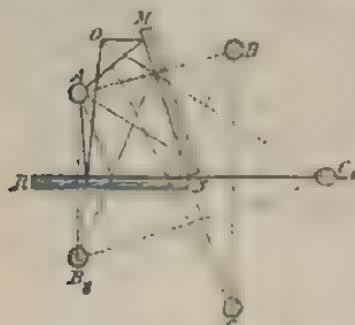


FIG. 301.—Inclined Mirrors.

the mirrors, sees the images and the object arranged regularly upon a circumference described about the point of intersection of the mirrors, and at each angular interval, equal to the angle between the mirrors, an image is formed. The eye at O sees, therefore, the object repeated as many times as this angle is contained in 360. Upon this regular arrangement of the images of angular mirrors depends the beautiful effects of the *kaleidoscope* (Brewster, 1817). This consists of two long reflecting strips, inclined to each other at an angle of 60°, the strips being inserted within a blackened tube. At one end of the tube is a small hole for viewing, while at the other, between two plates of glass, the outer of which is rendered translucent by grinding, is put a number of coloured fragments of glass, bits of feather, sprigs of moss, small seeds, etc. Looking through the small view aperture, the eye sees six images of each of these objects blended together in the form of a six-pointed star. On shaking the tube these images readjust themselves and offer to the eye an inexhaustible series of highly ornamental figures.

**384. The Sextant.**—In Fig. 302, let A and B be two small plane mirrors with their reflecting surfaces facing each other. Let L and R denote the positions of two objects, the former of which is visible to an eye at O, over the mirror, B, in the direction OB. The mirror, A, may be given such a position, that the light coming from R, after two reflections, may reach the eye along the path, RABC, so that both objects will be seen in the same direction, OB, the one directly, and the other by reflection. From the law of reflection it follows that the angle, LOR =  $\alpha$ , included between the lines of sight toward L and R, is twice as large as the angle at B =  $\beta$ , between the planes of the mirrors. If the normals AB and BD be supposed drawn, which intersect in D

at the angle  $\beta$ , and if  $\phi$  and  $\psi$  denote the incident angles of the rays, RA and AB, upon the mirrors, A and B it follows from the relations of the triangle, ABD, that  $\beta = \phi - \psi$ , and from the triangle, AOB, that  $\alpha = 2\phi - 2\psi$ , whence  $\alpha = 2\beta$ . To measure the angle,  $\beta$ , the mirror, A, is made movable about the centre of a graduated arc, MN, and connected with an arm (alidade, AZ, carrying a pointer above the graduations. The mirror, B, is fixed upon the plane of the circle and parallel to the radius, AM, passing through the zero of the graduation. Looking now along the direction, OL, through a telescope attached to the instrument toward the object, L, and at the same time turning the alidade, together with the mirror, A, the image of B will appear at length in this same direction. When both images are made to coincide, double the value of the angle indicated by the alidade gives immediately the angle,  $\alpha$ . This ingenious instrument, invented by Newton, and first constructed by Hadley (1731), is called the *sextant*, because to measure angles up to  $120^\circ$ , the arc, MN, must equal  $\frac{1}{2}$  of a circle (i.e. a sextant). The chief distinguishing characteristic of the sextant as an instrument for angular measurement is that it does not require a fixed mounting, but may be held freely in the hand during measurement. For this reason it is the only useful apparatus for measuring at sea the angles from which the seafarer determines the geographical longitude and latitude of his vessel.

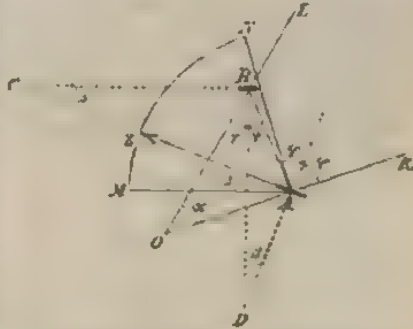


FIG. 302.—Principle of the Sextant.

**325. Spherical Mirrors.**—A spherical shell, polished on its inner surface, forms a *concave mirror*. The centre of the sphere, of which the shell is a part, is called the *centre of curvature*, and lines drawn through this point are called *axes of the mirror*. That particular axis which passes through the lowest point, or *vertex* (O, Fig. 303), of the concave surface is designated the *principal axis*, or *axis*, simply. The angle,  $\text{MCM}'$ , included between the lines drawn from the centre of curvature to two diametrically opposite points of the perimeter of the mirror, is called the *opening*, or *aperture*, of the mirror.

A ray, passing along an axis (an *axial ray*), meets the surface perpendicularly to the mirror, and is therefore reflected into itself. If a beam of parallel solar rays (Fig. 303) falls upon a concave mirror of *small* opening, and at a small angle with the principle axis, the rays will be reflected in the form of

a cone, whose vertex, *F*, lies in front of the mirror, upon the axis parallel to the incident rays. This point, *F*, through which all incident rays parallel with an axis pass after reflection, is called the *focus* of the corresponding axis, that lying upon the principal axis being called the *principal focus*.

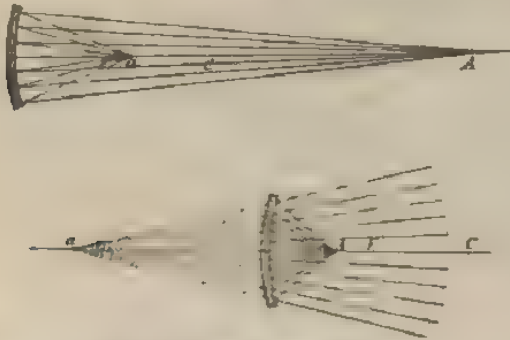


FIG. 303.—Focus of a Concave Mirror.

When caught upon a piece of white paper the principal focus appears as a white spot of blinding brightness, until, at length, the paper catches fire under the powerful thermal effect of the condensed rays, thereby showing that the name focus, meaning “burning-point,” is well chosen. From this effect, the concave mirror is sometimes called also the *burning mirror*. The focus lies upon the axis, exactly midway between the mirror and its centre of curvature, or the *focal distance* is exactly one-half the radius of curvature. All the foci lie upon a surface perpendicular to the principal axis, which, when the angle of incidence is small, may be regarded as a plane, and be termed the *focal plane*.

Rays which do not pass through the centre of curvature (Fig. 303, *C*) strike obliquely against the surface of the mirror, and are reflected so that the normal, erected at the point of incidence to the surface, forms equal angles with the incident and reflected rays. The normal is everywhere the radius drawn from the centre of curvature to the point of incidence, i.e. the normal is more inclined to the axis, the farther the point at which it is drawn lies from the axis. Therefore, any ray parallel to the axis must be bent toward the axis from its original direction by a greater amount, the farther from the axis it strikes the mirror. It is thus apparent that all the rays striking the mirror parallel to the axis must pass, after reflection, through the same point. If, at the focus, *F*, a source of light is placed, its rays will be reflected in directions parallel to the axis, since they must traverse the same path as before, but in the opposite direction. If a beam of rays from a luminous point, *a* (Fig. 304), lying between the focus, *F*, and the centre, *C*, fall upon the mirror, the individual rays impinge less

obliquely against the mirror than would be the case if they came from the focus, and will, consequently, be turned from the axis less strongly after reflection. They will proceed, therefore, from the mirror obliquely to the axis, cutting it at a point beyond the centre, *C*. Since their deflection is greater the farther the point of incidence lies from the axis, they will intersect at the point, *A*, called the image of the point, *a*. If a luminous point is placed at *A*, its rays, passing along the same path in the opposite direction, will meet at the point, *a*. The points *A* and *a* correspond, therefore, in a certain sense, so that each is the image of the other, and is called its *conjugate point*. The



FIGS. 304, 305.—Conjugate Points.

focus has its conjugate point upon the axis and at an infinite distance, while the centre of curvature is self-conjugate.

When the luminous point (Fig. 305, *A*) lies nearer the mirror than at the focal distance, the strongly divergent pencil can no longer be brought to a focus at a point in front of the mirror. The reflected rays pass away from the mirror as though they had been emitted from a point, *a*, behind the mirror. Conversely, since rays directed toward the point, *a*, behind the mirror are gathered together at the point, *A*, in front of the mirror, the points, *A* and *a*, are in this case also to be considered as *conjugate*.

It may then be said, generally, that all rays which, coming from a point, or being directed toward a point (*homocentric*), impinge against a spherical mirror at small angles of incidence,

pass after reflection again through a single point (i.e. either the rays themselves, or their prolongations), which lies upon the axis corresponding to the first point. *Homocentric* rays remain, therefore, *homocentric* after reflection.

To find by construction the point, *b*, conjugate to the luminous point, *B* (Fig. 306), only two of the reflected rays need be drawn, and these may be so selected as to be most convenient.



FIG. 306.—Conjugate Points and Planes.

For where these two rays intersect, all others must meet. For example, as in Fig. 306, the axial ray, *BC*, which returns into itself, and the ray, *BFM*, which is reflected parallel to the principal axis toward *MB*, may be most conveniently used. If planes be passed through the points, *B* and *b*, perpendicularly to the principal axis meeting it in *A* and *a*, *A* and *a* are also conjugate points, because, by reason of the small angle, *ACB*, the distance, *AO*, does not appreciably differ from *Bo*, and *aO* differs but little from *bo*. These planes are also said to be *conjugate*.

Since, to every point of a luminous, or of an illuminated



FIG. 307.—Formation of a Real Image.

object, lying in front of a concave mirror, corresponds an image lying in the conjugate plane upon the corresponding axis, there arises out of the combination of all the elementary punctual images, an image of the object. If an object, *AB* (Fig. 307), is situated between the focus, *F*, and the centre of curvature, *C*, the image of the point, *B*, lies upon the axis, *BC*, at *b*, and that of

the point, A, upon the axis, AC, at *a*, and so forth. An *inverted, magnified* image, *ab*, is, therefore, produced *beyond* C. If *ab* designate an object situated farther from the mirror than twice the focal distance, it would give rise to an *inverted, minified* image at *b*, upon the axis, AB, *between* the focus, F, and the centre, C. It is apparent from the drawing (Fig. 306), that image and object are similar—this is seen from the similarity of the triangles, ABC and *abC*—and that their magnitudes are to each other as their distances from the mirror, since the triangles, AOB, and *aOb*, are similar. Image and object move in opposite directions. If the object is moved from the focus toward the centre of curvature, C, its inverted and magnified image approaches from an infinite distance toward C. At twice the focal distance from the mirror, image and object exactly coincide. If the object continues to move beyond C, its inverted and *minified* image approaches the focus. The image of an infinitely remote object, *e.g.* of the sun, or other heavenly body, is formed at the focus itself.

These images differ very materially from those formed by plane mirrors. They arise by the *real condensing, or collecting* of the rays emitted by each point of the object into a single point *before* the mirror. Such an image may, therefore, be caught upon a screen, and it is visible from all directions. Images of this sort are for this reason called *real images*. Images due to plane mirrors, on the other hand, are produced by rays which diverge after reflection in such way as to appear to emanate from a point behind the reflecting surface. Such images are seen only when these particular rays enter the eye. They are therefore *apparent, or virtual images*. Real images are visible upon a screen whenever the eye is brought into the path of the rays emitted by the image after its formation from the points (Fig. 307 at *b*). The image appears to *float* in the air before the mirror.

Real images are produced by a concave mirror, whenever the objects are farther from it than the focal distance. From an object nearer than this to the mirror (Fig. 308, AB) the divergent pencil of rays can form only a *virtual image, ab*. To an eye looking into the mirror the image appears *erect*.

behind the mirror, and larger than the object. The figure shows the path of the rays in this case. On account of this magnifying effect, concave mirrors are sometimes called *magnifying mirrors*, and are extensively used for toilet purposes. A spherical surface polished on its convex side forms a convex



FIG. 308.—Formation of a Virtual Image.

mirror. Since a convex mirror reflects the rays from a point (Fig. 309, B) into a more rapidly divergent pencil than before, the reflected rays produced backward will again appear to issue from the point, *b*. Such a mirror produces a virtual image, *ab*, of the object, *AB*. This image is seen erect and behind the mirror. Since the image is always smaller than the object, convex mirrors are sometimes called *minifiers*, or



FIG. 309.—Formation of a Virtual Image with a Convex Mirror.

*reducers*, and, on account of their diminutive sizes, they are extensively used as pocket mirrors. Rays falling upon such a mirror, parallel to an axis, pass back from it as though they emanated from a point at a distance of one half

the radius behind the mirror. This point is called the *virtual focus*, and it plays the same part in constructing the images geometrically as does the real focus of the concave mirror (Fig. 303).

The position of the conjugate points may be easily found by computation. Any arbitrary ray, as *AM* from *A* (Fig. 310), which makes the angle,  $\alpha$ , with the corresponding axis, is reflected at the point, *M*, so that the angles of incidence and reflection are equal (both equal  $\delta$ ). Suppose the reflected ray to cut the axis in the point, *B*, at the angle,  $\beta$ . If the angle formed by the axis and the normal drawn from *C* to *M* be designated by  $\gamma$ , it is evident that  $\beta$  equals  $\gamma + \delta$ , and  $\alpha = \gamma - \delta$ . Adding these equations, we have  $\alpha + \beta = 2\gamma$ . If all

these angles are taken small, the perpendicular,  $MJ = k$ , dropped from  $M$  upon the axis, differs imperceptibly from the arc,  $MO$ . If the radius of the sphere be designated by  $r$ , the distance of the luminous point,  $OA$  (which, by reason of the small size of the angle, is almost equal to  $dA$  and  $MA$ ) by  $a$ , the distance



FIG. 310. — Determination of the position of Conjugate Points.

of the image,  $OB$  (nearly equals  $dB$  and  $MB$ ) by  $b$ , regarding  $Md = k$  as a small arc, belonging successively to the radii,  $a, b, r$ , the angles,  $\alpha, \beta, \gamma$ , may then be thus expressed—

$$\alpha = \frac{k}{a}, \beta = \frac{k}{b}, \gamma = \frac{k}{r}.$$

The above equation,  $\alpha + \beta = 2\gamma$  becomes, when we substitute these values in it,  $\frac{k}{a} + \frac{k}{b} = \frac{2k}{r}$ , or dividing out the common factor  $k$ —

$$\frac{1}{a} + \frac{1}{b} = \frac{2}{r}.$$

If the ray,  $AM$ , is parallel to the axis, i.e. if  $A$  is at an infinite distance ( $a = \infty$ ),  $\frac{1}{a} = 0$ ,  $\frac{1}{b} = \frac{2}{r}$ , or  $b = \frac{1}{2}r$ , i.e. the condensing point of the incident rays parallel to the axis is at a distance of half the radius from the mirror, or the focal distance,  $f$ , equals half the radius. If then, in the foregoing equation, we put  $\frac{1}{2}r = f$ , it becomes  $\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$ .

The circumstance that  $k$  (the only magnitude in the equation referring to the point of incidence,  $M$ ) falls out of the equation, is a mathematical representative of the principle that all rays emitted from a point, wheresoever they strike the mirror, meet again in one and the same point.

We notice, furthermore, that the equation is in no wise altered by interchanging  $a$  (distance to the object), and  $b$  (distance to the image), or that the luminous point and its image may be mutually interchanged.

The equation which was derived specially for concave mirrors, holds moreover for convex mirrors, by merely regarding the virtual focal distance as negative, i.e. by setting  $-f$  in place of  $f$ .

What has been thus far said of spherical mirrors, holds, however, only when the aperture is small, and the incident rays form small angles with the axis. With concave mirrors of wider aperture, the rays parallel to the axis and striking near the edge are turned toward a point of the axis lying nearer the mirror than the focus for the rays impinging at points nearer to the centre. The rays reflected from the various points of the surface do not, therefore, coalesce into a single point, but the numerous points of intersection form a continuous surface, called the *caustic surface*. The intersection of such a surface through the axis forms a *caustic line*, or *caustic*, simply. To unite the peripheral with the central rays into a single point, the spherical mirror must be replaced by one of parabolic form. This imperfection which prevents the

formation of sharp images, is called "the aberration due to spherical form," or *spherical aberration*.

**326. Refraction—Total Reflection.**—If a slender beam of light (AM) is thrown by a small mirror (Fig. 311) upon the surface of water contained in a glass trough, a portion of the incident beam is bent according to the law of reflection toward MN.

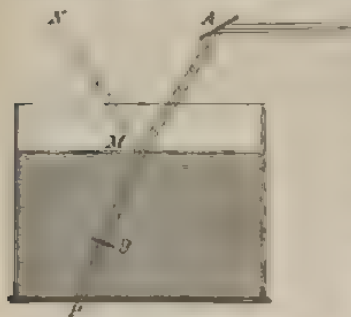


FIG. 311. Refraction

while another portion, MP, passes into the water. The portion of the beam beneath the surface does not form a direct continuation of that above. On the contrary, the lower portion of the beam is bent abruptly downward at the surface. Since the ray undergoes a *breaking*, or *refraction*, the phenomenon is designated by the latter term. The devi-

ation of the refracted beam from the direction of the incident beam, diminishes as the latter is turned into a more nearly perpendicular position by properly rotating the mirror, A. No change of direction occurs when the incident ray is perpendicular to the surface of the water, the incident and refracted rays in this case forming a continuous straight line. If the beam within the water impinges perpendicularly upon a small mirror, B, beneath the surface, it returns by the same path,

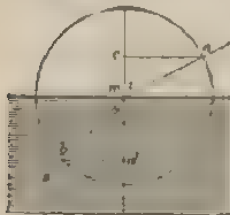


FIG. 312. Law of Refraction

BMA. In pure air and in pure water, the beam is invisible. It becomes perceptible, however, when particles of smoke and dust are mixed with the air and the liquid. If a small quantity of eosin (a fluorescent substance) be dissolved in water, the refracted ray shines with a greenish light.

To specify the path of the incident (am, Fig. 312) and of the refracted rays (mb, imagine a perpendicular, me, erected at the point of incidence, m, and continued into the water (toward md).

This perpendicular is usually termed the *normal*. It is readily seen that the plane containing the incident ray and the normal (the plane of the drawing), contains also the refracted ray. This plane is, therefore, called the *plane of refraction*. The directions of the rays are determined by the angles which they form with the *normal*, viz. by the *angle of incidence*,  $i$ , and the *angle of refraction*,  $r$ . To every angle of incidence corresponds a definite angle of refraction. The following table of corresponding angles may be verified by measurement—

Angle of incidence		to the angle of refraction	
"	15°	"	11° $\frac{1}{2}$
"	30°	"	22°
"	40°	"	32°
"	60°	"	40° $\frac{1}{2}$
"	75°	"	46° $\frac{1}{2}$
"	90°	"	48° $\frac{1}{2}$

In Fig. 312, according to this table, to the angle of incidence,  $i$  (equal to 60°), the corresponding angle of refraction,  $r$ , equals 40° $\frac{1}{2}$  has been drawn. If now we describe in the plane of refraction, a circle of arbitrary radius with its centre at  $m$ , and draw from the points  $a$  and  $b$ , at which the incident and refracted rays intersect the circumference, the right lines,  $ac$  and  $bd$ , perpendicularly to the normal, it will be seen that  $bd$  is  $\frac{3}{4}$  of  $ac$ , or that  $ac$  is  $\frac{4}{3}$  of  $bd$ . By proceeding similarly with all the corresponding angles of the above table, we should always find that the perpendicular corresponding to the angle of incidence is  $\frac{4}{3}$  of that of the angle of refraction. The number  $\frac{4}{3}$ , or 1 $\frac{1}{3}$ , which may be taken as a measure of the refraction on the passage of light from air into water, is called the *coefficient*, or *index*, of *refraction*, or the *refractive index* of water. From air into glass, the rays are more strongly broken, the ratio of the perpendiculars to the normals in this case being expressed by the number  $\frac{5}{3}$ , or 1 $\frac{2}{3}$ . Similarly, every transparent body possesses its own characteristic refractive index. They have been collected for a few substances in the following table—

Water	...	...	...	1.333
Alcohol	...	...	...	1.365
Canada balsam	...	...	...	1.530
Carbon disulphide	...	...	...	1.631

Crown glass	...	...	1.530
Flint glass of Fraunhofer	...	...	1.635
Flint glass of Merg	...	...	1.732
Diamond	.	.	2.487

As is well known, the perpendiculars,  $ac$  and  $bd$  (Fig. 312), with a circle of radius equal to unity, are called the *sines* of the corresponding angles,  $i$  and  $r$ . We may, therefore, state the law of refraction thus: *The sine of the angle of incidence has a constant ratio to the sine of the angle of refraction.* If the index of refraction be designated by  $n$ , this law is expressed by the equation (Snellius, 1620; Descartes, 1637)—

$$\frac{\sin i}{\sin r} = n, \text{ or } \sin i = n \cdot \sin r.$$

When light passes from air into a liquid, or a solid, the refracted ray is bent toward the normal. When a ray passes out of the water in the direction  $bm$  (BM, Fig. 311), it suffers the same reflection as the ray entering the water in the direction  $mb$  (MB). It passes out of the water in the direction  $ma$  (MA), and is accordingly bent away from the perpendicular. Precisely the same values as before, for the corresponding angles,  $r$  and  $i$ , hold here, with the difference that the incident angle within the water equals the former angle of refraction, while the present angle of refraction equals the former angle of incidence. If, therefore,  $\frac{3}{4}$  (or in general  $n$ ) is the refractive index for the passage from air into water (or other substance),  $\frac{4}{3}$  (or  $\frac{1}{n}$ ) is the refractive index for the passage from water (or

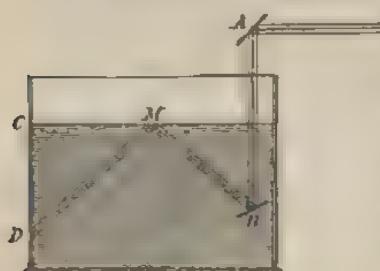


FIG. 313.—Total Reflection.

other substance) into air (*law of reciprocity*). To make the path of the rays coming from the water visible, a small movable mirror, B (Fig. 313), is sometimes placed beneath the surface of the water in the glass trough. The beam of rays bent vertically downward by the mirror, A, impinges

upon the movable mirror and is reflected toward the surface of

the water. If the ray, BM, passing out of the water, is made to strike the surface more and more obliquely, the portion above the surface of the water becomes more and more inclined to the surface, forming with the normal an angle which is always greater than that of the portion, MB, while, at the same time, the portion of the rays above the surface approaches continually toward the surface of the water. Finally, when the angle of incidence in the water has reached the value  $48^{\circ}5'$ , the issuing ray just grazes the surface. The angle of refraction now amounts to  $90^{\circ}$ . No greater angle of refraction than this is possible. At  $90^{\circ}$  the limit is attained. If now the ray falls a little more obliquely upon the surface, the light will no longer pass out into the air. For rays striking the surface at this obliquity, the surface is perfectly opaque. It will be noticed, also, that at the instant when this limit is passed, the ray, MB, which is reflected inward, and which, until the limit was reached, was considerably fainter than the incident ray, BM, suddenly increases in intensity and becomes as bright as the incident ray. While with the less oblique rays, the light was divided between an emergent ray and a ray reflected into the water, beyond this limit the emergent ray no longer exists, and the light is turned into the refracted ray without diminution. With all angles of incidence greater than  $48^{\circ}5'$  the light is thus *totally reflected*. *Total reflection* can, however, occur only when the ray coming from the more highly refractive medium, impinges upon the surface of a less powerfully refractive medium. The angle of incidence at which the outward passage of light ceases and total reflection begins, that is, the angle which corresponds to the external angle of  $90^{\circ}$ , is called the limiting, or *critical, angle*. For water it is  $48^{\circ}5'$ , for glass  $40^{\circ}45'$ , for diamond  $23^{\circ}45'$ . Since to the limiting angle,  $r = 90^{\circ}$ , corresponds the external angle,  $i = 90^{\circ}$ , and  $\sin 90^{\circ} = 1$ , there results from the law of refraction,  $1 = n \sin \gamma$ , or  $\sin \gamma = \frac{1}{n}$ .

By measuring the limiting angle, it is thus possible to find the index of refraction of any substance (Wollaston, 1802).

A glass surface from which light is totally reflected shines with a bright metallic lustre. It forms the clearest and most

perfect mirror known to science. In the construction of optical instruments, *totally reflecting prisms* are, therefore, frequently used (*reflecting prisms*, Fig. 314) to turn the rays of light without appreciable loss of intensity into any desired direction.



FIG. 314. Reflecting Prism.

Such a prism consists of a piece of glass having two surfaces, AC and BC, ground accurately at right angles to each other and a third surface, AB, inclined to each of the former at an angle of  $45^\circ$ . Rays striking the surface, AC, perpendicularly, pass without deflection into the glass and fall upon the surface, AB, at an angle of incidence of  $45^\circ$  (which exceeds the limiting angle of only  $40^\circ 75'$ ). The rays are here totally reflected, no light whatever passing through AB into the air, and after reflection they pass out of the prism through the space, BC.



FIG. 315.—Camera Lucida.

Wollaston's *camera lucida* depends upon this principle of total reflection. This camera is an apparatus for copying the objects of nature. It consists of a four-sided piece of glass, *abcd* (Fig. 315), with a right angle at *b*, and with the obtuse angle of  $135^\circ$  at *d*. A ray of light, *x*, coming from an object, and striking the front surface, *bc*, enters the prism and is then totally reflected, first at the surface, *cd*, and then at *da*. After passing out through the surface, *ab*, near the edge, *a*, it reaches the eye from the direction of the dotted line in Fig. 315. Looking past the edge, *a*, downward toward the sheet of paper for the reception of the drawing, in such a way that half of the pupil, *pp*, is covered by the piece of glass, the eye perceives the image of the object as though it were drawn upon the sheet. The outlines of the image may, therefore, be easily traced with the point of a lead-pencil seen simultaneously with the image.

By means of the law of refraction, the angle of refraction corresponding to any angle of incidence may be easily determined, and conversely, either by computation, or graphically. The problem may be solved graphically, as is illustrated in Fig. 312. The construction given in Fig. 316, however, is more convenient. Let two circles be described about the point of incidence,  $a$ , the one of radius unity and the other of radius,  $n$ , where  $n$  denotes the index of refraction on the passage of light into the second medium. Let the incident ray,  $la$ , be prolonged to its intersection,  $m$ , with the first circle, and through  $m$  let a parallel,  $pmq$ , be drawn to the normal,  $ka'k'$ . The connecting line of the point,  $p$ , where this parallel cuts the second circle, with the point of incidence,  $a$ , gives the refracted ray,  $ap$ . For, since angle  $qma$  equals the angle of incidence,  $i$ , we have  $\sin i = qa$ , and, further, since angle  $qpa$  equals the angle  $r$ , also  $n \sin r = qa$ , and, therefore, as the law of refraction requires:  $\sin i = n \sin r$ . For a ray,  $pa$ , coming from the second medium,  $a$ , parallel to the perpendicular which meets the circle at the point,  $m$ , is drawn through  $p$ , where the direction of the ray,  $pa$ , cuts the second circle. The line connecting  $m$  with  $a$  furnishes then the required emergent ray.

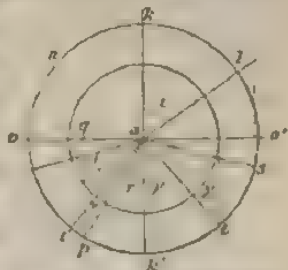


FIG. 316.—Law of Refraction

The latter construction is impossible, when, as with the ray,  $sa$ , the parallel to the normal does not meet the first circle. This circumstance, however, reveals the existence of the total reflection to which the ray is subject.

When the parallel touches the first circle exactly at the end of its horizontal diameter, as is the case with the ray  $ta$ , the refracted ray grazes the surface separating the two media along the line,  $ao$ , and  $ta'k'$  is the critical angle, for which we have from the figure, the equation  $n \sin \gamma = 1$ . If the ray passing out of the first medium is rotated 180 degrees out of the position  $oa$ , through  $ka$ , to  $o'a$ , the ray, refracted in the second medium toward the normal, is at the same time rotated from  $at$  through  $ak$  toward  $ar$  by double the value of the critical angle.

A luminous point beneath the surface of water, when viewed by the eye from above, is not seen in its true place, but appears to be lifted somewhat, because the rays passing from the water diverge more strongly from the normal than do incident rays within the water, and, consequently, they seem to come from a point lying nearer than the object to the surface of the water. This explains the fact also that a vessel of water whose bottom can be seen, appears shallower than it really is. For the same reason the portion of a vertical post beneath the surface of the water appears to be shortened, and a bar held obliquely in the water appears to be bent abruptly at the surface. A coin lying in a vessel of water, and viewed from above, is slightly magnified, because it appears to lie

nearer the eye, and is, therefore, seen under a greater visual angle.

Gases also refract light. The coefficient of refraction from *vacuo* into air of 0 and 760 mm. pressure equals 1.000294, and it diminishes with the density.

Since in the atmosphere the density of the air, and accordingly also its refracting power, under ordinary circumstances, increases from above downward, a ray of light passing obliquely downward from an elevated point is turned continually toward the normal, and the ray pursues a curvilinear path, which is concave downward, arriving at last at the eye, from a steeper direction than would have been the case had it been propagated rectilinearly in *vacuo*. Consequently, the point whence the light was emitted is seen at a position elevated somewhat above its true position (atmospheric refraction). The quivering of objects seen through heated air is due to the fact that the light from them is refracted by atmospheric currents of unequal density toward one direction and then toward another in rapid succession.

**327. Atmospheric Reflection** occurs when layers of air of different densities and consequently also of unequal refractive capabilities are superposed upon one another. If the lower layers are strongly heated, and, therefore, less dense than the higher, as frequently occurs above the hot sandy soil of deserts, a ray of light, passing from a lofty object toward the earth, assumes a path becoming more and more oblique in consequence of the decreasing refractive power of the air. It thus describes a curvilinear path concave upward. The ray continues to curve until finally it reaches the eye of the observer from below, as though it had been reflected from the horizontal surface of a mirror. The eye sees, then, beneath the real object its inverted image, and, since the light of the sky is also refracted by the hot layers of air, the image of the sky appears in the form of a surface of water. A similar refraction and reflection of light, by the atmospheric layers of varying densities, are frequently seen at sea, and the phenomena are known to the mariner as *mirages*. On the coasts of Sicily and Arabia, where such phenomena occur very frequently, popular tradition ascribes them to the magic of an evil spirit, the *Fata Morgana*, and they are with us frequently referred to as the *Fata Morgana*.

**328. Parallel Plates.**—When a ray of light passes through a plate (BB), bounded by parallel planes, it is bent on entrance toward the normal, and on exit by just as great an amount away from it (illustrated in Fig. 317). The ray *nT* does not form the rectilinear continuation of the entering ray, *nl*, although

the rays remain parallel to each other. The ray suffers no deviation from its original direction. It is merely displaced laterally by an amount which becomes smaller the thinner the plate. Thin plates, such as panes of window-glass, produce so slight a displacement of the rays, that objects seen through them appear in their true forms, sizes, and positions.

The direction of the emergent ray is not altered, even with a series of two or more parallel plates (Fig. 318), composed of any

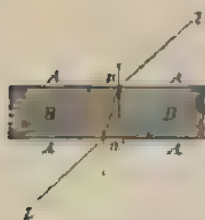


FIG. 317.—Refraction by a Parallel Plate.

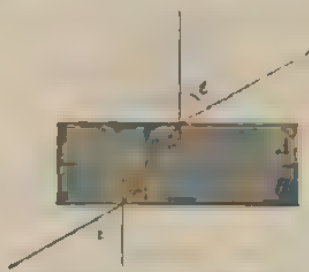


FIG. 318.—Refraction by two Parallel Plates.

sort of material whatever. Hence, it follows that the index of refraction on the passage of a ray from a medium, A, into a medium, B, is expressed by the quotient  $\frac{n''}{n'}$ , where  $n''$  designates the index of refraction of the medium, B, and  $n'$  that of the medium, A, both taken with respect to air.

For, on the entrance of the ray into the first plate, we have  $\sin i = n' \sin r$ , and on passing out of the second,  $\sin e = n'' \sin r$ , consequently  $n' \sin r = n'' \sin r'$ , or  $\sin r = \left(\frac{n''}{n'}\right) \sin r'$ . Since, at the passage from the first plate into the second,  $r$  represents the angle of incidence, and  $r'$  the angle of refraction,  $\frac{n''}{n'} = n$  is the corresponding index of refraction, or  $n'' = n'n$ .

The index of refraction of a body when light passes from a vacuum into it, is called its *absolute index*. It is found by multiplying the index of refraction,  $n$ , with respect to air by the index of refraction,  $n' = 1.000294$  of air with respect to *vacuo*.

**329. Prisms**, in the theory of light, are transparent bodies with two plane surfaces inclined to each other, through which light may pass inward and outward. The common glass prism

has the form of a triangular column, whose right section (*principal section*) is an equilateral triangle, ABC (Fig. 319). Only two surfaces of the prism (AB and BC, the "refracting surfaces") require polishing, the third surface, AC, lying opposite the "refracting angle," as also the triangular end surfaces are usually ground and blackened. A ray of light falling upon

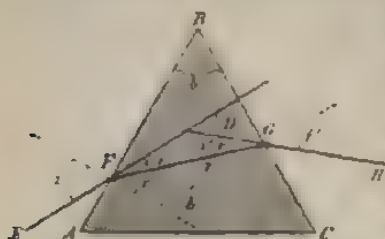


FIG. 319.—Path of Ray through a Prism.

the surface in the direction EF, in the principal section, passes along the path, EFGH. This path may be readily drawn from the law of refraction, since the ray is deflected by a known amount, both at entrance and at exit. As the drawing indicates, the ray is always bent *from* the edge, B.

toward the base of the prism. An eye, looking through the prism from H, sees an object situated behind the prism, displaced toward the edge in the direction HG.

The angle, D, formed by the direction of the ray, EF, with the direction, GH, of the emergent ray, gives the total deflection suffered by the ray at both refractions. This deflection is composed of the deviation  $i - r$  at entrance, and of  $i' - r'$  at exit, where  $i$  and  $i'$  denote the angles made by the incident and the refracted rays with the respective normals, and  $r$  and  $r'$  those made by the path of the ray in the prism with these same normals. The total deviation, D, equals, therefore,  $i - r + i' - r'$ , or  $D = i + i' - (r + r')$ . From the drawing it is evident that the sum  $r + r'$  is always equal to the refracting angle,  $b$ , of the prism, or that  $r + r' = b$ , and, accordingly, the deflection  $D = i + i' - b$ .

If the angle of incidence,  $i$ , is varied by rotating the prism back and forth, a position is readily found, where the deflection is smaller than in any other position. It may be readily proved that this smallest deflection, or the *minimum* value of the deflection ( $d$ , Fig. 320), occurs when the ray forms equal angles with the refracting surfaces both within and without the prism, or when it traverses the prism symmetrically. Since in this

case  $r' = r$  and  $i' = i$ , we obtain  $2r = b$  and  $d = 2i - b$ , and from these equations, the angle of incidence  $i = \frac{1}{2}(d + b)$ , and the corresponding angle of refraction  $r = \frac{1}{2}b$ . According to the law of refraction, the index of refraction,  $n$ , must equal the ratio of the sines of these two angles—

$$n = \frac{\sin \frac{1}{2}(d + b)}{\sin \frac{1}{2}b}.$$

If, therefore (by means of a *goniometer*, the refracting angle,  $b$ , of a prism and the least deflection,  $d$ , are measured, from this formula the index of refraction of the material composing the prism is easily computed. Bodies, whose indices of refraction are to be found by this very accurate method, are usually given the form of a prism. With liquids this is done by filling a hollow prism, whose refracting surfaces are composed of plane glass plates with parallel faces, which themselves produce no deflection of the ray.

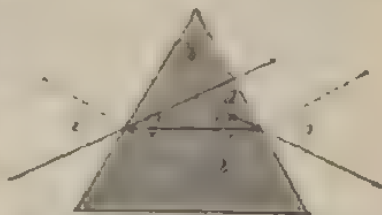


FIG. 320. —Least Deflection of a Prism.

That the deflection in a prism is least when the ray traverses it symmetrically, is readily proved as follows: From the vertex, B (Fig. 321), of the refracting angle, ABC ( $b$ ), let any arbitrary right line (BD) and two circular arcs of radii 1 and  $n$  respectively be drawn, the latter of which intersects the sides of the angle,  $b$ . If the angle, ABD, be taken as the angle of refraction,  $r$ , since  $r + r' = b$ , the angle, CBD, then equals  $r'$ . If now, from A and C parallels to BD are drawn, which meet the unit circle in A' and C', whereupon, according to the graphical construction of the law of refraction given above (Fig. 316), A'BD and C'BD are the angles  $i$  and  $i'$ , which correspond to  $r$  and  $r'$ ; then  $A'B'C' = i + i'$ . The deflection  $D = i + i' - b$ , varies only with the sum  $i + i'$ , and it is, therefore, a minimum, when the angle, ABC has its least value. If the angles  $r$  and  $r'$  are unequal, the chord, A'C', of the angle  $i + i'$ , in the unit circle, is always greater than the constant chord, AC' of the angle  $r + r' = b$ , in the circle,  $n$ , and becomes equal to the latter only when  $r = r'$ , and, consequently, also  $i = i'$ . The sum  $i + i'$ , and with it the deflection, is, accordingly, least in case of symmetrical passage. By means of this same construction, all possible cases of refraction in a prism may be studied graphically.

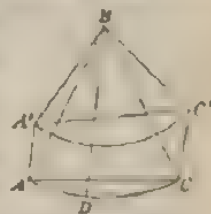


FIG. 321. —Least Deflection in a Prism.

With prisms of very small refracting angle, rays having small angles of incidence have always the same deflection, which is proportional to the refracting angle. If the angles,  $i$  and  $i'$ , and, *a fortiori*, the angles  $i$  and  $i'$ , are small, the circular arcs corresponding to these angles are not appreciably different from the sines, and may, therefore, replace them. The law of refraction then assumes the simpler form,  $i = nr$  and  $i' = nr'$  (Kepler). There results, then, for the deflection—

$$D = i + i' - b = n(r + r') - b = nb - b, \text{ or } D = (n - 1)b.$$

**330. Lenses.**—A transparent piece of glass, bounded by two spherically curved surfaces (or by one spherical and one plane surface), is called a *lens*. Seen from the surface, such a piece of glass appears circular. When cut through the middle, however, it would present some one of the forms shown in Fig. 322. Lenses, whose thickness increases toward the centre, are called *convex*. Among these we have the double-curved, or *bi-convex* lens (A, Fig. 322), which has a seed-like shape, from which resemblance the word *lens* was derived; the *plano-convex*-lens (B), curved on one surface and plane on the other;



FIG. 322.—Forms of Lenses.



FIG. 323.—Axis of a Lens.

the *concavo-convex* (C), sometimes called the *meniscus* (the "little moon"), is convex on one side and less strongly concave on the other. *Concave* lenses are thinner in the middle than at the edge, and include, likewise, three forms: the *double concave*, or *bi-concave* (D), the *plano-concave* (E), and the *convexo-concave* (F).

The discussion immediately following will be rigorously applicable only to lenses whose thickness is *small*, so that the vertices, S and S', may be regarded as coinciding with the point O, on the interior of the lens, which is called its *optical centre*. We shall assume, further, that all angles of incidence and of refraction are small. All right lines, MM, NN (Fig. 323), passing

through the centre, *O*, of the lens, are called its axes, and that particular axis (*AA*), which pierces the surfaces of the lens perpendicularly, and, hence, passes through the centres of curvature of the spherical surfaces, is termed the *principal axis*. A ray passing through the centre, *O*, suffers no deflection, because it meets the surfaces at points where they are parallel to each other. It traverses the lens along an axis, and is, for this reason, called an *axial ray*. All other rays, after passing through the lens, are changed in direction. They are deflected, or bent, to a greater extent the farther the point of incidence lies from the centre of the glass. The lens comports itself with respect to light like a prism of small refracting angle, the angle, and, therefore, also the deflecting action, increases gradually toward the edge. With convex lenses the angle of the wedge is turned away from the principal axis, while with concave, the angle points toward the axis. Inasmuch as a prism always bends a ray of light toward its base, rays of light passing through convex lenses must be bent toward the principal axis, while those traversing concave lenses are turned from this axis.

If a beam of parallel solar rays falls upon a bi-convex lens (*AB*, Fig. 324), it is refracted in such way that all rays pass through one and the same point, *FF*, lying on the other side of the lens, because each ray is bent toward the axis by an amount proportional to the distance of its point of incidence from the centre of the lens. If a sheet of white paper be held here, this point will appear as an intensely brilliant spot, into which not only the luminous, but also the thermal effect of the solar rays has been collected. The paper soon becomes so highly heated as to take fire and burn up. For this reason the point, *F*, is called the *focus* (burning point) of the lens, and the lens itself is sometimes called a *burning glass*. If the parallel beam falls upon the other side of the lens, its rays undergo the same reflection and are condensed on this side into a point at the



FIG. 324 — Parallel Rays pass through the Focus.

same distance from the lens as before. A lens possesses, therefore, two foci upon each axis, lying on opposite sides and at the same distance from the lens, called the *focal distance* of the lens. Rays emitted from a focus pass from the opposite side of the lens in directions parallel to the corresponding axis (Fig. 325).



FIG. 325.—Rays coming from the Focus are rendered Parallel.

When the focal distance of a lens is known the deflection suffered by any ray passing from the focus and falling upon the

lens is readily obtained. At any given point of the surface of a lens, a ray coming from any direction whatever, suffers the same deflection (provided its direction does not differ much from that of the principal axis). To illustrate: If a luminous point is situated at R (Fig. 326), farther than the focal distance from the lens, the ray, RA, falling at the edge of the lens is deflected by just as much as would have been the case had it proceeded from the focus, F, along the direction, FA, to the same point, A. Its change of direction, RAS, is then equal to



FIG. 326.—Conjugate Points.

the angle, FAN, and it intersects the axial ray, RS, passing through the lens without deflection at the point, S. At this same point all the rays from R, which impinge upon the surface of the lens, are condensed, because each is deflected proportionally to the distance of its point of incidence from the centre of the lens. A paper screen placed at S reveals a brilliant point as the image of the luminous point, R. Such an image formed by the condensation of the rays, and which may be caught upon a screen, as has been already stated, is called a *real image*. If the luminous point is placed at S, its

rays suffering the same deflections as before, must intersect at R, where the luminous point was previously situated. The points, R and S, then, in a certain sense, belong together. The image appears at one of them when the source of light is at the other, and they are for this reason called *conjugate points*. When one is more than twice the focal distance from the lens, the other lies beyond the lens at less than twice the focal distance, but always at more than the single focal distance from the lens. When the luminous point lies at exactly twice the focal distance from the lens, the image is also at the same distance from it. The foci are themselves conjugate to the points of the axes at an infinite distance from the lens.

If the luminous point, T (Fig. 327), lies between the focus, F, and the lens, AB, the deflecting power of the lens is no longer able to condense the strongly divergent rays (TA, TB) or even to make them parallel with one another. All the lens can do is to reduce somewhat their rate of divergence. A condensation of the refracted rays beyond the lens is im-



FIG. 327.—Conjugate Points.

possible. They diverge precisely as though emanating from a luminous point, V, lying upon the axis and on the same side of the lens. The eye looking from the opposite side of the lens sees, in place of the luminous point, T, a more remote point, V, as an image of T. Such an image produced by divergent rays when prolonged backward to a point is termed an *apparent*, or *virtual image*. This condition of things also occurred with mirrors. If, on the contrary, a convergent beam from the right (Fig. 327), whose rays intersect at the point, V, falls upon the lens, the rays will be drawn by the lens into a more strongly converging beam, collecting into a focus at T. To the point, V, which may be regarded as a "*virtual*" luminous point, the point, T, corresponds as a real image. The points, T and V, are here also so related, that the one is the image of the other, and they are consequently *conjugate points*. The positions of the conjugate points may be readily located in a

drawing as in Figs. 326 and 327, when the angle,  $FAN$  (Fig. 326), representing the total refraction suffered by a ray coming from any point and striking the edge,  $A$ , is cut from a piece of stiff paper, and its vertex is placed at the point,  $A$ , while the angle is turned about this point as a pivot. The sides of the angle cut every axis in two corresponding points, one of which is the image of the other. From this it is also clear that luminous point and image are displaced upon their axis always in the same sense.

As with mirrors, so with lenses, *homocentric* rays remain *homocentric* after refraction, and here also to find the image of a point any two rays may be selected, which may be most conveniently drawn. The rays most readily located are the axial ray and the ray parallel to the principal axis, both of which pass through the focus on the other side of the lens.

Since a lens produces an image ( $A$ ) of each point ( $a$ ) of an object lying in the plane,  $ab$  (Fig. 328), perpendicular to the



FIG. 328. —Formation of Real Image.

principal axis, i.e. in the conjugate plane (cf. 325),  $AB$ , at the point where the axis,  $aOA$ , corresponding to  $a$  pierces this plane, it produces an image,  $AB$ , of the object,  $ab$ , which is similar to the object and whose diameter is to that of  $ab$  in

the ratio of the corresponding distances from the lens. If the object is farther from a convex lens than the focal distance, the image is formed beyond the lens by the actual concentration of the rays from the various points of the object. Such an image may be caught upon a screen and has a position precisely the reverse of that of the object. When the object ( $ab$ , Fig. 328) is at less than twice the focal distance from the lens, its image appears *beyond* the lens *inverted*, *magnified*, and *farther than twice the focal distance*. If a well-illuminated, transparent painting (or photograph) is placed in an inverted position at  $ab$ , it will be reproduced upon a screen at  $AB$  magnified and in an erect position (*magic lantern*, *stereopticon*, *skiopticon*). The same thing happens with the *solar microscope*,

where a small convex lens of short focus is placed in front of a small object (usually held between two glass plates and placed slightly outside of the focus of the lens) is strongly illuminated by sunlight concentrated upon it by a large lens. A highly magnified image of the object may be then caught upon a screen.

If the object at AB is more than twice the focal distance from the lens, an *inverted* and *minified* image (*ab*) is formed on the *opposite side*. To screen these images from the disturbing effect of outer luminous sources, a box or case coated black on its inner surface is used. This case is called a dark chamber, or *camera-obscura*, in the front side of which the lens, O, is placed, while an adjustable plate of ground glass forms its back surface.

When an object, AB (Fig. 329), lies at less than the focal distance from the lens, the rays from one of its points (A) cannot be condensed into a single point beyond the lens. They pass away in a diverging pencil, just as though they had started from a point, *a*, on the same side of the lens as A, though at a greater distance from it. An eye, viewing the object from



FIG. 329.—Formation of Virtual Image.

the opposite side, sees the *magnified virtual image*, *ab* of AB. This image has the same position as the object, or it is *erect*. On account of this well-known property of convex lenses they are frequently termed *magnifying glasses*. An apparatus provided with such a lens, and designed to show near objects magnified, is called a *magnifier*. It is held close before the eye, at such a distance from the object as to bring it just within the focal plane, whereupon the image will lie at a convenient distance for viewing it. The magnification is then almost equal to the ratio of the distance of distinct vision to the focal distance.

*Concave lenses* act oppositely to convex. They deflect the rays from the axis by amounts varying with the distance of the point of incidence from the centre of the lens. When a

beam of parallel solar rays impinges upon the surface of such a lens (Fig. 330), it emerges in a divergent pencil of such form as to appear to come from a point,  $F$ , lying on the same



FIG. 330.—Virtual Focus of a Concave Lens.

side of the lens and in the corresponding axis. This point is designated the *apparent* or *virtual focus*. A concave lens possesses two such foci upon each axis, one upon either side and both at the same distance from the lens. These points have the same

significance for the concave lens as do the real foci for the convex. The focal distance is here also the controlling consideration for the deflection to which the rays from the various points are subject.

Rays falling upon a concave lens from a point,  $A$  (Fig. 331), of an object, are refracted as though they came from the point,  $a$ ,



FIG. 331.—Virtual Image of a Concave Lens.

on the same side of the lens and lying nearer to it than  $A$  does. An eye looking from the other side through the lens receives the rays from the object,  $AB$ , as though they had been sent out

from the *minified, erect, virtual image*,  $ab$ . Concave lenses furnish only virtual images of objects, because they increase the divergency of the pencil of rays emitted by a point, or, technically, they "disperse" the rays. For this reason they are sometimes called *dispersion lenses*. Only convex lenses are capable of gathering the rays from a point and bringing them again into



FIG. 332.—Computation of Focal Distance.

a single point, and they have from this property been distinguished as *condensing lenses*.

The angle which the front surface of the lens (Fig. 332) forms at the point,  $K$ , lying at a distance,  $KP = k$ , from the axis, with the opposite point,  $K'$ , of the back surface, or, what is the same thing, the refracting angle corresponding to the point,  $K$ , is equal to the angle,  $\angle CKL$ , between the radii,  $CK = r$  and  $C'K' = r'$ , drawn from the point,  $K$ , to the respective centres of

curvature,  $C$  and  $C'$ . The angle,  $CKL$ , being the external angle of the triangle,  $(CKC')$ , equals  $\gamma + \gamma'$ . When these (and all other) angles are small, and the thickness of the lens is inconsiderable in comparison with the radii of curvature, these angles may be expressed thus:  $\gamma = \frac{k}{r}$  and  $\gamma' = \frac{k}{r'}$ . The

refracting angle, characterizing the point,  $K$ , is, therefore,  $k\left(\frac{1}{r} + \frac{1}{r'}\right)$ . The deviation produced by a prism is now equal to  $(n - 1)$  times its refracting angle (329). Each ray striking the surface at the point,  $K$ , suffers the deflection  $k(n - 1)\left(\frac{1}{r} + \frac{1}{r'}\right)$ , which is proportional to the distance,  $k$ , from the principal axis.

The ray,  $SK$ , parallel to the principal axis, being deflected to the focus,  $F$ , undergoes the change of direction  $\phi = \frac{k}{FK}$ , or  $\phi = \frac{k}{FM}$ , or  $\phi = \frac{k}{f}$ , since, by reason of the small values of the angle, and of the thickness of the lens, the focal distance,  $FM = f$ , may be substituted for  $FK$ . There results then—

$$\frac{k}{f} = k(n - 1)\left(\frac{1}{r} + \frac{1}{r'}\right),$$

and, for the determination of the focal distance, we have the equation.

(1)  $\frac{1}{f} = (n - 1)\left(\frac{1}{r} + \frac{1}{r'}\right)$ , which is in no way altered by interchanging  $r$  with  $r'$ , showing thereby that the focal distances on both sides of the lens are equal.

The formula shows, further, that the focus depends not only upon the form of the lens, but also upon the index of refraction of the substance composing it. For a bi-convex lens, with equal surfaces ( $r' = r$ ) composed of ordinary glass ( $n = 1.5$ ), for instance, we find from this formula  $f = r$ , and, for a plano-convex ( $r' = \infty$ ),  $f = 2r$ .

A ray from the point,  $R$  (Fig. 333), of the axis, and passing toward the conjugate point,  $S$ , undergoes, at the point,  $A$ , of the lens, the change of



FIG. 333. — Determination of the position of Conjugate Points.

direction,  $\gamma = \alpha + \beta$ , which is also equal to the deflection of the ray,  $NA$ , parallel to the axis, and passing through the focus,  $F$ , on the other side. We have, then,  $\alpha + \beta = \phi$ . Designating, now, the distance of the point,  $R$ , from the lens by  $a$ , that of the point,  $S$ , by  $b$ , the focal distance by  $f$ , and, finally, as before, the distance of the point,  $A$ , from the axis by  $k$ , we have  $\alpha = \frac{k}{a}$ ,

$\beta = \frac{k}{b}$ ,  $\phi = \frac{k}{f}$ , and, consequently,  $\frac{k}{a} + \frac{k}{b} = \frac{k}{f}$ , and we obtain, as the relation between the conjugate points, the equation—

$$(2) \quad \frac{1}{a} + \frac{1}{b} = \frac{1}{f}.$$

which is the same as was found above (325 for the spherical mirror, and hence it brings out the analogy between mirrors and lenses.

Equations (1) and (2) hold not only for the convex lens, but for any form of lens whatever, provided, that for the radius of curvature of a plane surface infinity ( $\infty$ ) is substituted; for a concave surface, a negative; and, for the convex, a positive value is taken. According as the value of  $f$  is found to be positive, or negative from formula (1), the lens possesses real, or virtual foci.

**331. Refraction by a Spherical Surface.**—The proposition that homocentric (passing through a single point) rays remain homocentric after refraction holds, not alone for lenses with two spherical surfaces, but it is equally true for each individual spherical surface, provided its curvature is not too abrupt.

For, if two different transparent media, having the respective indices of refraction,  $n$  and  $n'$ , are separated by a spherical surface,  $MS$  (Fig. 334), with the centre,  $C$ , and radius,  $CS = r$ , the central ray,  $AC$ , from the point,



FIG. 334.—Refraction by a Spherical Surface.

$A$ , strikes at the "vertex,"  $S$ , perpendicularly to the surface, and suffers no deflection. On the other hand, the ray,  $AM$ , inclined to the axis,  $AC$ , at the angle  $\alpha$ , is refracted at the point,  $M$ , toward  $MB$ , and intersects the central ray (the axis) in  $B$  at the angle,  $\beta$ . Its path is then found by determining the angle of refraction,  $\delta$ , from the law,  $n \sin \gamma = n' \sin \delta$ , corresponding to the angle of incidence,  $\gamma$ , between  $AM$  and the normal  $CM$ , which is inclined to the axis at an angle,  $\rho$ . If all these angles are small, the simpler law,  $n\gamma = n'\delta$ , will suffice. From the figure, it is seen that  $\gamma = \alpha - \rho$ , and  $\delta = \rho - \beta$ . Consequently,  $na + n\rho = n'\rho - n'\beta$ , or  $na + n'\beta = (n' - n)\rho$ . Designating  $AS$  by  $a$ ,  $BS$  by  $b$ , and the small arc,  $MS$ , which may be regarded as a straight line perpendicular to the axis, and denoting the distance of the point,  $M$ , from the axis, or the vertex,  $S$ , by  $k$ , the small angles may be again expressed as follows:  $\alpha = \frac{k}{a}$ ,  $\beta = \frac{k}{b}$ ,  $\rho = \frac{k}{r}$ , and we obtain  $\frac{n}{a} + \frac{n'}{b} = \frac{(n' - n)k}{r}$ .

$$\text{or } \frac{n}{a} + \frac{n'}{b} = \frac{(n' - n)}{r}.$$

The disappearance of the arc,  $k$ , from the equation, is a mathematical statement of the principle that all the rays from the point,  $A$ , pass, after refraction, through the point,  $B$ , under the assumption, however, that the arc,  $k$ , is sufficiently small.

The latter equation may be also written thus—

$$\frac{n r}{n - n' a} + \frac{n' r}{(n' - n) b} = 1,$$

or, if for brevity, the constant magnitudes are symbolized thus—

$$\frac{n r}{n - n'} = f \text{ and } \frac{n' r}{n' - n} = f',$$

we have—

$$\frac{f}{a} + \frac{f'}{b} = 1.$$

For  $a = \infty$  it follows from this that  $b = f'$ ; for  $b = \infty$ ,  $a = f$ ; that is, the point of condensation of rays parallel to the axes, when coming from the

left, lies to the right and at a distance,  $f'$ , from the vertex, and, similarly, the point conjugate to that at an infinite distance toward the right, lies at a distance,  $f$ , toward the left of the mirror. The distances,  $f$  and  $f'$ , are called the *first* and the *second focal distances*.

**332. Systems of Lenses.**—Any arbitrary number of spherical surfaces with centres lying upon a straight line, called the *axis*, the intervening spaces of which are filled with any refracting media whatever, constitutes a *system of lenses*; for, each portion being included between two consecutive spherical surfaces, may be regarded as a lens of arbitrary thickness. With ordinary optical instruments (microscope, telescope, etc.), which are composed of glass lenses having a common principal axis, the refracting media are alternate layers of glass and air. On the supposition that all the rays form small angles with the axis, a homocentric beam in the first medium passes homocentrically into the last, since, at each refraction on the consecutive spherical surfaces, the beam remains homocentric. To the incident ray,  $AM$ , parallel to the axis in the first medium (Fig. 335), suppose the conjugate ray,  $M'F'$ .



FIG. 335.—Principal Points.

to correspond, and to the ray,  $A'M$ , parallel to the axis, and coming from the opposite direction out of the last medium, let  $MF$  correspond as conjugate ray in the first medium. Since each ray pursues necessarily the same course when passing in the opposite direction, it may be said that the rays,  $AM$  and  $MF$ , passing through the point,  $M$ , in the first medium, pass also through the point,  $M'$ , in the last medium.  $M$  and  $M'$  are, therefore, conjugate points, and the planes,  $MH$  and  $M'H'$ , passing through them perpendicularly to the axis, are *conjugate planes*. There are, accordingly, in every lens-system, two conjugate planes, perpendicular to the axis, and of such character that the point,  $M'$ , of the one, which is conjugate to a point,  $M$ , of the other, lies in a straight line,  $MM'$ , drawn through  $M$  parallel to the axis. Any figure located in one of these planes, gives rise to a congruent image similarly situated in the other. Gauss (1840) called these planes *principal planes*, and their points of intersection with the axis,  $H$  and  $H'$ , *principal points*. The distances,  $FH = f$  and  $F'H' = f'$  of the foci,  $F$  and  $F'$ , from the corresponding principal points, are the *first* and *second focal distances* of the lens-system. With a double convex lens having equal faces and an index of refraction of 1.5, the principal points lie inside of the glass, and about  $\frac{1}{2}$  of the thickness of the lens from its vertex.

If the principal points and foci of a lens-system are given, it is easy to find for any point,  $A$  (Fig. 336, in the first medium, the conjugate point,  $A'$ , in the last; for the ray,  $AMM'$ , parallel to the axis, passes from  $M$  through the focus,  $F'$ , while the ray,  $A'F$ , passing through the focus,  $F$ , proceeds from  $N$  parallel to the axis, and cuts the ray,  $M'F'$ , in the point,  $A'$ , which is sought. If, now, we indicate the distances,  $AM$  and  $A'N'$ , of the points,  $A$  and  $A'$ , from the corresponding principal planes, by  $\alpha$  and  $\alpha'$ , and, further, if we denote

MH by  $h$ , NH by  $h'$ , there result from the triangles, FHN and AMN, F'H'M' and A'N'M'—

$$\frac{f}{a} = \frac{h'}{h + h'}, \quad \frac{f'}{a'} = \frac{h}{h + h'}$$

consequently—

$$\frac{f}{a} + \frac{f'}{a'} = 1.$$



FIG. 336.—Conjugate Points in a System of Lenses.

If the first and last media are of the same constitution (e.g. air),  $f' = f$ , and the foregoing equation becomes—

$$\frac{1}{a} + \frac{1}{a'} = \frac{1}{f}.$$

For such a lens-system (and, accordingly, for a lens of arbitrary thickness also), the same equation holds for the conjugate points, which was derived above for thin lenses, *provided that the distances  $a$ ,  $a'$ , and  $f$ , be reckoned from the principal points, or principal planes.*

In FIG. 337 suppose as before the point,  $A'$ , conjugate to  $A$ , to be known. Draw  $MA''$  parallel to  $M'A'$ , and also the diagonal,  $AA''$ , of the quadrilateral

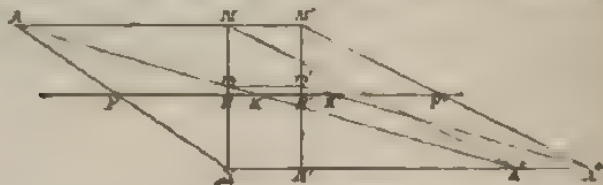


FIG. 337.—Nodes.

$AMA''N$ . To the point,  $m$ , where these meet the first principal plane, the point,  $m'$ , in the second plane will be conjugate,  $mm'$  being drawn parallel to the axis. Since, however,  $A'$  is conjugate to  $A$ ,  $m'A'$  is the emergent ray, corresponding to the incident ray,  $Am$ . Since  $mm'$  is parallel and equal to  $A'A''$  (or  $MM'$ ),  $mm'A'A''$  is a parallelogram, and, consequently,  $m'A'$  is parallel to  $Am$ . The points,  $K$  and  $K'$ , where the conjugate rays,  $Am$  and  $m'A'$ , cut the axis, are called *nodes*. They possess the characteristic that, to every incident ray passing through the first, there corresponds an emergent ray, parallel to the incident, and passing through the second node. From the figure, it is evident that  $KK' = HH'$  and  $HK = H'K'$ . If the first and last media are of the same constitution, the nodes and principal points coincide. Foci, principal points, and nodes, are called the *cardinal points* of the lens-system.

**333. Spherical Aberration.**—What has been said thus far of lenses holds good only in case the curvature of the surfaces is slight, or, what amounts to the same thing, in case the aperture is small. By the *aperture* of a lens surface is meant the angle intercepted by straight lines drawn from two diametrically opposite points of the edge to the centre of the sphere, of which the surface is a part. When the aperture is large, the rays striking the lens at its edge (VW, Fig. 338) are more strongly deflected than are those at the centre. They intersect, therefore, at a point, G, of the axis lying nearer the lens than the focus, F, of the central rays. The distance, FG, is called the *longitudinal spherical aberration*. Since the refracted beam is no longer homocentric, but the least possible cross-section of it is a small



FIG. 338.—Spherical Aberration.

circle (*circle of least confusion*), such a lens cannot produce distinct images. To bring the peripheral rays to the point, F, the surfaces of the lens must be given a form differing somewhat from a sphere. The imperfection due to this cause is called the error of *aberration due to the spherical form*, or *spherical aberration*. But, since for each distance of the luminous point the surfaces would require a special form, the spherical surfaces are preserved, but, by a suitable choice of the radius of curvature, the deviation of the rays are made as small as possible.

**334. Microscopes** are apparatus for magnifying small objects situated near the eye. Since a convex lens of short focal distance magnifies an object placed within its focus, such a lens is also called a *microscope*, but is distinguished from others by the adjective *simple*. The *compound microscope* (Jansen, 1590) has a far higher capability of performance. It consists essentially of two concave lenses (*ab* and *cd*, Fig. 339), one (*ab*) has a very short focal distance, and being turned always toward the object, is called the *objective*. It forms at RS an inverted, magnified, real image of the small object (*rs*), situated in a position a little beyond the focus. This image is viewed by means of an eye-

lens, or *ocular* (*cd*), which stands nearer the image than its focal distance. The image then appears precisely like a self-luminous object, and is seen a second time magnified at *R'S* as a *virtual image*. The image *R'S* finally seen has a position opposite to that of the object, *rs*, so that objects seen in such a microscope are *inverted*. Fig. 340 shows the external arrangement of an

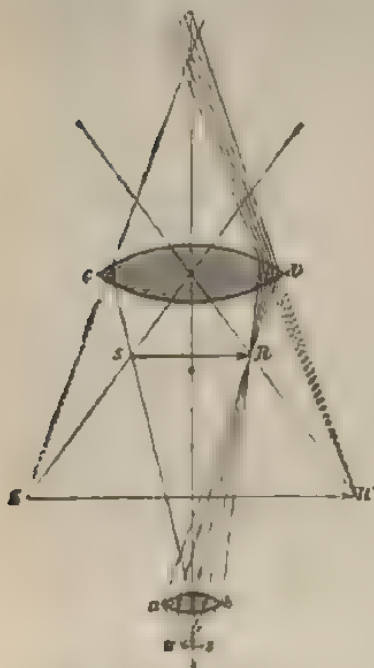


FIG. 339.—Path of Ray in a Microscope.



FIG. 340.—Compound Microscope.

ordinary microscope. The ocular, *a*, and the objective, *b*, are encased in a vertical brass tube, which is movable with gentle friction within the brass cylinder, *f*. The instrument is roughly focussed by sliding the former tube back and forth in the latter. Finer focussing may be accomplished by turning the milled head, *c*. Ordinary transparent objects are placed upon a plate of glass which lies upon the little table, *cc*, and are illuminated from below by means of a mirror.

**335. Telescope.**—Any instrument by which remote objects are made visible under a larger visual angle than with the naked eye, is called a *telescope*. The *Keplerian* (1611), or *astronomical telescope*, consists essentially of two convex lenses, a larger ( $oo$ , Fig. 341), of longer focal distance, and attached to the front end of a tube of suitable length, and a smaller ( $vv$ ), of short focal distance, which is movable within a small tube at the rear end of the large tube. The former lens, which is turned toward the object viewed, and called the *objective*, casts an inverted image,  $ab$ , of the remote object,  $AB$ , near its focus. The rays from every point,  $A$ , of the object are united at the corresponding point,  $a$ , of the image. This image seen at  $a'b'$  by the second lens, the *ocular*, is magnified, because it lies within the focus of  $vv$ . The inversion of the images of objects is of no serious disadvantage in observing the heavenly bodies, in surveying, etc. The value of the Keplerian telescope for these purposes is very materially increased by the so-called *reticle* (Luzout, 1667). Within the tube of the ocular, at the place where the image,  $ba$ , is formed, two perpendicular spider-lines are stretched, which intersect exactly upon the axis of the telescope. When the image of a remote point, e.g. of a star, is seen at the point of intersection of the threads, the axis of the telescope is directed exactly toward that point, and the position of this axis locates the line of sight drawn from the eye toward that point. Kepler's telescope forms, therefore, the *view-tube* in all instruments for the measurement of angles. As an example, the *theodolite*, used to measure both vertical and horizontal angles, may be cited (cf. Fig. 136). A horizontal disk, movable about its centre, carries a telescope which may be revolved about a horizontal axis. Two diametrically opposite verniers of the movable circle indicate upon a circular ring surrounding it and graduated to degrees and their fractional parts, the value of the horizontal angle through which the instrument has been turned. A vertical circle attached to the horizontal axis in a similar way permits the reading of vertical angles.

While the inverted position of the images is a matter of no consequence in astronomical observations, and for sighting, it is, nevertheless, a drawback in the direct scrutinizing of remote

terrestrial objects. This difficulty is obviated by replacing the astronomical ocular by the "terrestrial," which is a feebly



FIG. 341.—Action of Kepler's Telescope.



FIG. 342.—Action of Galileo's Telescope.

magnifying microscope, composed of four convex lenses placed within the same tube. These lenses again invert the image.

The telescope provided with such an eye-piece is called a *terrestrial telescope* (Schyrl, 1645; De Rheita, 1665). Objects are also seen erect with the *Galileian*, or *Holland telescope* (Jipporshay, 1608; Galileo, 1609). In this instrument the real image, *ba* (Fig. 342), which the objective, *oo*, tends to form, is not actually produced, because the rays converging toward the image fall, before reaching it, upon the concave lens, *vr*, acting as ocular. This lens, when so placed that *ba* lies outside of the focal distance, spreads out the rays until they appear to come from the erect image, *ab*. In Fig. 342 the paths of the rays for the point, *A*, of the object are clearly shown. Since no real image is formed, no reticle can be used, and, therefore, Galileo's telescope is not adapted to precise measurement. Here, as with Kepler's telescope, the focal distance of the ocular must be less than that of the objective. Since, with his instrument, the two glasses are separated approximately by the difference of their focal distances, Galileo's telescope differs from Kepler's, where the objective and the ocular are separated by a distance equal to the sum of their focal distances, in that it is considerably shorter, and is therefore specially well suited for use as a weakly-magnifying pocket telescope, or as an opera-glass (magnifying two or three diameters), or as a field-glass (20-30 fold magnification). Fig. 343 shows the arrangement of the ordinary opera-glass used at theatres. Within a tube carrying at its remote end the objective lens, *oo*, a collar, *bb*, called an *adapter*, is screwed, into which the tube, *c*, with the ocular, *aa*, slides with slight friction. The nearer the observed object is to the observer, the farther must he draw out the ocular tube to obtain a distinct image.

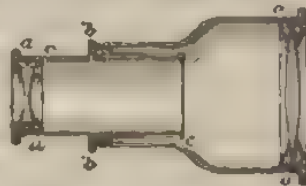


FIG 343.—Opera-glass.

These telescopes, which are constructed of lenses made of glass, are called *dioptric telescopes*, or *refractors*, the latter term being more specially applied to the large astronomical instruments of this pattern. On account of the similar action of convex lenses and concave mirrors, telescopes are constructed in which

a concave mirror assumes the rôle of the objective. Such instruments are called *reflecting telescopes*, *cathoptric telescopes*, or *reflectors*.

The arrangement of the *Newtonian reflecting telescope* (1663) is evident from Fig. 344. The concave mirror, placed at the

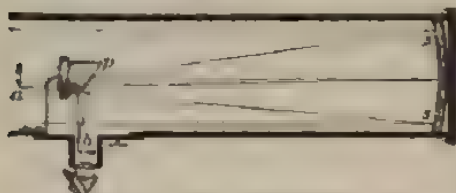


FIG. 344.—Newton's Reflector

bottom of an open tube of suitable length, tends to converge the rays of light from a distant object into an inverted image at *a*. Before the rays are focussed, however, they are reflected from a plane mirror, *p*, inclined at an angle of  $45^\circ$  to the axis of the tube, so that the image actually lies at *b*. It is then viewed by means of a convex ocular placed in the latter orifice. The reflection of the image toward the side prevents the loss of light from the mirror, *ss*, which would ensue if the head of the observer were in position to view the image, *a*, directly from the front. In the *giant telescopes* of Herschel (1795) and Lord Rosse, whose mirrors had diameters of from one to two meters, the second mirror, and consequently, also, the loss of light due to it, was avoided by the following simple and effective artifice. The



FIG. 345.—Herschel's Reflector.

concave mirror (*ss*, Fig. 345) is inclined slightly to the axis of the tube, so that the image forms near the edge of the tube, and is there observed by means of an eye-piece, *a*. With this construction, of course, the head of the observer intercepts some of the light entering the tube, but by reason of the large diameter of the mirror, this loss is of little consequence. Herschel called his instrument the *front-view telescope*. With Newton's reflector the observer has the object at his side, while with the front-view telescope he turns his back to it. The circumstances that direct vision is impossible, and also that the images are reversed, make these instruments unsuitable for observing terrestrial

objects. These inconveniences are avoided in the *Gregorian reflector* (1663; Fig. 346). The concave mirror, *ss*, is perforated at its centre, and the ocular is placed in the small tube behind this aperture. An inverted image of a distant object is formed at *a*, a little beyond the focal distance of a small concave mirror, *v*. The latter forms, at *b*, a second inverted image, which is *erect* with respect to the object. The image at *b* is then viewed through an ocular serving merely as a magnifier. The instrument is sharply focussed by means of the rod, *mn*, which carries the small mirror, *v*, back and forth by means of an endless thread cut on its upper end.

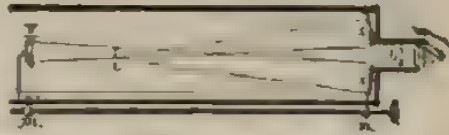


FIG. 346. —Gregory's Reflector

In the construction of very large instruments, reflectors offer some advantage over refractors. Small reflecting telescopes were in general use earlier than refractors, for it was not then understood how to secure the desired degree of perfection with the latter. Reflectors, however, do not furnish sufficient intensity, and they cannot now compete with refractors, although with the invention of silvered glass mirrors in recent times, their efficiency has been very materially heightened.

The *magnifying power* of a telescope is expressed by the ratio of the visual angle,  $a'mb'$ , or  $amb$  (Figs. 341, 342), under which the image,  $a'b'$ , is seen by an eye held near the ocular, to the visual angle,  $AcB = acb$ , under which the object is seen without a telescope. If, as is always the case, these angles are small, they are to each other in the ratio of the distances of the image,  $ab$ , from the objective on the one hand, and from the ocular on the other; or, since  $ab$  is near the foci of these lenses, the angles are, with sufficient approximation, to each other in the ratio of the focal distance,  $F$ , of the objective to the focal distance,  $f$ , of the ocular. The magnifying power is, therefore,

nearly equal to  $\frac{F}{f}$ .

Since the image of a point always lies upon the corresponding

axial ray, only such points can be seen by a telescope as have axial rays passing through the ocular. The *field of view*, or simply *the field* of a telescope, is, therefore, bounded by a cone of rays, having the optical centre of the objective for its vertex and the ocular for its base.

**336. Dispersion.**—Let a beam of sunlight be passed through a small opening, *b* (Fig. 347), in a window shutter, into a darkened room, and let the opening be covered with a piece of red glass. The beam will be coloured red, and will produce a

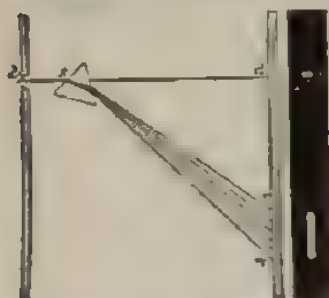


FIG. 347.—Formation of a Spectrum

bright red spot at *d* upon a screen of white paper interposed in its path. If now a prism (represented in outline at *a*) be placed in the path of the beam, the latter will be refracted toward the base of the prism, and the red spot will appear upon the screen at *r*, far to one side of *d*. When the opening is covered with violet glass there appears upon the screen a violet spot of light, *v*, displaced

laterally still further than the red, and if a piece of green glass is used, the green spot of light will appear between *r* and *v*, where the red and the violet spots appeared formerly. Hence, it appears that light of different colours is refracted by different amounts with the prism, and that green light is bent more than red, and violet more than green. If now a beam of white sunlight is allowed to fall upon the prism without the interposition of coloured glass, a coloured band appears upon the screen stretching continuously from *r* to *v*. This band is red at the place where the red spot formerly fell, and violet where the violet spot was seen. In this band, stretching from *r* to *v*, the following succession of colours is seen: red, orange, yellow, green, bright blue, dark blue, violet (the well-known colours of the rainbow). This coloured band is called the *spectrum*. From this experiment it is seen that white sunlight is composed of light of various colours. These fundamental, or *primary*, colours are refracted by the prism by different

amounts, the refraction increasing by a regular gradation from red to violet. Since these primary colours fall upon different places on the screen corresponding to their respective refrangibilities, they are separated from one another. This resolution of white, or, indeed, of any composite light into its variously coloured constituents, by virtue of their different refrangibilities, is called *dispersion*. The individual colours of the spectrum cannot be further decomposed, for if the spectrum is caught upon a screen, AB (Fig. 348), provided with a small aperture, so placed as to permit only the rays



FIG. 348.—Irreversibility of the Colours of the Spectrum.

of one colour (*monochromatic light*) to pass through, these rays will be refracted by a second prism, *p*, but not again spread out into a spectrum. The colours of the spectrum, therefore, being incapable of further resolution, are called *simple*, or *homogeneous colours*. Each simple colour corresponds to a definite refrangibility, and is hereby assigned a definite position in the spectrum. There are then as many simple colours as there are refrangibilities within the region of the spectrum. The number of simple colours is thus seen to be infinite, and they follow one another at inappreciable intervals, forming thus a continuous band of colour. The above enumerated seven colours are only the principal shades, which are distinguishable to the eye. If, now, white light is a mixture of the variously coloured rays of the spectrum, these rays, if compounded, must reproduce white light. If, indeed, the spectrum be allowed to fall upon a large condensing lens, *l* (Fig. 349), it focuses the



FIG. 349.—Recombination of the Spectral Colours

convergent coloured beam upon a screen at *f*, where they reproduce the image of the front surface of the prism, in the form of a *white spot* of light. This spot, however, ceases to be white if any colour is kept out of the mixture. When, for instance,

a slender wedge-shaped piece of glass is interposed before the lens and allowed to catch up the red rays of the coloured pencil, these rays will be turned aside, producing a red image upon the screen at the side of  $f$ . The image,  $f$ , in which the yellow, green, blue, and violet rays are now blended, has the appearance of a green spot. The red spot mixed with this greenish colour must again produce white. This occurs instantly at the point  $f$ , when the prism is removed. Two colours which thus complement each other to white, are called *complementary colours*. By moving the small prism gradually through the entire length of the spectrum, other colours may be turned aside successively, and the two images upon the screen will show an entire series of pairs of complementary colours. It is found in this way that red and green, yellow and blue, greenish yellow and violet shades are mutually complementary.

To produce the impression of white light in the eye, the combined effect of all the colours of the spectrum is by no means necessary, for it may be readily shown that the combination of *two simple colours* may produce white. Of the simple colours the following are complementary: red and greenish blue, orange and bright blue, yellow and dark blue, or indigo, greenish yellow and violet. In general, for every point of the spectrum from the red end to the beginning of the green, a complementary point may be found in the portion of the spectrum extending from the beginning of the blue to the violet end. The spectral green alone possesses no simple complementary colour. Its complementary is a reddish purple, which is compounded of red and violet.

**337. The Rainbow.**—The phenomena of dispersion are displayed before our eyes on a large scale in the rainbow, which may be seen by turning the back to the unclouded sun and looking toward a mass of clouds from which raindrops are falling. The formation of the bow may be best understood from the following experiment: Upon a hollow sphere, a (Fig. 350), of glass filled with water, suppose a horizontal beam of sunlight to impinge, the diameter of which is equal to that of the sphere, or greater. Upon a screen, as, placed before the sphere and provided with a central aperture for the passage of the incident rays, there will be seen around the opening at a distance from it equal to that of the sphere from the screen, a coloured circle, or rather a circular spectrum with its colours concentrically arranged, the red forming the outermost, and the violet the innermost ring. Still farther from the centre of the screen a second circle may be seen, whose

colours are considerably fainter, and arranged in the reverse order, the red here constituting the innermost, and the violet the outermost circle. The former spectrum is produced by rays, which, after entering the sphere, are reflected at its rear surface and pass out through its front surface. During this twofold refraction and single internal reflection, as represented in Fig. 350, the ray is deflected from its original direction, varying in amount with the distance of the incident from the central ray. By the *central ray* is meant the ray passing directly toward the centre of the sphere. It is reflected into itself at the back surface of the sphere, and suffers no deflection. Passing away from this central ray, the deflection increases at first, until at a certain distance a maximum value is reached; from here to the ray which passes tangent to the external surface of the sphere, the deflection diminishes. The rays most strongly deflected strike the circumference of the colour-ring upon the screen and produce here an illumination, which is considerably stronger than that of a point inside of the colour-rings. Passing from the rays, which are most deflected, either toward the central ray, or toward the tangent ray, the deflection changes at first very slowly, but later quite rapidly. For this reason the rays adjacent to those most strongly deflected on entrance, remain near them also after exit, and consequently intensify their luminous effect. On the contrary, those rays which impinge at other points of the sphere lying near each other, pass out after the second refraction, in a divergent pencil and produce upon the screen no considerable illumination. If, therefore, the experiment be performed with monochromatic light, i.e. by covering the aperture of the heliostat with a piece of red glass, the luminous appearance upon the screen reduces to a feebly illuminated circular area, surrounded by a bright red circumference. The maximum deviation for red rays, the angle between  $ok$  and  $kl$  is a little greater than  $42^\circ$ . By virtue of their greater refrangibilities, the other rays are drawn nearer to the direction,  $ok$ , of the incident ray, and produce circles whose radii are smaller in the order of refrangibility. The deviation of the violet rays is about two degrees less than that of the red. When white day-light is used, therefore, the circular spectrum must be surrounded by red at its outer limit.

The second coloured circle is caused by rays which, as shown in Fig. 351, are twice refracted, and twice internally reflected. The least deflection of such rays is about  $51^\circ$ , for the red it is slightly less, and for the violet somewhat greater.

Each falling raindrop acts precisely as the water-filled sphere. An eye, or (Fig. 352), looking in the direction of a cloud floating in the regions of the sky opposite the sun, will therefore receive the light reflected once on the

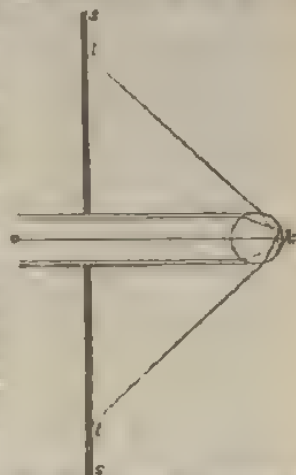


FIG. 350.—Refraction and Internal Reflection within a Sphere of Water.

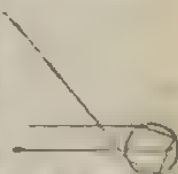


FIG. 351.—Refraction and double Internal Reflection within a Sphere of Water.

inner surface of the drops with sufficient intensity for visibility from only those drops situated at an angle of about  $42^\circ$  from a point, S, of the sky lying exactly opposite to the sun. The rays coming from other drops pass the eye unobserved. Since the drops, AA', sending the red rays toward O are somewhat farther removed from the point, S, than the drops, BB', whence the less strongly deflected violet light proceeds to the eye, there is seen a circular arc with its centre at the point, S, in which the colours of the spectrum are arranged concentrically in the order of their refrangibilities, and thus we see the *first* or *primary bow*. The much fainter secondary bow, with its reversed succession of colours, lies at an angle of  $51^\circ$  from the point, S. It is produced by the rays which have suffered a least deviation in the rain-drop after two refractions and two reflections, as in Fig. 351. The portion of the sky between the two bows is darker than the rest of the sky, because from it no rays enter the eye after having been reflected and refracted within the drops.

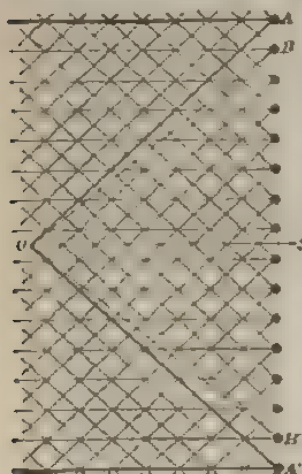


FIG. 352.—Formation of the Rainbow.

The higher the sun stands above the horizon, the shorter is the visible portion of the bow. No bow at all can be seen when the altitude of the sun is greater than  $42^\circ$ , because, in such a position, the entire arc of colour lies below the horizon. At sunrise and sunset both arcs appear as semicircles.

Upon high mountain-tops, and from balloons, they may sometimes be seen almost as complete circles.

**336. Halo.**—The bright ring which is frequently seen surrounding the moon, and less frequently, the sun, at a distance of  $22^\circ$ , is called the *halo*. It shows the colours of the rainbow, although in case of the moon's ring, of course, very feebly and indistinctly. The arrangement of colours is, however, opposite to those of the primary bow; the red being within, and the violet without. The phenomenon occurs when the sky is covered with light flaky clouds as with a semi-transparent veil, and its explanation lies in the effect produced upon light by the fine crystals of ice, of which these clouds consist. The small needles of ice have the form of regular hexagonal prisms, where the surfaces, which are neither parallel nor adjacent, form angles of  $60^\circ$  with one another. The rays of light are refracted and dispersed by such a pair of surfaces in the same way as by a glass prism. In the particular position in which the ray is perpendicular to the edge of the prism, and forms equal angles with the surfaces, the deflection is less than in any other. For a prism of ice of  $60^\circ$  the least deflection is  $22^\circ$ . If, therefore, a countless number of such small prisms are floating in the air in all possible positions, the eye can receive refracted light only from directions which make angles greater than  $22^\circ$  with the line of sight to the object. The space within a circle of  $22^\circ$  radius, not being illuminated by refracted light, appears relatively dark. A distinguishing characteristic of the position of least deflection is that from this position a prism may be turned considerably in either direction without perceptibly changing the direction of the refracted rays. Among the numerous ice

crystals there will doubtless be many exactly in the position of least deflection, but a far greater number will lie not exactly, but very nearly in it. All these prisms deflect the refracted rays toward a single direction, from which the eye will receive, from the combined effect of many such rays, a more vivid impression than from any other direction. Since the different colours of white light are refracted by different amounts, the bright ring thus produced is coloured like the rainbow, and the least deflected red appears at the inner edge. Since the needles of ice, under gravity, sink slowly downward, in quiet air they assume vertical positions, because in these positions they experience least atmospheric resistance. In still air, filled with needles of ice, those occupying a vertical position will then be most numerous. The ring frequently shows at the two points of its circumference which receive light from the vertical prisms, i.e. at the ends of its horizontal diameter, areas of particularly intense brightness. These spots of the solar ring, which show vividly the colours of the rainbow, are called *mock suns*, or *sun-dogs*. Less frequently than the *halo* of  $22^\circ$ , another of  $46^\circ$  is seen, which, as also various other similar phenomena, are likewise explained by the refraction of light in these prisms of ice. In polar regions, where the atmosphere is frequently filled with ice-needles, the halos of both sun and moon are very beautifully displayed. In temperate zones the ring about the moon shows the more frequently, because its rays are accompanied with so little heat as to permit the crystals of ice to form.

**339. Pure Spectrum.**—When the prismatic spectrum is produced in the above manner by dispersing a ray of sunlight admitted through a small aperture by means of a prism, the primary colours are not perfectly separated from one another.

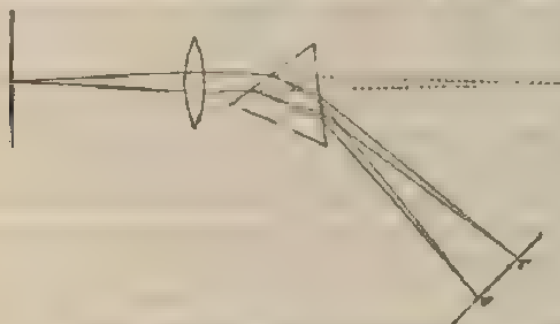


FIG. 339.—Formation of a pure Spectrum.

Since each primary colour produces its own image of the sun (317) which is deflected proportionally to its refrangibility, these images overlap at their edges, and partially intermix. To obtain a *pure spectrum*, the rays are admitted through a *narrow slit* (Fig. 353, seen from above) and caught upon a lens situated at a distance from the prism greater than

the focal distance of the lens. This image forms upon a screen, placed at the proper distance, a sharply defined, real image of the slit. The prism is now placed in front of, or behind, the lens with its edge parallel to the slit in the position of least deflection, as the prism forms perfect images only in this position. To each primary colour a deflected image of the slit will then correspond, and the numerous slender images of the slit lying beside each other, overlap much less, and form, accordingly, a purer spectrum the narrower the slit is made. A pure spectrum is also seen on looking, either with the unaided eye, or with the telescope, through a prism toward a narrow slit, situated parallel to the edge of the prism. If, however, a wide opening is subdivided into narrow strips parallel to the edge of the prism, each of these strips will give its own spectrum. Since these spectra overlap, a long image of the opening is produced, which is red at the less refrangible and violet at the more refrangible end, while at the middle, where all the colours mix, it is white.

**340. Fraunhofer's Lines.**—In a solar spectrum thus produced, a series of fine dark lines parallel to the slit is seen. These lines, first (1817) carefully studied by Fraunhofer, are called *Fraunhofer's lines*. They are distributed at unequal distances along the entire spectrum. Many of them are very fine and difficultly perceptible, while others are stronger and more readily seen. Their formation is independent of the material of the prism, for they show the same appearance and arrangement in all solar spectra. They are accordingly merely small gaps in the succession of spectral colours, the presence of which indicates that the simple colours corresponding to them are wanting for some reason. They form, within the regular succession of colours of the spectrum, useful distinguishing characteristics, each corresponding always to the same sort of light, thus making the identification of any desired point of the spectrum with certainty. Fraunhofer designated eight of the most conspicuous of these lines with the letters of the alphabet from A to H (Fig. 354, and the spectral chart, Fig. 1). The line, A, lies in the extreme dark red, B in the high red, C between the red and the orange, D between the orange and the yellow, E in the yellowish-green, F between the green and the blue, G between

the dark blue and the violet, and the double line, H, lies toward the end of the violet. By Fraunhofer's lines, it was made possible to determine with great accuracy the indices of refraction of different substances, for different parts of the spectrum, lying between B and H inclusive. Thus these lines immediately acquired great importance in practical



FIG. 354.—Solar Spectrum with Fraunhofer's Lines.

optics; for it was only upon the basis of this accurate knowledge of the refraction and dispersion of different sorts of glass that Fraunhofer was able to manufacture lenses free from dispersion (342), and, accordingly, also telescopes far surpassing in perfection any that had been previously constructed. For some liquids and for certain varieties of glass, the indices of refraction determined for the lines, B, D, E, and H, are given in the following table:—

	B.	D.	E.	H.
Water	1.3309	1.3336	1.3359	1.3412
Alcohol	1.3628	1.3654	1.3675	1.3761
Carbon disulphide	1.6182	1.6308	1.6438	1.7019
Crown glass	1.5258	1.5296	1.5330	1.5466
Flint glass of Fraunhofer	1.6277	1.6350	1.6420	1.6711
Flint glass of Meiss	1.7218	1.7321	1.7425	1.7895

The difference between the indices of refraction for the outermost rays (for the lines B and H) may be regarded as a measure of the dispersion. While for crown glass (the glass ordinarily used for optical purposes) the dispersion is 0.021, for flint glass (lead glass) it is 0.043, or a little more than double the former value. The index of refraction for the line E is ordinarily selected for an average value.

**341. Spectrometer.**—Measurements of the spectral lines are made by the method of least deflection with goniometers, which, from this special application of them, are called *spectrometers* (Fig. 355). A (Keplerian) telescope, provided with a reticle, is directed toward the centre of a horizontal graduated circle and has its vertical axis of rotation rigidly fixed. A second tube, also having its axis directed toward the centre of the circle, the *collimator*, carries a vertical slit at its outer end, and at its inner a convex lens, in whose focal

plane the slit lies, so that the rays from any point of the slit leave the collimator lens in a parallel beam. The slit is by this device removed to an infinite distance, and on the reticle of the focussed observing-tube its image is distinctly seen provided the axes of the two tubes have the same position (the setting, 0). Placing a prism, then, upon a small stand at the centre of the graduated circle, to see the deflected image of the slit, or, rather its spectrum, the observing-tube, together with the graduated circle, need only to be rotated

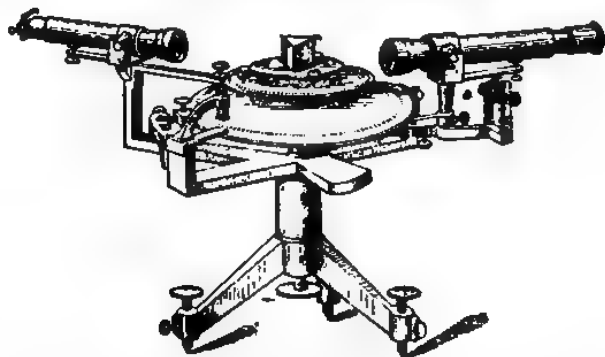


FIG. 355.—Spectrometer.

into the proper positions. By rotating the stand, the prism may be easily (for each particular Fraunhofer's line) brought into the position of least deflection, and the amount of rotation to the position of perfect focus and setting is then read from a fixed vernier. It is evident, also, that the instrument may be used as a reflecting goniometer (Fig. 300) for measuring the refracting angle of the prism, thus furnishing the two quantities needed to compute the index of refraction, viz. the least deflection and the angle of the prism (329).

**342. Achromatism.**—We have seen, then, that a beam of sunlight is not merely refracted by a prism, but that it is at the same time dispersed into a coloured band, so that, instead of a white spot of light, a spectrum appears upon an interposed screen. The distance of the centre of the spectrum from the place where the spot of light would appear may be taken as a measure of the deflection produced by the prism while the length of the spectrum furnishes a criterion for the dispersion. If a second precisely equivalent prism is placed behind the first, with its edge directed oppositely, the latter turns the beam back to its original position, and at the same time condenses the band of colour into a point. A white spot of light now appears upon the screen in the direction of the incident rays. The

second prism has, therefore, neutralized both the dispersion and at the same time the refraction of the first prism. Newton thought that, as in this instance, dispersion and refraction were always simultaneously produced, and that it would be impossible to remove the one without destroying the other also. In reality, however, a flint glass prism gives twice as long a spectrum as a prism of crown glass having an equal refracting angle, but by no means double the deflection. A prism of flint glass, with an angle about half as great as that of a crown glass prism, produces thus just as long a spectrum, but only about half as much refraction as the latter. When the prisms are placed together in opposite positions, the dispersion will be wholly removed, while the refraction, though diminished, will not be entirely destroyed. Such a combination of two prisms forms a prism free from dispersion, or an achromatic prism, which produces upon the screen a white spot of light deflected somewhat laterally. The achromatism would be perfect if the two spectra whose total lengths are equal exactly coincided in all their parts. With the older varieties of glass, however, this is not the case. With Fraunhofer's flint glass, the less refrangible rays are more condensed, while at the same time the more refrangible ones are more dispersed than with crown glass. Consequently a feeble dispersion (a secondary spectrum) still remains. But in the optical laboratory at Jena (Abbe and Schott) varieties of crown and flint glass are produced which show a uniform rate of dispersion throughout the entire spectrum, and prisms constructed of such glass, when combined, are perfectly achromatic (apochromatic).

In consequence of the unequal refrangibilities of the different colours, an ordinary double-convex lens cannot recondense the rays from any point exactly into another point; for the more refrangible violet rays unite at a point,  $v$  (Fig. 856), lying nearer to the lens, while the less refrangible red collect at the remoter point,  $r$ . Since to each point of the object there corresponds in the image, not a point, but a circle of dispersion with coloured edges, the images cast by this lens are not sharply defined, but show colour fringes. This imperfection is called *colour aberration*, or the *chromatic aberration*.

of the lens. A telescope, or microscope, having an objective of this sort would have very little value in consequence of the indistinctness of its images. For this reason it was impossible to construct really valuable refracting telescopes until it had become possible to manufacture lenses free from chromatic



FIG. 356.—Chromatic Aberration.

aberration (Dollond, 1757). The achromatic prism suggested the solution of this problem. To cure a convex lens made of crown glass (AB, Fig. 357) of chromatic aberration, a concave lens of flint glass (CD), with half the deflecting and the same dispersing power, is placed immediately behind the crown

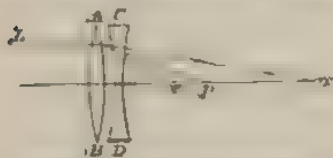


FIG. 357.—Achromatic Lens.

glass. In this position the deflections and the dispersion of the lenses are both opposed to each other. The ray of white light,  $L$ , is dispersed by the crown glass lens into a coloured band, whose red ray meets the axis at the re-

motor point,  $p$ , and whose violet ray meets it at  $v$ . The flint glass deflects the rays again from the axis, the violet more strongly than the red, so that both reunite with one another and with the intermediate rays of the band of colour into a single white ray which cuts the axis at the more remote point  $p'$ . The two lenses combined thus (they are frequently cemented together with a transparent substance such as Canada balsam, into a single piece), form an *achromatic lens*, which unites all the rays from a white point again into a white point in the image. The objectives of telescopes, microscopes, and of photographic cameras are always such achromatic combinations of lenses, in which, by a proper choice of the radii of curvature, spherical aberration is also reduced as much as possible (333). Such a combination of lenses is said to be *aplanatic*. Oculars of telescopes and of microscopes also consist of a number of lenses

made of the same glass, the radii and distances of which are so chosen that both spherical and chromatic aberration are reduced to a minimum.

To any given prism of crown glass a prism of flint glass may be readily adapted which will retrace a ray of any particular colour—e.g. the green of the Fraunhofer's line, E—just as strongly as does the prism of crown glass, and therefore, united with it in the reversed position, will destroy the refraction of this particular ray. But since the deflection of the remaining rays is not simultaneously neutralized, the two prisms combined form a *direct vision combination of prisms*, or a *direct vision spectroscope*, which will produce a spectrum to be viewed in the precise direction of the incident rays.

**343. Absorption of Light.**—If a complete spectrum is formed upon a white screen by means of a slit, lens, and prism, and the slit is covered with a dark red disk of glass, red and orange alone will remain on the screen. The other colours from yellow to violet are extinguished. Of the entire succession of colours composing white light, the red glass transmits only the red and orange. All other colours are *absorbed* by the glass. For them the glass is *opaque*, or *non-transparent*. The glass acts as a sieve, allowing the red and orange rays to pass, but holding back all the rest. This is why the image appears to be of the reddish hue produced by a mixture of the spectral red and orange. It is also to this circumstance that green or blue glass owes its colour. The former transmits the green rays with especial readiness, and the latter the blue, while the remaining rays are more or less perfectly absorbed. A window glass, on the contrary, appears *colourless*, because it permits the passage of all colours composing white light with equal facility. The rays after passage, of course, combine again into white light.

When a spectrum is thrown upon a red screen as in the foregoing experiment, with a red glass only the red end of the spectrum remains visible. The rays of light falling upon the rough surface of the screen penetrate it to a slight depth before they are diffused in all directions, and beneath the surface they undergo the absorption which is produced by the colouring

matter of the paper screen. This colouring matter, however, emits only the red rays and absorbs all the rest. From this it is clear also why such paper appears red when illuminated by the ordinary white light of day. When the spectrum is thrown upon a screen of yellow, green, or blue paper, it is noticed that each of the screens obscures, or extinguishes, different portions of the spectrum, but that it always leaves that particular colour unimpaired which is exhibited by the paper in daylight. White paper absorbs none of the primary colours with any special preference, but throws them all back in their original proportions. For this reason the paper appears *white* when illuminated by daylight. Surfaces which possess relatively low dispersive powers for all colours of light are called *grey*, and a body appears *black* whenever it absorbs all the rays which fall upon it. In this way the rich multiplicity of coloured bodies (natural colours) is explained by the absorption of light due to the bodies. The colour of a body is merely that mixture of all the coloured rays which remain from the white light illuminating it after extracting from this light the rays which the body absorbs. It is evident, therefore, that a body illuminated by diffusely reflected light can exhibit only such colours as are contained in the light incident upon it. For a piece of red paper to appear of this colour, red rays must be contained in the light illuminating it. Candle-light, for example, contains such rays. But if the red paper is illuminated by an alcohol lamp, whose wick is saturated with salt, or with the flame of a Bunsen burner in which a bead of molten salt is held by a platinum wire (sodium flame), and which emits only yellow light, the paper appears black. In this monochromatic (yellow) illumination it is impossible longer to distinguish the different primary colours. With it only brightness and darkness are distinguishable. The human face appears ghastly pale, and the most richly decorated oil-painting resembles a drawing in sepia. If the sun were a ball of glowing sodium vapour, all nature would appear clothed in raiment of this monotonously dull colour. White sunlight in which countless colours are combined is necessary to disclose to us the rich variety of colours of the physical universe. The light of gas flames

and of candles contains all colours of the solar spectrum, but in somewhat different proportions from sunlight. Yellow rays are here very richly represented, while the blue and violet are relatively scarcer than in sunlight, and for this reason the former sources appear yellow in comparison with daylight. This also explains the well-known fact that, by candle-light, white and yellow are easily mistaken for each other. Green and blue articles of wearing apparel are also distinguishable with difficulty. Green substances reflect green pre eminently, though somewhat mixed with blue, and blue substances reflect in addition to green a large proportion of blue. Since now blue is contained in but relatively small quantities in candle-light, while green is present in considerable quantity, both green and blue substances will be intensified when illuminated by the light of a candle.

The spectrum of light transmitted by a coloured body, or dispersed by it (the *absorption spectrum*), is not always so simple as it is with red glass, or red paper. There are many coloured substances which select from the rays of the spectrum one or more colours and absorb, or *quench*, them, while other adjacent, or intervening, radiations are left untouched (*selective absorption*). This behaviour is revealed in the spectrum by numberless dark *absorption bands*, some of which are broad and some narrow, and whose position in the spectrum is characteristic of the chemical constitution of the substance. These lines, therefore, furnish a means of recognizing and distinguishing substances. For example, light transmitted through the green leaf of a plant shows in the spectrum a black band in the high red (between the Fraunhofer lines, B and C). The middle red is absorbed by the chlorophyl of the leaf, while this is not the case with the extreme red and the orange red. The colouring matter of the blood absorbs the violet end of the spectrum and produces, in the yellow green (between D and E), two dark absorption bands separated by a bright yellowish green space. Many gaseous bodies, such as nitrous acid, vapour of iodine, etc., produce numerous slender dark absorption lines in the spectrum of light which has been transmitted through them.

The sodium flame mentioned above, emits simple yellow

light, which is not dispersed by a prism, but is refracted and produces a bright yellow line in the exact position where, in the solar spectrum, the dark line, D, would appear. If now the light from an incandescent body (*e.g.* Drummond's, or the electric light) be passed through this yellow flame, and the transmitted light be spread out into a spectrum, there appears, in the position of the former yellow line, a dark line against the bright ground of the otherwise continuous spectrum. The sodium vapour of the yellow flame has, therefore, permitted all rays from the glowing body to pass through it without hindrance, with the exception of the particular rays which it is itself capable of emitting. These are absorbed by the flame, and hence for them alone the flame is impermeable. The law revealed in this fact holds generally: *A body absorbs precisely those radiations which it is itself capable of emitting, or the absorptive power of a body for any particular radiation is directly as its radiating power for the same radiation (Kirchhoff's law, 1860).*

**344. Spectrum Analysis.**—Various forms of the *spectroscope*, as such spectral apparatus are termed, are used in observing and studying the spectrum. In *Bunsen's spectroscope* (Fig. 358) a flint glass prism, P, whose lateral faces are inclined at an angle of  $60^\circ$  to each other, is supported at the centre of a cast-iron stand. Three horizontal tubes, A, B, and C, are directed toward the prism. The first (A) is the slit-tube, or the *collimator*, and has at the end toward the prism a lens, *a* (Fig. 358, shown in plan), in whose focal plane the slit, *l*, is placed vertically, and hence, parallel with the edge of the prism. The rays from any point of the illuminated slit are rendered parallel by the lens, *a*, since they come from a point in its focal plane. After being deflected by the prism placed in the position of least deflection (for central rays), the rays pass through the objective, *b*, of the telescope, B, and are condensed at the corresponding points of the focal plane of *b*. If the rays incident on the slit are red only, a small red image of the slit is formed at *r*. But if the violet rays from the slit also pass through the prism, they will be more strongly deflected by it and will form a violet image of the slit at *v*. If now white light, which is composed of rays of all intermediate refrangibilities, passes through

the instrument, a countless number of corresponding images of the slit will lie beside each other in an unbroken series, and will form in the focal plane of *b*, a complete, or continuous, spectrum, *rr*, which is observed through the lens, *o*, used as an eyepiece, or ocular. To compare the spectrum with a graduated scale, Bunsen hit upon the following ingenious device. A third tube, *U* (the scale tube), carries at its outer end at *s*, a small photographed scale having transparent graduation marks, and at its inner end it carries a lens, *c*, situated at a distance from the scale equal to the focal distance.

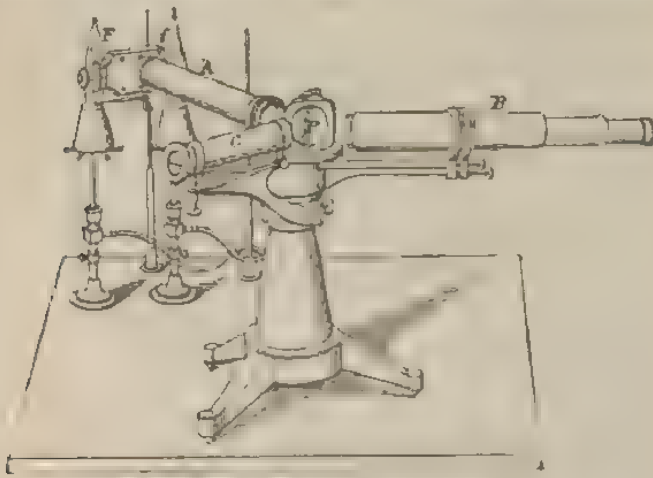


FIG. 358.—Bunsen's Spectroscope

The scale is now illuminated by the flame of a lamp. The rays emitted by one of its points, rendered parallel by the lens, *c*, are reflected from the front surface of the prism upon the lens, *b*, of the telescope. The lens then focuses them at the corresponding point of its focal plane, *rr*. Looking through the ocular, *o*, of the telescope, the eye then sees simultaneously with the spectrum, a sharp image of the scale lying along the spectrum parallel to its length, so that each point of the spectrum is designated by the corresponding graduation of the scale (*vide* spectral chart, 1, Bunsen's scale). On account of the deflection due to the prism, the collimator and telescope

of Bunsen's instrument form an angle with each other, so that the line of sight is broken. By a proper arrangement of the flint and crown-glass prisms, however, direct vision prisms (cf. 342) are constructed, which neutralize the deflection of

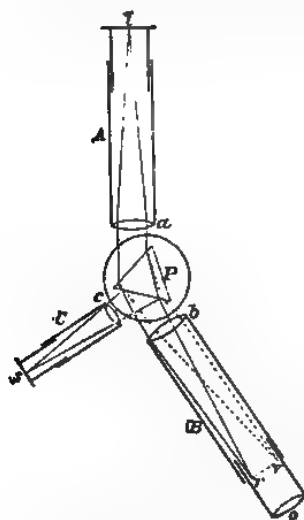


FIG. 359.—Arrangement of Bunsen's Spectroscope.

the rays, but do not entirely destroy their dispersion. By means of such prism-trains, *direct vision spectroscopes* are constructed, which permit the source of light to be viewed directly. Browning's pocket spectroscope, represented in Fig. 360, is such an apparatus. The slit is at *s*, an achromatic lens is at *C*, and *p* is a prismatic body composed of 3 flint-glass and 4 crown-glass prisms. The ocular is at *O*.

In the spectrum of sunlight, or daylight, the Fraunhofer lines peculiar to this light are seen by the aid of a spectroscope. Incandescent solids and the bright flames of candles, lamps, and of illuminating gas, in which solid particles of

carbon float in an incandescent condition, give spectra with no dark lines, and in which all colours from red to violet are represented in unbroken succession. These are called *continuous spectra*. The spectra of luminous gases and vapours, on the



FIG. 360.—Browning's Pocket Spectroscope.

contrary, consist of separate bright lines (slit images) seen upon a dark, or a feebly illuminated, background. The position and arrangement of these lines is due to the chemical constitution of the gas. If, for example, a small quantity of salt (sodium chloride) melted into the eye of a bent platinum wire is brought

into the feeble flame, F (Fig. 358), of a Bunsen burner, the flame assumes a beautiful golden yellow colour, producing in the spectroscopie a slender yellow line (spectral chart, 3) at the place where the dark line, D, appears in the solar spectrum. The sodium flame (343) emits therefore *monochromatic* (yellow) light of definite refrangibility, which is peculiar to the vapour of sodium. The existence of this yellow line in the spectrum, therefore, reveals the existence of sodium even though only the slightest traces of it are present. If a small quantity of dust is thrown into an alcohol flame toward which the spectroscopie is directed, the bright line of sodium will flash up instantly, because the dust particles, floating in the air and settling everywhere, contain traces of salt. Lithium salt brought into the flame of a Bunsen burner, gives a faint orange yellow and a beautiful red line (spectral chart, 4). Potassium salts show a faint continuous spectrum with a bright line in the extreme red, and another in the violet (2). The metals of the alkaline earths, calcium (7), strontium (8), barium (9), show more numerous lines than do these same alkaline metals. Every substance is thus characterized by definite bright lines, when the substance is in the condition of an incandescent vapour. When several substances contained in a mixture are vaporized in the Bunsen flame, the characteristic lines of each individual constituent are seen side by side in the spectrum, each in its own place, and thus at a glance the presence of those constituents is recognized in the mixture. This process of determining the elementary constituents of a body by observing its spectrum, which was developed by Bunsen and Kirchhoff, is called "*spectrum analysis*." Bunsen discovered by this process the alkaline metals, cesium (5), and rubidium (6), and other investigators discovered in a similar way the substances thallium (10), indium, and gallium. Substances which cannot be vaporized by the heat of a Bunsen burner are volatilized in the electric spark by discharging a sparking inductor between poles made of the metal to be investigated, or coated with the compound to be tested. The spectra of the heavy metals are thus rendered visible. They are characterized by numerous bright lines peculiar to each metal. In the spectrum of iron.

for example, there are about 460 of these lines. To render a gas luminous, the spark of an inductor is discharged through a Geissler's *spectral tube* which contains platinum wires soldered into its ends, *a* and *b* (Fig. 361), and which contains the gas to be investigated in a rarefied condition. When, for example, hydrogen is in the tube, its central narrow portion shines with a beautiful reddish purple colour, and gives a spectrum consisting of three bright lines: A red one which lies in the same place as



FIG. 361.  
—Spectral  
Tube

the Fraunhofer line, C, a greenish blue one coinciding with F, and a violet one which coincides with a dark line of the solar spectrum near G. The dark absorption bands seen against a bright background produced by sunlight, or lamplight, passed through coloured bodies, are also characteristic of the chemical constitution of these bodies and furnish a method of detecting their presence spectroscopically. The spectroscope may, therefore, be used in numerous instances to test the genuineness of foods, medicines, dye stuffs, etc. Spectral analysis has also found extensive application in medical jurisprudence, since it is capable of proving the presence of the least trace of blood by means of the absorption bands (343) peculiar to it.

Fraunhofer observed that the bright yellow double line<sup>1</sup> of sodium light in the spectrum occupies the place of the dark double line, D, of sunlight. But Kirchhoff's law states that a gaseous, or vaporous, body absorbs those radiations, which, when in an illuminated condition, it emits, at the same time allowing all other radiations to pass through with undiminished intensity. If, for instance, a spirit lamp whose wick is covered with salt, is brought between the eye and a pocket spectroscope, looking through this flame toward the flame of a lamp, the reversed spectrum of sodium may be seen, i.e. the sodium line may be seen dark upon a bright ground, since the sodium flame is opaque to the rays of the refrangibility of those it emits, but is

<sup>1</sup> The sodium line, as also the D-line of the solar spectrum, consists of two very close lines, D<sub>1</sub> and D<sub>2</sub>, which can be seen separate only under a high dispersion.

transparent to all others. Supposing, now, the sun to be a glowing body, whose surface radiates white light, which would of itself give a continuous spectrum, and suppose this body to be surrounded by an envelope of cooler gases and vapours containing sodium, the latter must then produce in the spectrum of sunlight a dark line in the position of the D-line. From the presence of this D-line in the solar spectrum, the existence of sodium vapour in the solar atmosphere may be inferred. From a detailed comparison of Fraunhofer's dark lines with the bright lines of terrestrial substances, it has been found that a large number of the former exactly coincide with the latter. Each of the 460 bright lines of iron has its dark representative in the solar spectrum, whence it is concluded that iron vapour exists in the solar envelope. Most of Fraunhofer's lines are merely fine bands, produced by the absorption which the gases and vapours contained in the solar atmosphere<sup>1</sup> exercise upon the light radiated from the intensely heated body of the sun. They are the "reversed" bright lines, characteristic of these gases and vapours. If this view is correct, this atmospheric envelope should exhibit, instead of the dark Fraunhofer's lines, bright lines upon a darkened field, when the spectroscope is directed toward the edge of the sun, where the envelope rises high above the incandescent body. Even before these views were held certain red *prominences*, or *protuberances*, sometimes resembling in appearance snow-covered mountains illuminated by the setting sun and sometimes looking more like floating masses of cloud, had been observed near the edge of the sun during solar eclipses. Spectroscopic study (now possible with an uneclipsed sun) showed that the spectrum of the light radiated by these formations consists of bright lines, among which the three lines of hydrogen (C, F, and a line near G) are the most conspicuous. The protuberances consist, then, in the main, of glowing hydrogen whose presence in the solar atmosphere has

<sup>1</sup> Some of the lines of the solar spectrum are produced by the absorption of the earth's atmosphere. This is proved by the fact that when the sun approaches the horizon, and its rays have to traverse thicker layers of our atmosphere, these lines become darker. They are, therefore, called "atmospheric" lines, or bands. To these belong the lines, A and B.

already been inferred from the coincidence of its bright lines with the Fraunhofer lines, C, F, and the line a little below G. Spectral analysis, therefore, gives information not only concerning the chemical constitution of the sun, but also of the other heavenly bodies to which its methods have already been applied with great success.

**346. Fluorescence—Ultra-Violet Rays.**—When the sun shines on petroleum, the feebly yellowish oil radiates a violet-blue light in all directions and becomes *self-luminous* through being illuminated. Water into which pieces of the bark of the horse-chestnut are thrown, shimmers in the sunlight with a bright blue colour by reason of the dissolving aesculine, and the same is true of a quinine solution. Yellow uranium glass (canary-glass) exhibits a bright green light, and certain varieties of fluor spar (calcium fluoride) a blue shimmer, when illuminated by daylight. From the latter body the phenomenon has been called *fluorescence*. When powdered leaves of plants are moistened with alcohol, the chlorophyll is dissolved, and the green solution resulting, when illuminated by sunlight, shines with a blood-red hue. A blue solution of litmus fluoresces orange, and the same is true of the purple red solution of naphthaline red. After sunlight has been passed through a flask of petroleum it cannot produce the same blue shimmer in a second flask of petroleum, even when the light is much brighter than ordinary daylight. Those particular rays which possess this peculiar property must, therefore, be absorbed in the petroleum of the first flask, or they are exhausted in exciting the blue light. Only those rays are capable of producing fluorescence in a substance which are absorbed by it, and their action is stronger the more powerfully they are absorbed. To determine more accurately what radiations produce this self-luminosity of the petroleum, let a solar spectrum, formed by means of a slit, prism and lens, be thrown upon the liquid contained in a glass trough, and let it be further noted at what parts of the spectrum the blue shimmer appears. The red and all succeeding colours to the violet are seen to pass through without effect. The bluish shimmer begins high up in the violet, covers the entire violet portion

of the spectrum, and extends far beyond the violet end to a distance equal approximately to the length of the ordinary visible spectrum. There are, therefore, rays which are more refrangible than the violet, but which, under ordinary circumstances, are not seen. They are called *ultra-violet rays*. They become visible in petroleum because they have sufficient energy to excite its blue fluorescent shimmer. Upon the bright blue ground of the fluorescing spectrum the Fraunhofer lines from G to H are visible, but the ultra-violet region also appears filled with numerous similar lines, the most conspicuous of which have been designated with the letters I. to S (Fig. 362). Rock crystal, or quartz, possesses the property of transmitting the violet rays much more perfectly than does glass. When a spectrum is produced by a prism of rock crystal, the ultra-violet portion of the spectrum appears upon the petroleum



FIG. 362. Solar Spectrum with the Ultra-violet Region.

considerably brighter and more extensive. The ultra-violet rays may, moreover, be seen directly by means of a prism of glass, or of quartz, without the use of a fluorescing substance. By screening off the ordinary visible spectrum these rays are seen in a bluish grey (lavender-grey) colour. These rays of high refrangibility are also perceptible to the eye, although under ordinary circumstances they are not seen, since they are very feeble in comparison with the bright rays.

A fluorescent body is excited to most vigorous self-luminosity by those radiations which it most powerfully absorbs. Colourless, or feebly yellowish, substances, such as sulphate of quinine, extract of the bark of the horse-chestnut (aesculine), petroleum, etc., which absorb only the feeble violet and ultra-violet rays, can, of course, fluoresce only under the influence of these highly refrangible rays. A coral red solution of eosine, which fluoresces pea-green, is most powerfully excited by the green rays; naphthaline red, by the yellow-green, and the green of chlorophyll by the high red rays. In

every case fluorescence is excited by those radiations to whose absorption the colour of the body is due. This fact is recognizable in the spectrum of transmitted light (absorption spectrum) by a black absorption band in the position which corresponds to the missing radiations.

When the light of a fluorescent body is studied through a prism (by the spectroscope), it is found to be compound, even when the exciting light is simple. The fluorescent light of petroleum, for example, produced by the simple violet light from the end of the spectrum, is spread out by the prism into a spectrum containing red, orange, yellow, green, blue, and violet, but in such relative proportions that the colour compounded from them all appears blue. With colourless, or inappreciably coloured, fluorescent bodies, which, like petroleum, sulphate of quinine, etc., absorb only the more refrangible rays of ordinary daylight, the radiated fluorescent light contains only rays which are less refrangible than the exciting simple light (Stokes' rule). But with fluorescent substances having strong absorption bands in the region of the less refrangible rays and appearing, therefore, vividly coloured, rays may also be contained in the fluorescent light which are more refrangible than those of the exciting light. When naphthaline red is excited by light that has passed through red glass and contains only red and orange rays, it is found that the excited fluorescent light is composed of red, orange, yellow, and yellow-green, and that, therefore, the more refrangible yellow-green rays have been produced by the orange light (Lommel, 1871).

**346. Phosphorescence—Ultra-Red Rays.**—The feeble glowing of a body produced by any cause whatsoever is called phosphorescence. The glowing of phosphorus is the result of its slow combustion; that of rotten wood, decaying fish, and of the glow-worm are due also to a *chemical* process. The development of light often results from *mechanical* effects, e.g. from striking together two pieces of flint, from the crushing of chalk, pulverizing of sugar, and from cleaving mica. Chlorophane, a species of fluor spar, glows on *warming*, and the same is true of many diamonds.

The glowing of bodies after they have been previously

illuminated is especially noteworthy. The so-called Bologna stones, sulphur compounds of the alkaline metals (calcium, strontium, barium), prepared by heating the corresponding earths (calc, strontia, or baryta) with sulphur, phosphoresce very beautifully in this way. By allowing the whitish powders thus obtained to stand for a few moments in sunlight, or daylight, they will emit a feeble glow of mellow light for hours or even for days. The colour of the light depends upon the process of preparation and upon the freedom from contamination with foreign substances.

To study the effect of the various radiations upon these substances, a solar spectrum is cast upon a screen, whose surface is coated with one of these phosphorescent powders (*e.g.* with the commercial *Balmain's colours*), and which has been made to glow by previous exposure to daylight. After the rays of the spectrum have acted a few moments, if they are screened off, a peculiar "phosphorographic" image of the spectrum is seen in the dark upon the screen. Where the blue and violet rays fell, the screen shines with a brilliant phosphorescent light against the feebly luminous background of the screen. But where the green, yellow, and red acted, the image appears dark upon a bright ground. It seems then that only the more refrangible rays (from Fraunhofer's F-line on) have the power of exciting phosphorescence, while on the other hand the less refrangible rays extinguish the phosphorescent light which had been previously excited. This quenching action extends quite beyond the red end of the spectrum (line A), whence it follows that invisible rays of *less* refrangibility than the red exist. These are called *ultra-red*, or *intra-red* rays. The complete solar spectrum consists, therefore, of the following three parts: of the *invisible ultra-red* portion, of the *visible* portion, between the Fraunhofer lines, A and H, and of the *invisible ultra-violet* portion.

The extinction of the phosphorescent light by the rays of the lower portion of the spectrum is always preceded by an increased radiation of light from the substance. These rays compel the phosphorescent body to surrender somewhat suddenly and, therefore, in a condensed form, the supply of energy it had previously taken up by the absorption of the more refrangible

rays exciting it. If, then, a spectrum be thrown upon a phosphorescent screen, the region of the ultra-red rays shines with a bright greenish blue light, because here the phosphorescent light is fanned to higher luminosity (Lommel, 1883).

E. Wiedemann designates all luminous processes not accompanied by heat *luminescence*, and the fluorescence and phosphorescence produced by absorbed light, he specifies as *photoluminescence*. Luminous radiation produced by the action of the cathode rays (342) is designated by him *cathodoluminescence*, and that due to chemical action, *chemiluminescence*.

**347. Thermal Effect of the Rays.**—The ultra-red rays were discovered by Herschel as early as 1800, by moving the black-

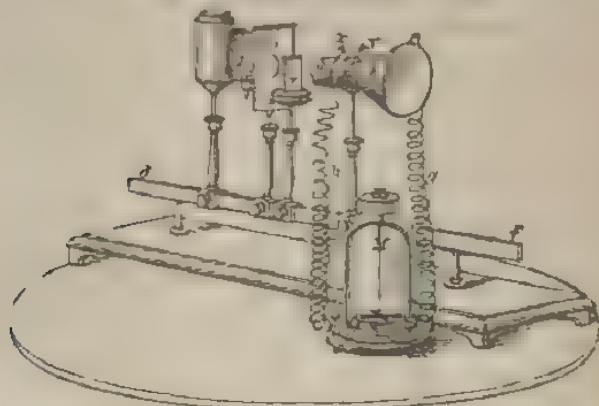


FIG. 363. — Thermo-multiplier.

ened bulb of a thermometer along the spectrum and observing that the thermal action increases from the violet toward the red end and reached its greatest value in a dark region beyond the red end. As a more accurate means of proving the existence and studying the nature of these *dark heat radiations*, Melloni (1834) made use of a *thermopile* in connection with a galvanometer, called the *thermo-multiplier*, Fig. 363 (cf. 237). This consists of a thermoelectric pile, *p*, whose blackened terminal surfaces are provided for the reception of the rays, at one end with a cylindrical tube (*a*), and at the other with the funnel-shaped mouth (*b*). A very sensitive galvanometer (multiplier),

M, is connected with the thermopile by means of the binding screws, *x* and *y*, and the connecting wires, *y* and *h*. The heat radiated from the lamp, L, passes through the hole, *s*, of the metal screen to a terminal surface of the thermopile. Here it excites a thermo-electric current, which produces a deflection of the needle of the galvanometer by an amount which is proportional to the rate of radiation. Thermopile, lamp, screen, and a small stand to support the object (*r*) to be studied, are movable along a brass bar. By means of the thermo-multiplier the thermal effects of the different regions of the spectrum may be accurately measured. More recently the *bolometer* (230) has also been used for this purpose.

The dark heat rays follow the same laws of reflection and refraction as the visible light rays. If two large concave mirrors are placed in the positions indicated in Fig. 364, and a heated iron ball is put at the focus of one of the mirrors, the rays emitted by the ball are reflected parallel to each other against the other mirror, whence they are again reflected and collected into its focus. A thermometer, with its bulb coated with lampblack, when placed at this point, rises, and a feebly phosphorescent plate set up here shines brilliantly. A bi-convex lens forms an invisible thermal image of the heated ball, which likewise renders a phosphorescent plate visible.

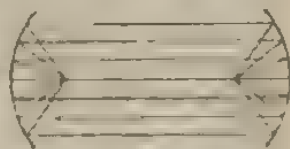


FIG. 364.—Reflection of Heat Rays.

The invisible radiations emitted by a warm body, e.g. by an iron ball, are refracted by a prism less strongly than the red rays, and are, therefore, of the same nature as the ultra-violet rays of the sun. With rising temperature, the rapidity of radiation increases, and a point is soon reached, where more highly refrangible, luminous rays are added to the dark ones, whereupon the heated body becomes visible—it *glows*. At 540° the red extends to B (dark red heat), at 700° (bright red heat) the spectrum of the emitted rays extends beyond F, and finally, at white heat (1200°), they extend even beyond H.

As with the bright rays, the transparency of different bodies is very different for the dark heat rays. Pure air transmits

solar rays, dark as well as light, almost perfectly. It is therefore heated but little by them. The higher atmospheric layers, though receiving the solar layers directly, remain nevertheless so cold that even in the torrid zone the summits of high mountains are covered with perpetual snow. The atmosphere is heated in much the greater part, not by the solar rays directly, but indirectly by the heated surface of the earth, which in turn obtains its heat by absorbing heat from the lower layers of the atmosphere in contact with it. These layers, becoming lighter, rise and carry the heat to higher altitudes. Neither water, nor clouds, nor any of the constituents of the earth's solid crust are as transparent to the rays as is the atmosphere. All absorb a greater or less proportion of the solar rays incident upon them, and become heated thereby. Melloni called those bodies which transmit the dark (ultra-red) rays of heat as transparent bodies transmit luminous rays *diathermanous*, or *diathermal* bodies. The term *athermanous*, or *athermal*, he applied to those bodies which absorb the dark heat rays. Rock salt transmits all dark heat radiations (as well as the bright ones), and it accordingly comports itself with respect to them as a colourless transparent body does to luminous radiations. Alum, on the contrary, which is just as transparent to light as rock salt, is well-nigh opaque to the ultra-red rays. Other bodies absorb definite portions of the ultra-red region of the spectrum, and comport themselves with respect to the dark heat rays after the analogy of coloured transparent bodies, which transmit only light rays of a certain colour, and absorb all rays of a different colour. Melloni designated this behaviour of substances as *heat colouration*, or *thermo-chrose* action.

It follows from these facts that the only difference between the dark heat rays and the light rays is the difference in their refrangibilities. The dark rays differ no more from the red rays than the latter differ from the yellow, or the yellow from the green. The invisibility of the one set, and the visibility of the other, consists not in the nature of the rays, but in the constitution of the eye, which is incapable of perceiving the ultra-red rays. These are immediately perceptible to us only through the sense of feeling, as heat, while the light rays act simultaneously

upon two senses, affecting the nerves of feeling, as heat and the eye, as light. A light ray is also at the same time a heat ray. It is impossible to separate the effect of heat, which resides in the simple yellow light of the sodium flame, from its luminous effect. There are no rays of this refrangibility which produce thermal effects without the simultaneous production of luminous effects. Light and radiant heat, therefore, differ from one another not in themselves, but only for us, as forms of sensation. They are merely different effects upon us of one and the same cause. The same ray evokes in us sometimes the sensation of light, and sometimes that of heat, according to the nerve through which the impression produced by it is transmitted to the seat of consciousness, just as a vibrating tuning-fork awakens in the ear the sensation of sound, and in the hand a buzzing sensation.

**348. Radiometer.**—To prove the thermal effect of the rays, the radiometer may be used. This apparatus was devised by Crooke, and is set in motion by light and heat radiations. It consists of a small wheel (Fig. 365) of aluminium wire, with four arms, and free to rotate about the point of a needle. Each arm carries a vertical fan of annealed mica, one surface of which is black, the fans being so placed that all the blackened surfaces face in the same direction. The entire apparatus is enclosed in a glass globe of from 5 to 6 cm. diameter, within which the air has been highly rarefied. Rays of light, or heat, falling against the fans, set the wheel in rotation in a direction opposite to that in which the blackened surfaces face, precisely as though the blackened surfaces were repelled by the rays. The rotation increases the more powerful the thermal effect of the radiation becomes. This phenomenon may perhaps be best explained on the kinetic theory of gases (129). We need only imagine that the rapidly moving molecules of the air strike more quickly and frequently against the blackened surfaces which are the more highly heated by absorption than they do on the white surfaces, and that these molecules impart to



FIG. 365 —  
Radiometer.

the blackened surfaces more powerful impulses than to the opposite faces.

When a radiometer is moved gradually from the violet toward the red end of the spectrum, it is observed to rotate with increasing velocity, and continues to rotate even when carried beyond the red end into the ultra-red region.

**349. Chemical Effect of Light -Photography.**—It has been long known that there are bodies which suffer a permanent alteration of their properties under the action of light, i.e. a change in their chemical composition takes place. The bleaching of linen cloth and of wax, and the so-called fading of coloured cloth and of water-colour paintings, the turning brown of fir wood, etc., are well-known examples of the chemical action of light. If a flat object, *eg* the leaf of a plant, be laid upon a sheet of paper covered with silver chloride and then exposed to daylight, the silver chloride blackens over the uncovered portions of the paper by the light, and a bright image of the leaf is obtained upon a dark background. Silver bromide and iodide are even more sensitive than silver chloride to the action of light. The art of *photography* is based upon the chemical action of light upon these silver salts. The photographer catches upon a glass plate coated with a film of gelatine or collodion, containing silver bromide, the image of a person, or of an object formed by a *camera obscura*. When the plate is first taken out no image is apparent, for the light during the short time of its action has only started the decomposition of the silver salt. To call out, or to *develop* the image, the plate is washed with a solution of some reducing substance (iron-vitriol, hydrochinon), which completes the decomposition. On the brightest portions of the image the silver bromide is hereby rendered perfectly black, on the partially darkened places a partial blackening occurs, the degree of which depends upon the intensity of the shadows, while at the dark places of the image the silver bromide remains unaltered. There is now upon the plate of glass a so-called *negative* image, which shows the bright places of the object dark and the dark portions bright. This image would remain for but a short time, because the silver salt, not altered by the daylight, would soon be likewise

decomposed, and the entire plate would become uniformly black. For this reason, the image is fixed by washing it with a solution of sodium sulphite, or of potassium cyanate, which dissolves out the undecomposed silver salt. A positive image, with its lights and shades at the proper places, is obtained by placing the negative upon silver chloride paper and exposing to sunlight, which produces behind the bright portions of the plate a dark area upon the paper. The positive image is then fixed by washing the paper with a solution of sodium sulphite, which dissolves the undecomposed silver chloride still remaining. The processes of developing and fixing the image must be carried on under yellow, or red, illumination, since silver salts are almost unaffected by yellow and red light.

It is known even from everyday experience that the blue rays are photographically more effective than the yellow and the red: for a blue article of clothing looks bright in a photograph, while a red one appears dark, although when observed directly, the former appears much the darker. Silver chloride paper covered partially with red and partially with blue glass blackens in light only beneath the latter. But the most direct information possible concerning the action of the rays of different colours is obtained in photographing the solar spectrum itself. The red, yellow, and a portion of the green rays have no effect at all upon an ordinary photographic plate, while the blue and violet regions are beautifully reproduced with all the Fraunhofer lines. The photographed spectrum does not terminate with the H-line lying at the end of the violet portion of the visible spectrum. The ultra-violet rays acting photographically also extend the spectrum far beyond the violet. When colouring matter (azaline, eosine, etc.) is added to the sensitized film, these materials absorb the less refrangible rays, and a so-called *orthochromatic* plate is produced. Upon such a plate, the green, yellow, and red rays are also chemically effective. By means of a specially prepared emulsion of silver bromide, Abney (1880) succeeded in photographing even the ultra-red portion of the spectrum. The less refrangible portion of the spectrum may be photographed far into the ultra-red with ordinary photographic plates as follows: Upon a

phosphorescent surface (Balmann's colours) the phosphographic image of the spectrum (346) is produced upon the dark ground of which the Fraunhofer lines shine with a distinct blue colour. Placing upon this image an ordinary gelatine dry plate, the spectrum with all its details will be distinctly copied (Lommel, 1890).

The more refrangible rays which act upon silver chloride, bromide, and iodide, *i.e.* the blue, violet, and ultra-violet rays, may with propriety be designated *photographic rays*. By calling them "chemical rays," as is frequently done, they are incorrectly ascribed the exclusive property of chemical action. The chemical effect of these rays, as the latter designation might lead one to think, depends not upon any special chemical, or, as we sometimes say, "actinic" power, inhering in them alone, in contradistinction to all other rays, but simply upon the circumstance that the easily decomposable silver salts absorb the more refrangible rays, and allow the less refrangible to pass through unaffected. *Only those rays can affect a body either chemically, or otherwise, which are absorbed by the body.* Upon a decomposable body, which absorbs the less refrangible rays in comparatively large quantities, these would therefore exert the strongest chemical action. Nature herself furnishes, on a large scale, an illustration of the chemical action of the less refrangible rays. Plants procure the entire supply of carbon needed to their growth from the air, by decomposing the gaseous carbonic acid mixed with the air into carbon, which remains in the plant, and oxygen, which returns in a gaseous condition into the atmosphere. This decomposition of the carbonic acid, with the accompanying *assimilation* of the carbon, takes place in the green portions of the plant as a result of the action of sunlight upon the chlorophyl. This result is due chiefly to the middle red rays which are absorbed by the chlorophyl (Lommel, 1871; Engelmann, 1881).

**350. Energy of Solar Radiation.**—If the solar rays were completely reflected at the surface of the earth, they could neither warm, nor otherwise affect it. The action of these rays is made possible by the absorptive power of terrestrial objects. Pure air transmits solar rays almost undiminished in intensity,

and is, therefore, but very slightly heated by them directly. On the other hand, the solid portions of the earth's surface, which possess considerable absorptive power, are very considerably heated. Furthermore, radiation from the soil gradually heats the air. This heating effect being different at different places on the earth's surface (e.g. at the equator, where the incident rays are more nearly perpendicular to the surface, a higher temperature is reached than in the polar regions), the equilibrium of the atmosphere is disturbed, and in the attempt to restore itself, air-currents, called *winds*, are produced. The movements of atmospheric currents are, therefore, due to the energy of the solar rays. In the breeze which swells the sail, as well as in the hurricane which uproots the forest, a small portion of the energy which the sun sends to the earth in its thermal radiations is revealed to us. In the process of evaporation, carried on under the influence of the solar rays at the surface of the sea, immense quantities of water vapour are raised to the higher atmospheric layers, whence after being condensed to water, they fall as rain, or snow, and, collecting into brooks and rivers, flow again to the sea. During this circle the water gives up the entire quantity of energy which it received from the sun at the outset. The falling raindrop, the ship-burdened stream, the waterfall which turns the millwheel, or drives the tunnel through alpine granite, all derive their energy from the sun. In the green foliage of plants, the carbonic acid taken from the air is decomposed by the solar rays absorbed by the plant, the oxygen being returned to the air while the carbon is assimilated by the plant. In the woody fibre of trees, the energy of the sun's rays, consumed in the formation of the fibre during the course of years—is stored up, as it were, in a passive or latent condition to reappear, however, without loss or gain, as kinetic energy in the form of light and heat, when by burning the wood, or rather the carbon it contains, it is again reduced to the condition of carbonic acid. In the seams of coal, which are merely the metamorphosed remains of primeval plants, we possess a well-filled storehouse of latent solar energy, which, liberated by the process of combustion, heats and illuminates our dwellings, swings the hammer in the workshop, turns the

spindle in the factory, and drives the locomotive upon its iron way. Among animals, some feed directly upon plants, others devour their herbivorous fellow-creatures, so that in either case the life of the animal is nourished directly or indirectly by the vegetable kingdom. Within the body of an animal, the carbon taken in with the food combines with the inhaled oxygen and is exhaled in the form of carbonic acid. The clock wound up in the vegetable kingdom runs down again in the body of the animal, i.e. the energy of the solar rays consumed by the plant in the separation of the carbon from the oxygen, is again liberated in the body of the animal in the form of heat and motion on the recombination of these constituents. The heat of the blood, the pulse-beat of the heart, and the capability of performance of the arm, are merely energy, which was originally given out by the sun. Through the mediation of its



FIG. 366.—  
Pyrheliometer.

rays the sun thus becomes the source of all heat, of all light, and of all motion upon the surface of the earth. To ascertain the amount of solar energy incident upon the surface of the earth, it is necessary to determine the amount of heat produced by the rays of the sun, when they are completely absorbed by a surface of definite size. To execute such measurements, Pouillet used an apparatus (Fig. 366), called by him the *pyrheliometer*. It comprises a thermometer whose bulb is enclosed within a cylindrical vessel of thin sheet silver, which is filled with water. The portion of the thermometer tube outside of the vessel is enclosed by a brass tube, provided with a slit

through which the thermometer is read. The brass tube carries also a metallic disk whose diameter equals that of the silver vessel. To cause the rays of the sun to impinge perpendicularly upon the bottom of the vessel, which is coated with lampblack, and consequently in the most favourable direction, the instrument is so directed that the shadow of the silver vessel falls exactly upon the metallic disk. The rise of the thermometer is now observed for five minutes, and since the weight of the water

contained in the vessel is known, the number of heat units may be obtained, which the blackened surface absorbs from the sun during this time. But during the experiment heat has also been lost by radiation from the coated surface. This loss is afterward obtained by observing in the shade the fall of the thermometer for five minutes. The amount of heat lost by this fall is then added to the quantity of heat found above. Adding, then, to this sum the loss of heat due to absorption during the passage of the rays through the atmosphere, it is found that the amount of heat incident upon the earth in the course of a year, and due to the solar radiation alone, would melt a shell of ice, enveloping the entire terrestrial globe, to a thickness of thirty meters.

**351. Fresnel's Experiment.**—From a slit, *P*, illuminated by the lens, *L* (Fig. 367), and standing perpendicularly to the

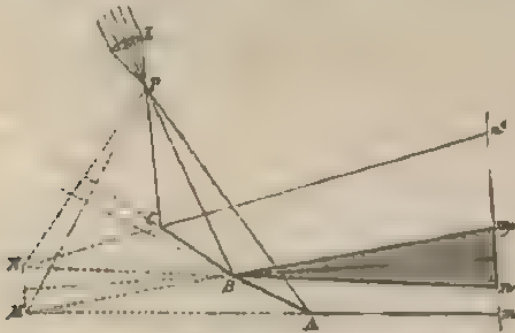


FIG 367 — Fresnel's Experiment.

plane of the drawing, a beam of light is made to fall upon two mirrors, *AB* and *BC*, which, to the end that each may have but a single reflecting surface and form but a single image, are made of black glass. They are placed with their edges together at *B*, and with their faces inclined at a very obtuse angle. The rays from the mirror, *AB*, are reflected as though they came from the point, *M*, and from the mirror, *BC*, as though coming from *N*. Consequently two luminous cones, *Mmm'* and *Nnn'*, proceed from the mirrors, and appear to

emanate from the points,  $M$  and  $N$ . They have the space,  $Bmn$ , in common, so that the portion,  $mn$ , of the screen,  $m'n$ , receives light simultaneously from both cones. In the centre of the field now is seen a series of alternately bright and dark streaks lying parallel with the edge,  $B$ . When one of the mirrors is covered, these streaks disappear, and the field,  $mn$ , now receiving light from the other mirror alone, appears uniformly bright. The streaks appear again as soon as the cover is removed, and the light from the point,  $N$ , is added to that from the point,  $M$ , upon the screen. Fresnel's experiment therefore proves that *when light is added to light*, the result may be either *intensified brightness* or *absolute darkness*, dependent upon the manner of the addition.

The assumption that light is matter (Newton's emanation, or emission theory, or corpuscular theory, 1669), whose particles are hurled from luminous bodies with a velocity of 300,000 km., is wholly incapable of furnishing a satisfactory explanation of the last mentioned fact. On the contrary, from the other



FIG. 368.—For Fresnel's Experiment.

possible assumption that light is a mode of vibration, propagated from molecule to molecule in a medium called *ether*, supposed to fill all space, including the interstices between the molecules of bodies and with the above mentioned velocity (undulatory theory, Huyghens, 1678), the phenomenon follows as a necessary consequence, viz. as an *interference* phenomenon (Young, 1801).

Let the two points,  $M$  and  $N$ , be regarded as the sources of two intersecting series of waves (waves of water), whose *crests* in Fig. 368 are indicated by full lines, and whose *troughs* are shown by dotted lines. At the points,  $0, 2$ , and  $2'$ , then, where two troughs, or two crests, meet, the motion will be intensified, while at the points,  $1, 1', 3, 3'$ , where a crest and a trough

intersect, no motion at all will exist. But what is called rest in case of the waves of water, is darkness with the luminous vibration of the ether. Hence, if we conceive light to be a mode of wave motion, we readily understand why bright and dark places alternate upon a screen placed at  $3$  and  $3'$ , or rather, inasmuch as light is propagated in the condition of spherical waves in the ether, why it is that bright and dark streaks appear in positions parallel to the common edge,  $B$ , of the mirrors.

Considering now, instead of the waves themselves, the rays proceeding from the two centres of excitation,  $M$  and  $N$ , toward each point of the screen, it is clear that the two waves, from  $M$  and  $N$  to the middle point,  $O$ , are of equal lengths. The vibrations proceeding simultaneously from the centres meet, therefore, at the point,  $O$ , in the same phase, and mutually intensify each other's effect. But at the point,  $1$  (and likewise at  $1'$ ), two rays meet whose paths,  $M1$  and  $N1$ , differ by half a wave-length. The impulses imparted by them to the point,  $1$ , are, therefore, equal and opposite, and the point remains at rest. The same thing happens at the points,  $3, 5$ , and so on, where the differences of the distances traversed by the rays amount to  $3, 5$ , and so on, in general to an odd number of half wave-lengths. At the places,  $2, 4$ , etc., where the rays meet after having traversed distances differing by  $1, 2$ , etc., or in general, by any number of entire wave-lengths, and are consequently in the same phase, the most vigorous vibration takes place. The intervening points, where corresponding rays meet in all possible stages of accordance and discordance, are in states of less vigorous motion.

It may perhaps be asked why the luminous centres,  $M$  and  $N$ , were formed in this circuitous way, by the use of mirrors, rather than directly by two luminous lines, for example, by two glowing platinum wires. By using sources of light of the latter sort it would be found that no interference bands would appear upon the screen. For, from the above discussion, it is evident that the two wave systems must proceed simultaneously and similarly from the two centres of excitation in order that the phenomena of interference may arise. But it is impossible for us so to control the luminous process in two glowing bodies

that the vibrations sent off by the one may be in exact accord with those of the other. After a short time there will occur in each of them interruptions of the motion, changes of phase and other disturbances, which will not occur simultaneously in the other. The processes of developing light in different sources are independent of one another (incoherent), and, therefore, dissimilar. The similarity, or coherency, demanded for interference is most certainly secured by producing the two wave systems in some way from the same source. For the irregularities of light development in the single source then occur simultaneously and accordantly in both wave systems, and will, therefore, have no influence upon the accordance, or discordance, of the rays. The latter will now be due wholly to the differences of the distances traversed by the vibrations.

In place of the mirrors, the obtuse-angled double prism of glass, shown in cross section in Fig. 369, may be used. From



Fig. 369 — Interference Prism.

the front surfaces of this prism, the rays coming from the source, T, will pass precisely as though they came from M and N.

If in Fresnel's experiment the aperture of the heliostat is covered in turn with red, green, or blue glass, it is noticed that with blue light the dark streaks stand closer together than with green light, and with green light, closer than with red. Consequently, blue rays require a smaller difference of distance from the respective sources to neutralize each other than

green and still less than the red, or the wave-length of blue light is less than that of green, and this in turn is less than that of red light. Generally, to each simple colour corresponds a definite wave-length which grows smaller in the succession of the spectral colours from red to violet. When, therefore, the interference experiment is performed with white light containing all the primary colours, in place of the alternating stripes of white and black, coloured stripes appear

upon the screen, because from the different wave-lengths the stripes of different colours do not coincide. Moreover, since with greater difference of distance from the sources to any given place of the screen, more numerous intensifications and enfeeblements of the various colours intermix, there are seen with white light, at both sides of the central bright streak, but few coloured stripes which fade outward and finally become lost in a uniform white. But when homogeneous light is used, the dark stripes are perfectly black and are present in great quantities.

**352. Wave-lengths—Vibration Frequencies.**—In addition to this general information concerning the ratios of the wave-lengths, Fresnel's experiment will also furnish measures of these ratios. The difference of the distances from the points, M and N (Fig. 368), to the first (or second, third) black stripe must equal one (or three, five, etc.) half wave-lengths of the homogeneous light used. Fresnel executed these measurements for light, which had been passed through red glass and found its wave-length equal to 638 millionths of a millimeter.

According to a method to be discussed later, the wave-lengths for definite rays, viz. for the Fraunhofer lines, have been measured with extreme accuracy. Light waves are extraordinarily small. Upon the space of one millimeter, 1315 waves of the extreme red (line A) would lie, 1698 waves of the yellow sodium light (B), and 2542 waves of the extreme violet (H).

The velocity of light is now known to be 300,000 km. per sec., and that in the ether of cosmic space (*in vacuo* and approximately also in air) it is the same for all kinds of light. After the wave-lengths of the different primary colours are known, their vibration frequencies are readily obtainable. These frequencies are expressed by the number of wave-lengths of each kind of light contained in the distance of 300,000 km. For the extreme red, 1315 of whose waves equal the length of one millimeter, it is found that the enormous number of 394,500,000 vibrations occur in a second. The smaller the wave-length the larger the frequency must be. In a ray of yellow sodium light each particle of ether makes 509 billion

(French) vibrations per second, and to the extreme violet, corresponds a vibration frequency of 763 billions.

The pitch of a tone is higher the greater its vibration frequency. Just as the ear perceives the frequency of sound vibrations as *pitch*, the eye takes cognizance of luminous vibrations as *colour*. To produce the sensation of the yellow of the sodium flame, exactly 509 billion ether waves must enter the eye every second and strike the retina. Similarly, the colour of each simple ray is determined by the number of its vibrations. The frequency is the invariable characteristic of what with the perception of light is called colour, and with the perception of sound is called pitch. The succession of colours of the spectrum may be regarded as a sort of *light scale*, or *gamut*, rising gradually from the lowest tone of colour perceptible to the eye, the extreme red, to the highest, the extreme violet. The low ultra-red shades, or *tones*, whose vibrations are too slow to awaken the sensation of light in the optic nerve, precede the visible colour scale, and the highest *tones*, the ultra-violet, producing in the eye only a very feeble luminous impression, follow it. In music at one is called the octave of another when its frequency is twice, or its wave length one-half as great as that of the latter. If we apply this nomenclature to the analogous realm of colour tones, we may say that the visible spectrum (from A to H) is not quite an octave. Considering the solar spectrum, however, in its entire compass, it may be said that the ultra-red comprises about four octaves, the visible portion not quite one, and the ultra-violet a little more than one; so that the entire scope of the solar radiations known to us embraces about six octaves.

**353. Huyghens' Principle.**—While a light wave is being propagated through a medium, each molecule, or particle of the medium, initiates the vibratory motion of the one exciting it. Since now each particle stands in the same relation to its neighbours as does the first with respect to those about it, the former must produce exactly the same effect upon its environment as the one first excited, and may just as well as any other particle be regarded as the source of the system of waves. The numberless simultaneous partial wave systems radiating from all

the particles in motion, produce by their combined action (their superposition) the principal wave system, which is the system actually observed to emanate at any instant from the centre of disturbance. This important proposition, called *Huyghens' principle*, reveals the true process of wave propagation in a medium having extent in all directions, since it takes due account of the mutual actions of all the particles surrounding each particular particle concerned in the vibration. In such a medium, the propagation of the vibratory motion along a single straight line obviously cannot occur. The subject of consideration must always be the propagation of a portion of a wave. But to every portion of a wave, howsoever small it may be, belongs a countless number of rays, which, in their totality, compose a beam, or *pencil*. Individual rays do not exist in nature. Beams are the simplest elements of light capable of physical existence. But the ideas heretofore advanced on the assumption of the possible existence of individual rays will be in complete accord with the phenomena, if we regard each light ray as the representative of the very slender beam to which it belongs.

Since in a homogeneous medium, the waves, *e.g.* the sound waves in the air, the light waves in the ether, etc., radiate from the centre of excitation in the form of spherical envelopes, each wave ray constitutes a radius of a sphere and is perpendicular to the corresponding portion of the wave. If this portion be regarded as small and remote from the centre of excitation, the rays may be regarded as *parallel* to each other, and the element of wave surface may be considered *plane*. In general, to every beam of parallel rays belongs always a plane wave lying perpendicularly to the direction of the rays.

#### 354. Explanation of Reflection and Refraction.—

Let us now inquire what happens when a beam of parallel rays, *amua''k*, falls upon the plane surface, *nk*, separating two different media (Fig. 370). The plane wave, *mn*, of the beam impinging against



FIG. 370.—Explanation of Reflection

the surface sets the superficial particles,  $mm'k$ , successively in vibration, and each of the particles (according to Huyghens' principle) sends its own wave system back into the first medium. At the instant when the point,  $k$ , of the surface is reached by the incident wave, the point,  $m$ , first reached, has produced a circular, or spherical, partial wave, which has spread out around  $m$  just as far as the primary wave has advanced meanwhile, so that the radius,  $mo$ , of this partial wave is equal to the distance,  $mk$ . The points,  $m$  and  $k$ , have in the mean time produced elementary waves whose radii are smaller the nearer they lie to the point,  $k$ . The point,  $m'$ , for example, produces a wave whose radius,  $m'o'$ , equals  $km'$ . The common tangent,  $ko$ , to all the elementary waves, along which all motions are in the same phase, represents now a principal wave passing from the surface into the first medium, or, technically, reflected from this surface. Manifestly, from the construction, the reflected wave,  $ko$ , is inclined to the reflecting surface,  $mk$ , at the same angle as the incident wave. The corresponding reflected beam,  $mlkr$ , whose rays,  $ml$ ,  $m's$ ,  $kr$ , are perpendicular to the wave,  $ko$ , forms, therefore, with the surface,  $mk$ , and consequently also with the normal, an angle equal to that of the incident beam.

From the points of the surface which have been set in motion by the incident ray, waves must also be produced in the second medium, which will be propagated with a velocity differing from that in the first. The elementary waves from the point,  $a$  (Fig. 371), which is first struck by the incident wave,  $ab$ , will then, at the instant when the incident ray reaches



FIG. 371.—Explanation of Refraction.

the point,  $b'$ , have a radius,  $ac$ . This radius will be to the distance,  $bb'$ , which has been traversed simultaneously in the first medium, in the ratio of the velocity of propagation in the first medium to that in the second. Since the tangent,  $bc$ , drawn from  $b'$  to this first elementary wave, touches also all the elementary waves thus far formed, and all motions in it are in

the same phase, it represents the plane primary wave entering the second medium. It is apparent that on passing from one medium into the other the wave suffers deflection. Its front advances now in a direction different from that of the incident wave. The corresponding beam,  $aEb'F$ , forms with the normal,  $lal'$ , an angle differing from that of the incident beam. It has suffered refraction. When, as is assumed in the figure, the velocity of propagation in the second medium is less than in the first, the angle of refraction,  $r$ , is less than that of incidence,  $i$ , or the beam is refracted toward the normal. If the distance,  $ab'$ , be taken as the linear unit,  $bb'$  is the sine of the angle of incidence,  $i$ , and  $ae$  that of the angle of refraction,  $r$ . But the lengths,  $bb'$  and  $ae$ , are to each other in the invariable ratio of the velocity of propagation of light in the first and in the second media. From the theory of wave motion follows, then, not only the law of refraction, that the sine of the angle of incidence has a fixed relation to that of the angle of refraction, but also the true significance of this relationship:—*the index of refraction is the ratio of the velocity of propagation in the first medium to that in the second.* It results, then, from the undulatory theory as a necessary consequence, *that light is propagated more slowly in a more highly than in a less highly refracting medium.* The corpuscular theory, on the other hand, which explains refraction as an attraction of the refracting body for the hypothetical luminous material, leads to the conclusion that light is propagated more rapidly in the more highly refracting medium than in the less. The contradiction brought to light in these opposed conclusions afforded the means of terminating the long contest waged between the corpuscular and undulatory theories. Foucault succeeded (1850), by the following ingenious experiment, which made possible the measurement of the velocity of light within a small room (320), in summarily settling the dispute. A beam of rays admitted through the aperture,  $O$  (Fig. 372), was thrown by the glass plate,  $P$  (inclined at an angle of  $45^\circ$ ), upon a small plane mirror,  $S$ . From here the beam was reflected to a concave mirror,  $H$ , of radius,  $HS$ , whence it returned along the line,  $SP$ , to the glass plate which deflected it laterally along

(Po) toward the observer. When the mirror was rotated rapidly about an axis perpendicular to the plane of the figure, so that after the light had traversed the distance, SH, in both directions, the position of the mirror had changed somewhat, the reflected ray then passed toward SI', and showed the image, o, of the aperture displaced to o'. From the

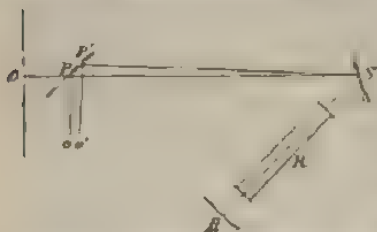


FIG. 372.—Foucault's Experiment.

displacement and the measurable velocity of rotation of the mirror, S, the time required for the light to traverse twice the distance, HS, was obtained. When a tube filled with water, and closed at its ends with glass plates, was inserted be-

tween S and H, the displacement, oo', was found to be *greater* than for air. Light is propagated, therefore, more *slowly* in water than in air, and it was found that the velocity in water is only three-fourths of that in air.

The proposition assumed above for empty space, that light of all wave-lengths is propagated with the same velocity, does not, then, hold good for the propagation of light in transparent bodies. For the fact of dispersion, interpreted on the undulatory theory, proclaims that in colourless transparent substances, rays of greater frequency are propagated with lower velocities. In air, of course, and in gases generally, the dispersion is so insignificant that we may, without appreciable error, ascribe the same velocity to all rays propagated through them. This velocity is less than that for empty space in the ratio of 1:1.000294. The latter number is the index of refraction from *vacuo* into air of 0 and 760 mm. pressure (326).

As for the pitch of a sound, so also for the colour of homogeneous light, the controlling consideration is the frequency, and this does not change on the passage of light from one medium into another. The wave-length, on the contrary, does vary during the passage, for it is always obtained by dividing the *variable* velocity of propagation (which varies with the medium and also with the colour) by the *invariable* frequency. The wave-lengths in air being directly measurable, are, however,

ordinarily given instead of the frequencies, to distinguish the several varieties of homogeneous light, and the wave-length in any given substance is then found by dividing the wave-length in air by the index of refraction of the substance.

**355. Doppler's Principle.**—A case is nevertheless conceivable in which a variation of the frequency does occur. Doppler, in 1841, called attention to the fact that the pitch of a tone, or the colour of light, must rise or fall according as the source approaches toward or recedes from the observer. In the former case the organ of sense (the ear, or the eye) receives within a second a greater number, and, in the latter case, a smaller number of waves than would impinge upon it if the source were at rest. When a train is passing a railway station, the pitch of the whistle of the locomotive rises perceptibly while the train is approaching the station, but falls while leaving it.

Suppose now that a ball of glowing sodium vapour is moving through interstellar space with high velocity toward the earth. Its light must necessarily appear more greenish than does that of a terrestrial sodium flame. When the body is receding, its yellowish light would appear more reddish than it would were the ball at rest. If now this light should be caught upon a prism, in the former case it would arrive at the prism with a greater, and in the latter with a smaller, frequency than that of a stationary sodium flame, and it would be refracted correspondingly more or less. In a spectroscope directed toward the moving source of light, the bright sodium line must then be shifted toward the more refrangible or toward the less refrangible end of the spectrum, according as the moving source approaches toward, or recedes from the observer. What is true of the bright sodium line in the example cited, is also true of the dark lines in the spectrum of a star. They will no longer coincide with the bright lines of the simple substance, through whose selective absorption they arise, if the star is moving with sufficient rapidity in the line of sight. From the direction and amount of the displacement, the direction and the magnitude of the component of the star's velocity in the line of sight are deducible. In this way Huggins found, by comparing the F-line of the spectrum of Sirius with the corresponding

bluish-green line in the spectrum of a Geissler's tube containing hydrogen, that Sirius is receding from the solar system with a velocity of 48 km. per second, and Lockyer concluded from the peculiar displacements and deformations of the F-line of the solar spectrum that the glowing masses of hydrogen in the "cyclones" of the solar atmosphere are rotating with a velocity of from 50 to 60 km. per second.

**356. Emission and Absorption.**—After having explained reflection and refraction by the undulatory theory, let us briefly indicate how the processes underlying the other luminous phenomena hitherto discussed may be conceived to take place. A body becomes a source of heat, or of light, by virtue of an extremely rapid vibratory motion of its particles. This motion is propagated in the form of waves in the surrounding ether, and is perceived by the nerves of feeling as heat, and by the optic nerve as light, in case the vibrations are sufficiently rapid. Every body is composed of molecules. It is solid if its molecules are drawn together by cohesion into certain definite positions of equilibrium, and such that, when these positions are disturbed, the molecules vibrate about them. In the liquid condition the molecules have no fixed positions. They move freely among each other from place to place, the force of cohesion preventing them always from passing beyond a certain limit. In a gaseous condition the molecules are released from their mutual bonds, and move freely and independently through space. Each molecule is built up in definite way of like or unlike atoms, held together by chemical affinity. By the mole, number, and grouping of the atoms composing a molecule, the chemical properties, both of the molecule and of the body, which is made up of an infinite number of such molecules, are determined. Precisely as a vibrating cord emits a definite fundamental tone in addition to its overtones, all of which are determined by the length, thickness, tension, and material of the cord, so also the atoms within each molecule are capable of only a definite series of vibrations, whose frequencies are prescribed once for all by the structure of the molecules, i.e. by the chemical constitution of the body. Just as we say a cord, or a tuning-fork, is pitched to a certain tone, so can we also say a sodium molecule is attuned to the yellow colour, D. It thus becomes comprehensible that the nature of a substance will reveal itself by definite bright lines in the spectrum of its light.

When a tone is sounded into the open case of a piano, the same tone is heard feebly in response. The cord which was strung to this tone begins to vibrate as soon as the tone is sounded elsewhere. But the second wave set in motion by a singer passes by all other cords without effect. This sympathetic vibration, awakened by the similarly pitched tone, is termed *resonance* (308). The wave, however, must give up a portion of the energy of its motion (kinetic energy) in the act of setting the cord into vibration. It passes, therefore, to the other side of the cord somewhat enfeebled. Conceive now of a harp, containing only cords attuned to the same pitch, and suppose a sound wave, similarly pitched, to be excited near it. This wave must be considerably weakened while passing through the harp, because its energy has been in great part absorbed by the cords. A wave producing any other tone, however, would pass the cords uninfluenced and advance beyond them without material loss of intensity. A Bunsen flame, in which glowing molecules of

sodium heat, is analogous to such a harp. It must, therefore, weaken, and may even extinguish that particular light, B, which it itself emits, while for all other rays it is transparent. The process of absorption of light is, therefore, comprehensible, and Kirchhoff's law becomes intelligible: *that every body absorbs precisely those rays which it is capable of emitting, or that the absorptive power of a body for any particular kind of rays is proportional to its emissive power for the same rays.*

In virtue of the undulatory motion, which is excited within each molecule by the absorbed light, the molecules themselves emit rays which, when visible, are perceived as *fluorescence* or *phosphorescence*, and the molecules respond with the particular colour peculiar to them by reason of their chemical constitution. But, since within the molecules of solids and liquids the vibrations are not free and independent as is the case within the free molecules of a gas, the fluorescent light emitted by them, even when excited by homogeneous rays, is not itself homogeneous, but furnishes a continuous spectrum, whose brightest part is less refrangible than the darkest portion which corresponds to the vibration of the molecule giving the absorption spectrum (Lommel, 1877).

When a body is heated, both the molecules and the atoms within them are set in vibration. Since the vibrations of the molecules do not depend upon their chemical constitution, they take place in the same way for all solids at the same temperature. At low temperatures bodies emit only invisible ultra-red rays. With increasing temperature the intensity of radiation increases, and, at the same time, other rays of higher refrangibilities are added to those already present. When the temperature has reached a point where all the visible rays appear, the body is said to glow.

**367. Diffraction (Inflection) of Light (Grimaldi, 1665).—**Looking, with the eyes partially closed, toward the flame of a remote candle, a series of images may be seen at both sides of the flame. Similar phenomena may be seen at night by looking through the meshes of the cloth of an umbrella toward street lamps, or by observing the bright image of the sun upon the face of a sundial through the vane of a sparrow's feather. In the latter case a luminous point is seen at the intersection of an oblique cross, whose arms are composed of a series of images tinted with the hues of the rainbow. To produce these phenomena at one side of the source, a portion of the light must have been deflected laterally from its rectilinear path to the eye during its passage through the narrow interstices between the eyelashes, or between the threads of the cloth, or finally between the filaments of the feather. Technically it must have been *diffracted*. The simplest, and therefore the most satisfactory, diffraction phenomena are obtained by allowing the rays of the sun, deflected by means of a mirror through a small vertical aperture into a dark room, to pass through a

narrow slit and be caught upon a screen situated at some distance behind the slit. Covering the opening with a piece of red glass, so that homogeneous red light alone may be admitted, one sees upon the screen, at both sides of the bright luminous streak, which appears in the direction of the incident ray, a series of alternately dark and bright streaks (Fig. 373),



FIG. 373.—Diffraction Image of a Slit.

the latter of which rapidly diminishes in intensity toward the sides. The appearance of perfectly dark streaks at places which are illuminated just as strongly as the bright intervening spaces, again furnishes proof that light is a mode of vibration.

For only on this assumption is it conceivable that light rays, acting in conjunction with light rays (*interfering*) can produce darkness. The undulatory theory gives a thoroughly satisfactory explanation of the phenomenon. All points of the portion,

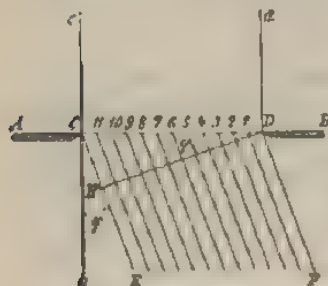


FIG. 374.—Explanation of Diffraction.

CD, of the wave (Fig. 374) which, coming from the aperture in the shutter, fills the slit, are in the same phase of vibration. Each of these points is, on Huyghens' principle, to be regarded as the source of a wave, issuing from it behind the slit. Each acts as the origin of rays, which radiate from it in all directions. The lateral spreading of the light, which is readily perceived upon the screen, is also explicable on the undulatory theory.

The rays, such as CG, which form the continuation of the incident rays,  $\infty C$ ,  $\infty D$ , are in the same phase. They will, therefore, produce upon the screen, where they strike simultaneously with their crests, or with their troughs, an intensely bright streak at the centre of the diffraction image. Consider now the diffracted beam, CEDF, which advances toward a point situated at one side of the central streak. (Inasmuch as this point is very far from the slit in comparison with its breadth, the rays of this beam may be regarded as practically

parallel.) The rays from the portion,  $C'D$ , of the wave have to traverse different distances to reach the point on the screen, and cannot, therefore, generally impinge upon it in the same phase. Drawing the line,  $DH$  from  $D$ , perpendicular to the ray,  $CE$ ,  $CH$  is the distance by which the outside ray,  $CE$ , remains behind the ray,  $DF$ . If this difference of their distance,  $CH$ , is an entire wave-length, the central ray ( $G$ ) of the beam is retarded by half a wave-length with respect to the wave,  $DF$ . It produces, then, a trough at the point upon the screen where  $DF$  produces a crest, and conversely. Since the difference of the lengths of their paths are equal to half a wave-length, these two waves are in exactly opposite phases, and destroy each other's effect; and generally, to every ray belonging to the half,  $VG$ , of the beam there is, in the other half,  $GC$ , a corresponding ray, which lags with reference to the former by half a wave-length. Such, for example, are 1 and 7, 2 and 8, 3 and 9, and so forth. The rays of this beam, then, destroy each other in pairs, and at the place on the screen where the beam falls total darkness must prevail. With a more inclined beam (directed toward a point of the screen lying farther toward the side) the difference of distances traversed by the outermost rays amounts to two whole wave-lengths. The beam may be again supposed divided into two halves,  $UG$  and  $GD$ , the distances travelled by whose marginal radiations differ by an entire wave-length, and the two halves again vanish independently. Continuing thus, it is seen that dark streaks appear at all those points of the screen for which the difference of the distances of the marginal rays equals some integral number of entire wave-lengths. At intervening points, however, where the difference of the distances travelled has some other value, the rays cannot be completely extinguished. Between the dark streaks, therefore, bright rectangles appear, whose intensities diminish rapidly outward. If green glass be used instead of red, green rectangles will be obtained in place of the red, but they will be narrower and more closely packed than the red, and with a blue glass the streaks crowd still more closely together. It is clear, that the shorter the wave-length, the smaller the inclination of the diffracted radiation need be to produce the difference of distance required for

the respective bands. The dark stripes lying nearer the centre of the field of diffraction show again that to the simple colours of the spectrum, there correspond wave-lengths diminishing in continuous succession from red to violet. The magnitude ( $\lambda$ ) of these wave-lengths may be easily obtained from these phenomena of diffraction. For, measuring micro-metrically, the breadth of the slit,  $CD = b$ , and the "diffraction angle,"  $ECG = CDH = \phi$ , which the diffracted radiations directed toward the ( $n$ th) stripe form with the direct rays (with a goniometer),  $CH$  must equal  $n$  wave-lengths, or  $n\lambda = b \sin \phi$ .

Admitting white light, composed of all colours, through the opening of the shutter, the lateral rectangles and the dark bands for the various colours cannot coincide, and upon the screen at both sides of the white centre there is seen a series of variegated bands, separated from each other by fainter bands of colour.

If the slit be gradually widened the same differences of distance occur with continually diminishing inclinations of the diffracted rays. The stripes crowd more and more closely together until they finally become so fine as to be imperceptible. To observe satisfactorily the phenomena of diffraction, therefore, narrow apertures must always be used. The images observed are of various shapes, and frequently of ornamental patterns, depending upon the form of the aperture used. If the brilliant image of the sun upon a polished metal knob is observed through a rhomboidal aperture, an oblique cross composed of rhomboids is seen, whose constituent parts are coloured with all the hues of the rainbow. If the aperture is circular a brilliant disk surrounded by several coloured rings is seen. Through a triangular aperture a six-pointed star is seen, in whose angles many smaller luminous images appear. Howsoever complicated and composite these images may appear, they can nevertheless be accurately computed on the undulatory theory (Schwerd, 1835), and serve as a most complete vindication of this theory. The so-called Babinet's principle that an opaque screen gives the same phenomena of diffraction as an aperture of the same form is verifiable with charming clearness.

**358. Gratings.**—The most beautiful of all diffraction phenomena are produced by *gratings* (Fraunhofer, 1821). This term is applied to a number of narrow parallel slits produced either by stretching fine wires in a light frame at equal distances apart (*wire grating*), or by ruling upon a blackened glass plate a number of fine parallel lines with a graduating engine. Very excellent gratings are also produced by scratching a large number of exceedingly fine lines with a diamond point upon a glass plate (*glass grating*), or upon a reflecting

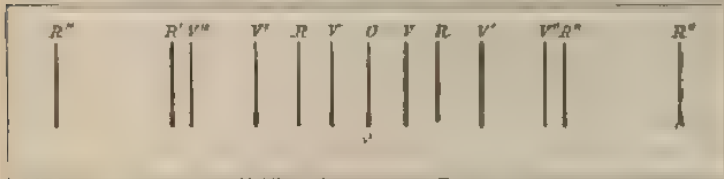


FIG. 375.—Formation of the Diffraction Spectrum

metallic surface (Roland's *reflecting grating*). When simple light, red light, for instance, falls upon such a grating, after after having been passed through a slit, a small condensing lens placed behind the grating collects the rays coming directly from the slit into a slender image, OO (Fig. 375), upon a screen placed conjugate (330) to the slit. Since a lens between conjugate points produces no differences of distance traversed by rays passing between them, all rays have equal distances to traverse before reaching the image, OO, and strike this image together in the same phase.

The diffracted radiations for each direction consist of as many homogeneous beams (Fig. 376) as there are apertures in the grating. Every pair of adjacent beams has a difference of distance,  $ab$ , which is greater the greater the amount of their deflection from the direct rays (the diffraction angle), or the farther the centre, OO, is from the position upon the screen, where all the

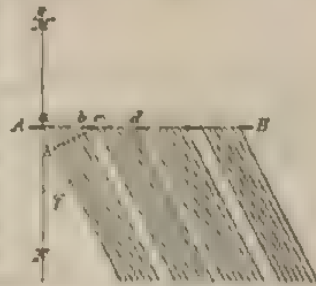


FIG. 376. Diffraction by a Grating.

rays having this direction are concentrated. There must now be a certain direction for which the difference of distance traversed by two adjacent beams is equal to an entire wave-length of red light. In this direction all beams mutually reinforce each other, and at the corresponding place upon the screen, a slender red image of the slit, R, appears. A little aside from this direction, the beams all destroy each other on combining, provided the grating is supplied with a sufficiently great number of lines. For, with a grating of 100 lines, if the diffraction angle increases only until the first beam is delayed by  $1 + \frac{1}{100}$  of a wave-length with respect to the second, it will lag behind the third by  $2 + \frac{1}{100}$ , behind the fourth by  $3 + \frac{1}{100}$ , etc., behind the fifty-first by  $50 + \frac{1}{100}$ , or by  $50 + \frac{1}{2}$  wave-lengths. The fifty-first beam is therefore in opposite phases with the first, similarly the fifty-second with the second, the fifty-third with the third, etc., finally the hundredth with the fiftieth. Hence it follows that the diffracted rays destroy each other for all directions, save those for which the difference of distance of each pair of neighbouring beams equals an integral number of wave-lengths. The diffraction pattern, or image, upon the screen is, therefore, for homogeneous red light of very simple form. In the centre appears the image, O, of the slit; then follows upon either side, at a distance corresponding to the difference of distances travelled by the waves equal to an entire wave-length of red light, a slender red line, R, then, at double the distance (corresponding to the distance of two wave-lengths), a second red line, R', and still beyond these at three times the distance (R''), and so on. For violet light a series of violet lines would be similarly obtained, which, however, in consequence of the shorter wave-length of this colour, would lie nearer the image of the slit, OO, at V, V', V'', etc. When white light is used the central slit image appears white, because all colours here lie superposed. The variously coloured lines arising from the diffraction and corresponding to a difference of distance of one wave-length, for example, lie beside one another in the order of their wave-lengths, and form at either side of the central image a band of colour, which presents from its outer toward its inner edge, the well-known

succession of rainbow colours—red, orange, yellow, green, blue, indigo, and violet—which is called the *diffraction spectrum*, VR (1st series). Similarly, the rays of higher differences of distance form the second (V'I'), the third (V''I''), etc., *diffraction spectrum*, or *series*. With a prism the relative distribution of colours will be found to depend upon the material of the prism. With a grating, on the contrary, the simple colours are arranged solely according to the differences of their wave-lengths, and hence according to a characteristic feature of the rays themselves. The spectrum of a grating may then be regarded as the *normal*, or *typical spectrum*. With sunlight the diffraction spectrum also shows the Fraunhofer lines, each in a position peculiar to it by virtue of its wave-length. By means of a telescope rotating above a graduated circle (341), the angular distance of each Fraunhofer line from the central image of the slit (the *diffraction angle*,  $\phi$ ) may be measured, and from its measured value in connection with the known distance of the rulings (*grating constant*,  $c$ ) the corresponding wave-length ( $\lambda$ ) may be obtained. For the  $n$ th spectrum ( $n$ th series) we have  $n\lambda = c \sin \phi$ . The following table contains the wave-lengths thus found for the Fraunhofer lines, expressed in millionths of a millimeter:—

A	760	b	..	..	518
a	718	F	...	...	486
B	687	G	...	...	431
C	656	H <sub>1</sub>	...	...	397
D	589	H <sub>2</sub>	...	...	393
E	527				

In reflected light, also, gratings and other finely ruled surfaces exhibit the phenomena arising from the interference of diffracted rays. Mother-of-pearl, for instance, is composed of extraordinarily thin layers of calcium deposited by the animal itself. These layers being inclined to the surface give rise to a number of extremely fine lines. That the delicate play of colours, characteristic of mother-of-pearl, is due to this peculiar surface structure is shown by the fact that when mother-of-pearl is copied upon black sealing-wax, the same colours are exhibited by the wax. By engraving fine lines upon metallic

surfaces (Barton's iris buttons) a play of colours resembling that of mother-of-pearl can be produced.

**359. Court.**—The coloured wreath, or garland, of light frequently observed around the disk of the sun and moon when the sky is covered with a filmy veil of clouds is called a *court*. Looking at the flame of a candle through a glass plate strown with lycopodium powder, the flame is seen surrounded by a bright reddish luminosity which is encircled by several rings, tinted with rainbow colours, and having their violet edges inward. This phenomenon is due to the diffracting effect produced by the grains of the powder upon the rays of light passing their surfaces. According to Babinet's principle, each of these grains acts as a circular dark screen, and produces the same phenomena of diffraction, viz., coloured rings (357), as would be produced by a circular aperture of the same diameter. If a still finer powder is used, for instance, the fine powder of a puff-ball, the rings appear larger, their diameters always being in the inverse ratio to the diameters of the grains of powder. The condition necessary to the formation of the colour rings is that all the powder grains shall be of equal size. When small particles of unequal size are mixed together, the variously coloured rings overlap and blend into a whitish shimmer. In this same way the courts about the sun and moon are produced by the diffraction of light due to the particles of vapour composing the clouds. The diameters of the particles producing the haze may be determined from the diameters of the rings, the first of which appears under an angle of  $1^\circ$  to  $4^\circ$ . It is found that these particles are on the average larger in the winter than in the summer. With approaching rain the little particles swell rapidly, and the lunar court contracts. Lunar courts are more frequently observed than solar, because the light of the sun is so blinding as to render the feebly illuminated rings beside it invisible. Solar courts may, however, be seen at once by observing the less brilliant image of the sun upon the surface of water, or upon a plate of glass.

Standing upon an isolated mountain-top, and surrounded by a fine, scarcely perceptible film of fog, with the sun at the

observer's back, and the waving film of cloud at his feet, he may see his shadow in giant proportions upon the background of the cloud, with the head of the shadow surrounded by coloured rings. These rings are produced by the diffraction of the solar rays from the little globules of moisture in the air floating about the head of the observer, the diffracted rays being subsequently reflected from the particles of vapour before him. The apparently gigantic proportions of the shadowy image are due to an optical delusion. The solar rays being particularly parallel, the shadow cannot be greater than the shadow-casting body. Despite the fact in the present instance, that the shadow lies against the cloud in its natural size, the judgment unconsciously locates the image at the greater distance at which the film acquires more definite outlines for the eye and presents a more palpable screen for the reception of the shadow. Since the visual angle now remains the same, the shadow at this misjudged distance, apparently assumes correspondingly large proportions. From the mountain, on the top of which these images were first seen, they have been termed *Brocken-spectres*.

**380. Colour Rings of Thin Plates—Newton's Rings.**—If a little oil of turpentine be poured upon water it spreads out over the surface into a thin skin with brilliant iridescent colours playing over it. Similar colours are observed in panes of old window-glass which have become "blinded" by constant exposure to the weather, and they are especially clear and distinct in soap-bubbles. They are always to be seen in thin, transparent plates of every kind, and are therefore termed "colours of thin plates." If a beam of light (AB, Fig. 377) falls upon a thin plate, a portion of it is reflected at the surface toward BC; but a larger portion, BD, penetrates the plate, and is reflected at the lower surface along DEF. Since the rays reflected from the back surface have traversed the thickness of the plate twice, they have suffered a retardation of phase with respect to the rays reflected from the front surface. The



FIG. 377.—Colours of Thin Plates.

retardation is greater the thicker the plate, and the two emergent beams are in condition to interfere. If the thickness of the plate is such that the difference of distance traversed by the two beams equals  $\frac{1}{2}$  of a wave-length of green light, the longer red rays will be delayed by about one, while the shorter violet waves will be retarded by almost two wave-lengths. The green rays mutually destroy each other, though the red and violet do not, and the plate exhibits the purplish colour peculiar to the mixture of red and violet light. Various colours are sifted out of the reflected light dependent upon the thickness of the plate, and the most variegated mixture of colours is thereby produced. If, however, the plate is not everywhere equally thick, it will appear to be striped with many colours, all places of equal thickness showing the same colour, giving rise to what are known as *isochromatic curves*. The uppermost



FIG. 378. Newton's Colour Rings.

and thinnest portion of a soap-bubble, for example, is encircled by a series of vividly coloured rings. Such colour rings (Fig 378) may be permanently produced, as was done by Newton (1675), by placing a plano-convex lens upon a plane glass plate and pressing the surfaces together. Between the two surfaces of glass a thin layer of air is thus produced which increases in thickness gradually outward from the

point of contact, and about this point coloured rings form in regular order.

Thicker plates do not show colours, because, with greater differences of distance, many of the simpler radiations are extinguished by interference, and, while many intervening radiations contain all colours of the spectrum and are intensified, their mixture appears as white light. If the white light, reflected from a plate of mica, or of glass, be dispersed by a prism, numerous dark bands appear in the spectrum, in positions corresponding to those radiations which have been quenched by interference.

**381. Stationary Light Waves.**—When a parallel beam of light rays (or a plane

light wave) impinges perpendicularly upon a plane mirror, by the interference of the incident with the reflected wave, a stationary wave is produced, of such character that, at a given distance from the mirror *i.e.* everywhere in a plane parallel to it, the same phase of vibration exists. The nodes and antinodes form two systems of planes parallel to the mirror, whose distances, for planes of the same system, equal half a wave-length, and are bisected by the planes of the other system. Suppose, now, that this system of stationary waves is cut by a plane inclined to the mirror. The two systems of planes meet then intersect this plane in two systems of parallel, equidistant straight lines, which correspond alternately to antinodes and nodes of vibration. If the cutting plane is perpendicular to the mirror, the distances of these straight lines will be only half a wave-length, and, hence, so small that the unaided eye cannot perceive them separately. The distances of the lines, however, become greater, the smaller the inclination of the cutting plane to the mirror, and this inclination may be so chosen, that the lines are separated by from  $\frac{1}{4}$  to 2 mm. To render these lines visible, Wiener (1890) used a plate of glass, upon which a thin, transparent, sensitized film of collodion was spread, and placed it with the film toward the mirror, at such an inclination that, between the film and the mirror, only a thin, wedge-shaped column of air remained, within which the stationary-wave was formed. The strongest photographic effect occurred along the line of antinodes, and the weakest along that of the nodes, so that after developing the plates there appeared upon the film a system of alternate dark and bright bands, the former corresponding to the nodes, and the latter to the antinodes.

**362. Colour Photography.**—With the help of stationary light waves, Lippmann (1891) succeeded in photographing the spectrum and other coloured objects in their natural colours. As a sensitive plate, he used a thin film of albumen, spread upon a glass plate, within which silver iodide and bromide were mixed uniformly, and in an extremely finely divided state. This plate formed, with its film toward, the front wall of a glass trough filled with mercury. The photograph of the spectrum taken upon it, when developed and fixed in the usual way, showed before the dark background, when seen in reflected light, the spectral colours, each in its characteristic place, and when seen in transmitted light the complementary spectral colours. During the photographing process stationary waves were formed within the sensitized film by the combined effect of the incident rays and of those reflected at the surface of the mercury, having the planes of their nodes and antinodes parallel to the surface of the film. Only those of greatest intensity acted upon the silver salt, and even they, after being fixed, leave transparent layers of silver more or less strongly reflecting. The film is thus divided into a series of very thin plates, whose thickness for each colour equals the distance of two antinodes, or equals half a wave-length of the respective colour. If now, white light falls upon such a plate, the light reflected at the front surface will interfere with that reflected from the back surface, by reason of the difference of distances traversed, which difference equals double the thickness of the plate. But this distance is equal to an entire wave-length, for only one of the colours contained in white light, *viz.* for that which during the photographing process reacted upon this part of the plate. While the interference of the two beams of light intensifies this colour, it weakens all the others. At this part of the image, therefore, the plate has exactly the necessary thickness to reproduce by interference in reflected light the colour photographed at this particular place. The colours shown by the image are nearly the colours of thin plates. They appear, however, much clearer and more thoroughly saturated than the latter. The reason for this is, that in the sensitized plate, by reason

of the extreme smallness of the light waves, a very large number of such thin plates be superposed (about 200, if the plate is  $\frac{1}{10}$  mm. thick). The more reflecting surfaces are present, the purer the reflected colour will be: for the surfaces, succeeding each other at equal distances, form a grating, which intensifies the beam of the corresponding colour, whose difference of distance equals an integral number of wave-lengths, while it destroys the rays producing other colours.

**363. Polarization of Light.**—Tourmaline plates, considered from now on, are cut from tourmaline, a half-precious stone which crystallizes in the form of a hexagonal prism. The plates are formed by cutting the crystals parallel to their axes. Light passed through such a plate, shows to the unaided eye no other modification than the brown or olive-green colouration peculiar to the crystal and produced by absorption. If a second tourmaline plate be placed upon the first, in such



FIG. 379. Tourmaline Plates.

position that the axes of crystallization of the plates are parallel, e.g. both vertically, as in Fig. 379 A, the light, passing from the first plate, continues through the second, and, by reason of the greater thickness traversed, suffers a somewhat deeper colouration. But if the plates be turned in their plane, the light transmitted through them gradually diminishes in brightness, and wholly disappears when the axes of the crystals are perpendicular to each other (Fig. 379, B). Turning the plate still farther, the light gradually reappears, and attains its original intensity when the axes are again parallel. A ray of light emitted directly from a luminous source is transmitted with uniform intensity in all positions of the second plate. The light transmitted through the first plate differs, then, very materially from natural light; for it passes with undiminished



FIG. 380.—Cross-sections of Light Rays.

intensity through the second plate *only when the axes of both plates are parallel*. It is wholly cut off by the second plate when these axes are mutually perpendicular. While, therefore, natural light shows the same properties, no matter which of the positions indicated in Fig. 380, A (in this figure the ray of light

is supposed to impinge perpendicularly to the plane of the drawing), the tourmaline plates are supposed to occupy, and possesses, consequently, the same constitution in all directions perpendicular to the plane of propagation; with the ray transmitted by the first plate, there is one among all these positions, viz. that parallel to the axis of the first plate, which is distinguished from all the rest (Fig. 380, B), in that the light passes through the second plate, or not, according as this direction is parallel or perpendicular to the axis of this plate. A ray of this sort which exhibits different properties in different directions from its centre of cross-section, is characterized by the rather unfortunate term, "*polarized*."

It is easy to account for the possibility of such behaviour of light on the basis of the undulatory theory. In a wave the vibration of the individual particles of matter concerned in the motion, may take place either in the direction of propagation, i.e. in the direction of the ray (longitudinal vibration), or perpendicularly to this direction (transverse vibrations). The former mode of propagation is exemplified by sound waves in the air, since they are propagated wholly by longitudinal vibration. Transverse vibrations, on the contrary, are observed in a long cord stretched between the points, A and



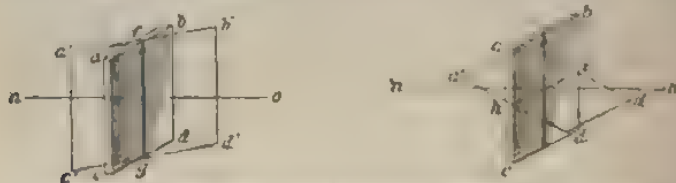
FIG. 381.—Polarized Ray of Light.

B (Fig. 381), when the cord is given an impulse in a vertical direction. Waves are then seen to pass along the cord in such way that every point of the cord vibrates upward and downward perpendicularly to the direction of propagation.

Looking along the cord from B toward A, an observer would see the vibrations in a vertical direction, as in Fig. 380, B, and he would find the upper and lower sides, toward which the vibrations are alternately directed, very essentially different from the right and left sides, toward which no vibrations occur. He may also readily see that, if the cord were passed through a slit, the vertical vibrations would pass unhindered, when the slit stands vertically, but that when it

is placed horizontally, the vibrations cannot pass through it. Since the wave ray,  $AB$ , has different properties in different directions, precisely as is the case with a light ray after having passed through a tourmaline plate, the wave in the cord may with equal propriety be called "polarized." The behaviour of a polarized ray ( $AB$ , Fig. 381) is thus easily explained on the assumption that it is propagated wholly by transverse vibrations, all of which take place in, or parallel to, the same plane through the ray. This plane, in Fig. 381, the plane of the drawing, is called its *plane of vibration*.

The experiment with the tourmaline plates is easily explained, by assuming that the vibrations of the polarized rays, coming from the first plate, are parallel, or perpendicular to the axis of the crystal. This experiment does not enable us to decide which of the two directions is the direction of vibration. The following simple experiment, however, permits us to decide at least with some probability upon the direction of vibration. If a tourmaline plate,  $abcd$  (Fig. 382), is rotated about an axis,  $fg$ , parallel to the axis of the crystals into the



FIGS. 382, 383.—Experiment to obtain the direction of Vibration.

position,  $a'b'c'd'$ , while a white spot is observed through it in the direction,  $on$ , the brightness of the field of view remains practically unaltered. But if the plate be so inclined to the direction of the ray,  $no$  (Fig. 383), that the line,  $hi$ , perpendicular to the axis of the crystal, becomes the axis of rotation, the field of view becomes very considerably darker. The fact is now obvious that a change of brightness can only occur when the angle formed by the direction of vibration with the axis of the crystal is altered. From the fact that the first mode of rotation (Fig. 382) produced no alteration in brightness, it may then be inferred that in this case the direction of the

axis of the crystal with respect to the direction of vibration remained the same, whether the plate were in the position *abcd* or *a'b'c'd'*. This direction must accordingly be that of the axis of the crystal, *fg*, which in this case is also that of the axis of rotation. There is, then, good reason to assume (with Fresnel) that the plane of vibration of the polarized ray coming from the tourmaline plate is parallel to the axis of the crystal, as is illustrated by Fig. 384.

The experiment with the crossed tourmaline plates (Fig. 379, B) proves that, in a polarized ray of light, transverse vibrations alone exist. If longitudinal vibrations were present, since the constitution of a ray is the same in all directions so far as longitudinal vibration is concerned, they would necessarily pass through both the first and the second tourmaline plates, no matter what position were given to the latter. Complete darkness, such as was observed in the crossed position of the plates, could then never occur. But if a polarized ray contains no longitudinal vibrations, it is highly probable that an unpolarized ray (emitted directly from a luminous source) consists of transverse vibrations only, and this assumption seems all the more justified by the fact that all known luminous phenomena may be satisfactorily explained by transverse vibrations alone. Since an unpolarized, or natural, ray shows the same constitution in all directions, it is necessary to suppose that the vibrations occur in all directions simultaneously in its different parts and in rapid succession at any given place. This is indicated in Fig. 380, A, which represents, in a sense, the cross-section of an unpolarized ray emerging perpendicularly from the page toward the eye of the reader, while Fig. 380, B, illustrates in a similar way the cross-section of a polarized ray. The correctness of this view is confirmed by the following experiment. When a tourmaline plate is rotated rapidly in its plane (about the direction of the ray, *no*, Fig. 382, as axis), the light coming from the plate behaves precisely as natural light, since its vibrations take place in very short intervals



FIG. 384 - Position of the Plane of Vibration with Tourmaline.

of time in all possible directions perpendicular to the ray. Selecting now in the cross-section of a natural ray any two directions perpendicular to each other (Fig. 283, C), each vibration may be resolved according to the rules of mechanics into two components in these directions. By compounding all the components in the same direction, the undulatory motion in a natural ray of light may thus be reduced to two equal vibrations perpendicular to each other, or, in other words, a natural ray may be regarded as composed of two polarized rays of equal intensity and vibrating perpendicular to each other. This view is justifiable experimentally also, for two equally bright rays polarized perpendicularly to each other give, when compounded, a ray of light, which exhibits the same behaviour as a natural ray. When light is reflected from a plane glass plate, or from any other smooth surface, and is studied through a tourmaline plate, on turning the plate in its plane about the reflected ray as an axis, it appears brighter in some positions and darker in others, though it never vanishes completely in any position of the plate. It appears brightest when the axis of the tourmaline is perpendicular to the plane of reflection, or to the plane of incidence, and is darkest when the axis coincides with this plane. The light reflected from a glass plate is accordingly not natural, nor is it completely polarized. But it acts as though it were a mixture of natural

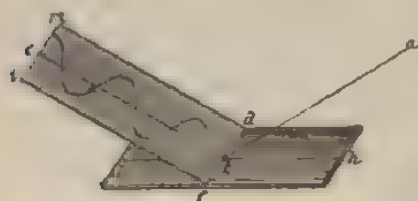


FIG. 385.—Polarization by Reflection.

and polarized light whose vibrations are perpendicular to the plane of reflection. It is, therefore, termed *partially polarized*. The ratio of the polarized to the unpolarized portion varies with the angle of incidence. When the beam strikes the surface perpendicularly its reflection contains no polarized light at all. But when the angle of incidence is  $57^\circ$ , or when the incident ray (*ab*, Fig. 385) forms an angle, *abn*, of  $33^\circ$  with the glass plate, the unpolarized portion is wanting. At this angle, which is called the *angle of polarization*,

and polarized light whose vibrations are perpendicular to the plane of reflection. It is, therefore, termed *partially polarized*. The ratio of the polarized to the unpolarized portion varies with the angle of incidence. When the

or the *polarizing angle*, the reflected light (*bc*) is *completely polarized*, there being now no vibrations whatever perpendicular to the *polarizing plane*, as the plane of reflection is called in this case. The position of the plane of vibration (*dfim*) is represented by Fig. 385. Instead of using a tourmaline plate to investigate the light reflected from the glass plate, it may be caught upon a second glass plate inclined at the same angle (Fig. 386). When the plates are parallel to each other, as in the figure, their planes of reflection coincide, and the ray, *bc*, polarized at the first plate, and having its vibrations perpendicular to the common plane of reflection, is reflected at the second plate toward *de*. If, however, the second plate is rotated out of this position, while it preserves continually the angle  $33^\circ$  with the ray, *bc*, the light reflected by it becomes weaker, and finally vanishes completely, when the two plates are perpendicular to each other; because with this position the vibrations of the ray, *bc*, are executed in the plane of reflection of the second plane. But at this angle of incidence the plate can reflect only those vibrations which are perpendicular to its plane of reflection. In this experiment the plates are usually blackened upon their back surfaces, or they are made of black glass to exclude the unpolarized foreign light passing through them.



FIG. 386 — Polarizing Mirrors.

Light transmitted at an oblique angle by a glass plate is also partially polarized, as shown by tests with a tourmaline plate. The vibrations of the polarized portion are in the plane of incidence, or the transmitted light is polarized perpendicularly to the reflected. Arago proved that with each value of the angle of incidence the quantities of light polarized perpendicularly in the reflected and refracted rays are equal. A refracted ray is never completely polarized, no matter what the angle of incidence may be. Nevertheless, an almost complete polarization of the transmitted rays may be attained, if, instead of a single glass plate, a laminated plate consisting

of a great number of glass plates, or of a glass pile is used. If a natural ray falls upon such a series of plates at the polarizing angle, the ray being resolved into equally bright rays, one of which vibrates in the plane of incidence, and the other perpendicularly thereto, the former passes through the plates without diminution, because its deviation renders it incapable of deflection. The latter, on the contrary, undergoes partial reflection at each surface, and is thereby rendered too feeble to be perceived. The glass pile, therefore, transmits at the polarizing angle only those rays whose vibrations are parallel to the plane of incidence. The polarizing angle differs with different substances. It differs also with the index of refraction, as was shown by Malus, the discoverer of polarization by reflection (1810), and for water it is  $53^\circ$ , for carbon disulphide  $59^\circ$ ,



FIG. 387.—Polarizing Angle—Brewster's Law.

for flint glass  $60^\circ$ , etc. The precise relation between the polarizing angle and the index of refraction was discovered by Brewster in 1815. He showed that the polarizing angle is the angle of incidence for which the reflected ray (*bc*, Fig. 387) forms a right angle with the refracted ray (*bd*). Since the angle of refraction,  $90^\circ - p$ , corresponds to the polarizing angle,  $p$ , there results from the law of refraction—

$$\frac{\sin p}{\sin (90^\circ - p)} = \frac{\sin p}{\cos p} = n, \text{ or } \tan p = n,$$

as the symbolic expression of Brewster's law. White light can, then, never be completely polarized by reflection, because for each homogeneous colour the index of refraction,  $n$ , and therefore also the polarizing angle, has a special value.



FIG. 388.—Double Refraction

**384. Double Refraction.**—All crystallized bodies not belonging to a regular system possess the property of dividing a ray of light, *ab*, transmitted through them into two components (*bc* and *bd*, Fig. 388). The tendency of crystals to cleavage in definite directions reveals a regularity of internal structure.

which finds its explanation in the regular arrangement, or in the *uniform orientation*, of its molecules. Each molecule is built up of atoms of definite constitution and number. These atoms are disposed, according to a definite law, about three axes perpendicular to one another. These three axes are generally unequal, so that forces acting in their respective directions upon the molecule encounter different resistances. An aggregation of equal molecules will compose a *crystal*, when they collect in such way that their equal axes are respectively parallel. As a consequence of this the crystal, as a whole, exhibits different physical characteristics in different directions, *e.g.* heat is transmitted through them with unequal rapidity in different directions; the amount of expansion due to heat varies with the direction, etc. If, however, the molecules are irregularly disposed, so that the corresponding molecular axes lie in all possible directions, they form an *un-crystallized*, or *amorphous* body. Such irregular orientation of molecules occurs with liquids. Since, in this case, no direction is distinguished in any special way, uncrystallized liquids and solids possess the same properties in all directions. This is true also of crystals of the *regular systems*, whose molecules have three equal axes. Those bodies, which are endowed with the same characteristics in all directions, are called *isotropic*. Crystals of the quadrangular and hexagonal systems have two axes which are equal and a third which differs from these two, while crystals of the rhomboidal, monoclinic, and triclinic systems have three unequal axes. Bodies like the crystals of these five systems, exhibiting different properties in different directions, are called *anisotropic*, or *heterotropic*.

A wave of light cannot be transmitted through the ether, which fills the interstices of bodies without both acting upon the molecule and experiencing from it a corresponding reaction. This effect is revealed, on the one side, by an enfeeblement of the wave (absorption), and, on the other, by an alteration of the velocity of propagation. In an isotropic body, the luminous vibrations are influenced in the same way, regardless of their direction of propagation. If, at a certain point of such a body (*e.g.* glass), vibrations are excited in any given direction,

they will be transmitted through the body with a lower velocity than through the ether, though the velocity will be the same in all directions, and the vibrations will produce spherical waves about the point of excitation. The *wave surface* of an isotropic medium is, therefore, said to be spherical. The mode of propagation of light in such a medium is completely specified by means of this form of wave surface. The motion of light in an anisotropic body is just as completely known as the former, so soon as its characteristic wave surface has been obtained, *i.e.* the surface, upon which all molecules, at any given instant, are in the same phase vibrations.

*Calc-spar*, called also Iceland spar, may be taken as an example of such bodies. This substance possesses in a remarkable degree this property of double refraction (Erasmus Bartholinus, 1669). Its transparent, colourless crystals cleave perfectly along three directions. Pieces may, therefore, be readily



FIG. 389.—Rhombohedaon.

obtained which are bounded by six equal rhomboidal surfaces, and are called *rhombhedra* (Fig. 389). Two of the opposite vertices, *a* and *b*, are enclosed by three obtuse face angles, and the remaining six are bounded each by one obtuse and two acute face angles. The straight line, *ab*, connecting the two obtuse vertices, is called the *principal axis*, or, more simply, the *axis* of the crystal. The surfaces, edges, and vertices are symmetrically disposed with respect to this line. Any plane passed through the axis, or a line parallel to it, is called a plane of *principal section*. A similar symmetry is exhibited by each molecule of the calc-spar. Each possesses a principal axis, distinguished from all other directions in that it lies parallel to the axis of the crystal, so that the molecule exerts upon luminous vibrations perpendicular to this principal axis an influence different from that exerted upon vibrations parallel to the axis, or at any other angle with respect to it. Suppose, now, in Fig. 390, the plane of the drawing to represent the principal section of a crystal of Iceland spar, and *ab* to indicate the direction of the axis. Suppose, also, that

vibrations originate at the point, *m*, and that these vibrations are partly in the plane of the principal section and partly perpendicular to it. The latter, which are also perpendicular to the axis, are transmitted in all directions with the same velocity, giving rise to the circular wave indicated in the figure. The

vibrations in the plane of the principal section are transmitted with different velocities in different directions, dependent upon the angle they form with the principal axis. Vibrations, for instance, parallel to the direction of the axis, give rise to a ray, *mb*, which, during the time required for the vibrations perpendicular to the axis to traverse the radius of

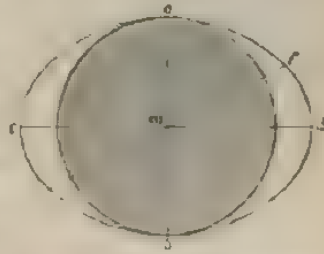


FIG. 390.—Propagation of Light in Calc-spar.

the former circular wave, pass over the greater distance, *md*, because in Iceland spar the vibrations parallel to the axis are more rapidly transmitted than are those perpendicular to it. But vibrations directed toward *ed*, being perpendicular to the axis, produce a ray, *ma*, which, during the time considered, advances only to the circle. Finally, the rays whose vibrations are oblique to the axis, have a velocity of propagation (e.g. *mf*) intermediate between *md* and *ma*. So that, as Huyghens (1678, has shown, the vibrations in the principal section produce a wave of elliptical contour, *acbd*, which is tangent to the circular wave produced by the vibrations perpendicular

to the principal section, at the extremities of the axis, *a* and *b*. What is true for this principal section, is true for all similar sections. It is then only necessary to imagine Fig. 390 turned about the axis, *ab*, to obtain the wave surface, which determines the nature of the propagation of light in calc-spar. These wave-surfaces consist of two shells:



FIG. 391.—Model of Wave surface in a uniaxial Crystal.

of a sphere for the vibrations perpendicular to the axis, and of a flattened ellipsoid of

revolution enveloping the sphere and touching it at the ends of the axis, for the vibrations oblique to this axis. Fig. 391 shows three sections mutually perpendicular to one another, viz. two principal sections, and a section perpendicular to the axis. The figure furnishes a simple model of the wave surface for this substance.

Imagine, now, a beam of parallel rays,  $abef$ , to fall upon the surface,  $MN$  (Fig. 392), of a crystal of calc-spar. Let a perpendicular,  $hg$ , to the direction of the ray be drawn from  $h$ , where the surface is first struck by the beam. This line,  $hg$ , will then represent that portion of the plane corresponding wave,

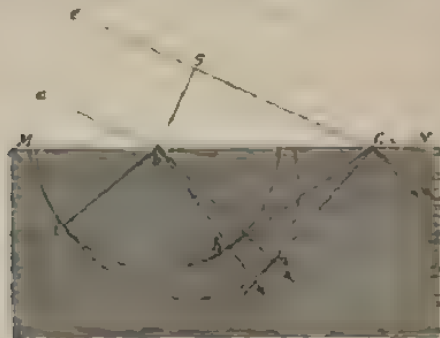


FIG. 392.—Double Refraction in a Calc-spar.

in which all particles of the ether are simultaneously in the same phase (cf. Fig. 371). As the wave,  $hg$ , impinges against the surface of the crystal, the particles of ether lying between  $h$  and  $f$  are successively set in motion, and each transmits a wave into the crystal. For simplicity, let it be assumed that the plane of incidence, i.e. the plane of the drawing, represents a principal section of the crystal. It is then necessary merely to conceive of each incident natural ray as consisting of two equally bright rays, one vibrating in the principal section, and the other perpendicularly to it. The latter vibrations being perpendicular to the axis,  $bi$ , of the crystal, while the wave,  $hg$ , passes from  $g$  to  $f$ , will have spread out into a circular wave,  $ih$ , whose radius,  $bh$ , is to  $gf$

as the velocity of propagation of these vibrations in the crystal is to the velocity of propagation in air. From each point of the surface of the crystal between  $b$  and  $f$ , a circular wave will be simultaneously emitted, whose radius is smaller the later the corresponding point is reached by the incident wave. All these circular waves have, at the instant the incident wave strikes the point,  $f$ , advanced to the line,  $fh$ , which is the common tangent to all the circular waves. The line,  $fh$ , accordingly, represents the plane wave, entering the crystal, and the straight line,  $bh$ , drawn from  $b$  toward the point of contact,  $h$ , gives the corresponding direction of the refracted ray. Since the wave contour used in this drawing, as in simple refracting (isotropic) media, is spherical, a ray, vibrating perpendicularly to the principal section, conforms to Snellius' law of refraction. If, in a similar way, it is desired to take account of the rays vibrating in the principal plane, the elliptical contour,  $ni$ , must be drawn about  $b$  ( $bi$  is the direction of the axis), and then, from  $f$ , the line,  $fn$ , tangent to this contour must be drawn. This line indicates the position of the refracted wave, and the line from  $bi$  to the point of contact shows the corresponding direction of the ray. By reason of the ellipsoidal form of its wave surface, this ray does not follow the ordinary law of refraction. It conforms to a much more complicated one. It is thus seen that a natural ray ( $ab$ ) incident upon a crystal of Iceland spar, is resolved generally into two rays, which are propagated with unequal velocities: an *ordinary ray* ( $bh$ ) and an *extraordinary ray* ( $bn$ ). Both are *completely polarized*, the latter with its vibrations in the primary section, while those of the former are perpendicular to it. If the plane of incidence is perpendicular to the axis, the rays will radiate in two concentric circles (cf. Fig. 391), and both waves follow Snellius' law of refraction. This occurs with a crystal of Iceland spar, whose refracting edge is parallel to the axis of the crystal. By means of such a prism, the two principal indices of refraction,  $n_o$  for the ordinary, and  $n_e$  for the extraordinary ray, may then be found by the method of least deflection (329). For sodium light,  $n_o = 1.6585$ ,  $n_e = 1.4865$ , and from these, if the velocity of light in air equals 1, the least velocity of propagation in

Ice-land spar equals  $\frac{1}{n_e} = 0.6030$  (semi-minor axis,  $ma$ , of the ellipse, Fig. 389), and the least equals  $\frac{1}{n_o} = 0.6727$  (semi-major axis,  $md$ ).

Since there is but a single velocity of propagation in the direction of the axis, a natural ray entering the crystal along the axis does not admit of resolution. Any direction in a double refracting prism along which double refraction does not occur is called an *optical axis*. All crystals of the square and hexagonal systems (to the latter of which calc-spar belongs) possess a single optical axis, which coincides with the crystallographic principal axis, and such crystals are called *optically uniaxial*. Crystals in which the extraordinary rays are transmitted more rapidly than the ordinary—in which the ellipsoidal wave envelope encloses the spherical wave—such as calc-spar, tourmaline, sodium nitrate, etc., are called *uniaxially negative*. When, on the contrary, the spherical wave includes the ellipsoidal, or when the ordinary waves have the greater velocity, the crystal is said to be *uniaxially positive*, as, for example, rock crystal, or quartz, zirconium, tin ore, ice, etc. In the crystals of the three remaining systems two rays polarized perpendicularly to each other are likewise propagated with unequal velocities, though neither follows the ordinary law of refraction. In each of these crystals two directions without double refraction, or *two optical axes*, may be found. The crystals are, therefore, termed *optically biaxial*. Such crystals are exemplified by aragonite, topaz, gypsum, etc.

Inasmuch as double refraction decomposes a natural beam of light into two perpendicularly polarized beams, it affords a convenient way of procuring polarized light, by merely turning aside one of the two beams. This deflection of the beams is necessary, because otherwise the beams would intermix, and give again unpolarized light. It is accomplished very ingeniously by means of the Nicol's (1841) prism (Fig. 393). This prism is prepared by properly splitting a rhombohedron of Ice-land spar. The natural end surfaces forming an angle of  $71^\circ$  with the blunt lateral edges,  $PH$ , are replaced by new

surfaces, PP, making an angle of  $68^\circ$  with these edges. The prism is now sawed in two, through the section, HH, perpendicularly to the end surfaces, and the surfaces of section, after being polished, are cemented together with Canada balsam. When a natural ray,  $ab$ , impinges upon the front surface, PP, it splits into an ordinary ray,  $bc$ , and an extraordinary ray,  $bd$ . The former, whose index of refraction (1.658) is greater than that of Canada balsam (1.53), impinges so obliquely against the cemented surface that it cannot pass through and out. It



FIG. 393.—Nicol's Prism.



FIG. 394.—Nicol's Prism.

suffers *total reflection*. The extraordinary ray, on the contrary, which is propagated in Iceland spar more rapidly than in Canada balsam, penetrates the latter section, and leaves its back surface in a completely polarized condition along  $def$ , the vibrations being parallel to the principal section, PHP, or parallel to the shorter side of the rhomboidal end surfaces, as indicated in Fig. 394. For rays which vibrate perpendicularly to its principal section, the Nicol's prism is *perfectly opaque*.

The polarizing property of *tourmaline* is, moreover, connected with its property of double refraction. As already indicated, in double refracting crystals both the velocity of propagation and the absorption of the luminous vibrations depend upon the

angle which these vibrations make with the optical axis, so that vibrations perpendicular to this axis suffer a different absorption, and appear, therefore, differently coloured from those parallel to the axis. This property is called *dichroism*. It is so conspicuous in many crystals as to be perceptible at first glance. Pennine, for example, seen in the direction of its axis, appears of a dark bluish green colour, though when viewed perpendicularly to this direction it appears brown; cordierite (*dichroite*) in the direction of its axis looks dark blue, but perpendicularly thereto, it looks yellowish grey. Tourmaline is also a "dichroitic" crystal, in which the vibrations of the ordinary ray perpendicular to the axis are almost completely extinguished by absorption and only the vibrations of the extraordinary ray parallel to the axis are transmitted.

**365. Polarizing Apparatus** are appliances used in studying transparent objects under polarized light. Since an apparatus for polarizing light may also be used for the converse purpose



FIG. 395 - Tourmaline Pincette.

of detecting the presence of polarized light, a combination of two polarizing plates, the first of which furnishes the polarized light, and is termed the *polarizer*, and the second plays the part of an *analyzer* and is called a *polariscope*, constitutes a *polarizing apparatus*. The simplest form of this apparatus is the *tourmaline pincette* (Marx, 1827; Fig. 395). Two tourmaline plates are held by means of disks of cork in such way that they may be turned within rings of wire. By means of a bent wire the plates are pressed gently against each other, so that a small object placed between them is held as by a pincette.

In Noerremberg's apparatus (Fig. 396) a transparent mirror (*AB*) forming with the axis, *Sc*, of the instrument an angle of  $33^\circ$ , forms the polarizer. The light coming, say, from the clouded sky, along the direction *ab*, is deflected downward (*bc*), and then upward by means of a mirror, *c*, imbedded in the base plate, so that after passing through the glass plate, *AB*, it may

fall upon the blackened mirror, *S*, which acts as a polariscope. The mirror, *S*, is supported by two columns upon a ring which moves within a second fixed ring graduated into degrees. Objects to be investigated are placed upon the glass plate, *A*. Either a glass pile, or a Nicol's prism, may be used as polariscope. If it is desired to adapt this apparatus, which in the form just described is specially adjusted for parallel rays, to



FIG. 396.—Noerremberg's Polarizing Apparatus.

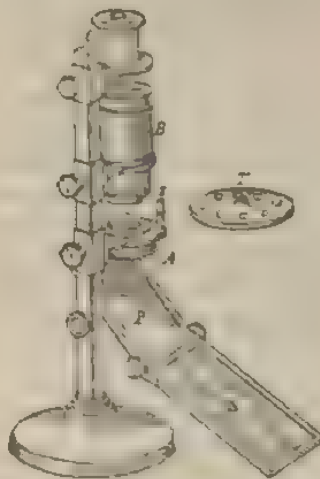


FIG. 397.—Noerremberg's Microscopic Polarizer.

use with a convergent beam of light, a suitable lens must be placed before and behind the object. Instruments have in this way been constructed, which, since they permit the study of very small objects, have been called *microscopic polarizing apparatus*, or *polarizing-microscopes*. Noerremberg's microscopic apparatus (Fig. 397) contains within the casings, *A* and *B*, combinations of lenses between which is placed the object to be observed, e.g. a plate of doubly refracting crystal. The instrument is sometimes provided with a wooden disk, *T*, within whose marginal openings various plates of crystal are placed, the disk being located with its perforated centre upon the pivot, *c*. The

light from the clouded sky is transmitted to the polarizing mirror, T, by means of the mirror, S, in the usual manner, and the Nicol's prism, C, serves as analyzer.

**366. Chromatic Polarization.**—Under this caption are included all those colour phenomena, shown by doubly refracting bodies in polarized light. The transparent crystals of gypsum are



FIG 398—Gypsum Plate.

so that this substance is especially well suited for use in the study of the colours of thin plates under polarized light. When a thin plate of gypsum is placed between the polarizer and the analyzer of a polarizing apparatus, for example, by laying it upon the disk, A, of Noerrenberg's apparatus

(Fig. 396), when not too thick it appears more or less vividly coloured. The colour disappears only in two special positions. When the planes of vibration of polarizer and analyzer are mutually perpendicular, the field of view of the analyzer looks perfectly dark. Inserting now a plate of gypsum, the field instantly appears brightly coloured against a dark ground. By rotating the plate, it may readily be brought to a position where it appears as dark as the rest of the field. This occurs when either the direction, *ab*, or the perpendicular direction, *cd*, viz. the directions of vibration of the two beams arising in the gypsum plate by virtue of its double refraction, coincides with the direction of vibration of the polarizer. On the contrary, it



FIG 399—Resolution of Vibrations.

appears most brightly coloured when these directions form angles of  $45^\circ$  with the direction of vibration. If, therefore, one of these directions is parallel to the direction of the luminous vibrations from the polarizer, this light passes without alteration of its direction of vibration, and is extinguished by the analyzer. But if the direction, *ab*, forms an angle with the direction of vibration, RS (Fig. 399), of the polarizer, the motion directed toward RS must be resolved into two components, one along *ab*, and one along *cd*,

and these components are propagated through the crystal with unequal velocities. Arriving at the analyzer, which transmits only vibrations directed toward PQ, each of these two component vibrations is again resolved into two components, one of which is directed toward PQ, and the other perpendicularly to PQ, along RS. The latter is consequently extinguished. The two component vibrations directed along PQ interfere with each other by virtue of the difference of distances they have travelled, this difference having resulted from their unequal velocities in the crystal. Through this interference, those colours have been sifted out of the incident white light, for which the difference of distance equals an integral number of half wave-lengths. The gypsum plate seen through the analyzer then shows a colour, which is compounded of all those component colours which have vanished through the effect of interference. If the analyzer is gradually rotated from the position PQ, into the position RS, the colour diminishes in brightness, and at  $45^\circ$  it becomes white. Still farther rotation brings out the complementary colour, and in the parallel position (RS) to the planes of vibration this colour attains its highest brilliancy. In this position the vibrations directed toward PQ are extinguished, and the components directed along RS interfere. The latter are, however, like directed when the former oppose each other, and conversely. In the parallel position, therefore, those colours will appear in greatest intensity, which vanished in the crossed position, and the colour of the plate in the one position must be complementary to that in the other.

The composition of the colour at any instant may be ascertained by decomposing with a prism the light which passes through a gypsum plate placed at an angle of  $45^\circ$  between crossed Nicol's prisms. Dark streaks then appear in the spectrum in the places of the colours destroyed by interference. These streaks grow fainter when the second Nicol is rotated, and vanish completely when the principal sections of the Nicols are inclined at an angle of  $45^\circ$  to one another. Turning the second Nicol until it becomes parallel to the first, dark streaks again appear, but now at the places, which in the crossed positions gave greatest brilliancy. The thicker the plate of gypsum

the more dark streaks appear in the spectrum and the more nearly does its interference colour approach white.

All gradations of colour, shown by gypsum plates of varying thicknesses, may be seen at once with a wedge-shaped, or concave gypsum plate. The colours arranged in regular streaks parallel to the edge of the prism, or in concentric circles around the thinnest parts of the plate, exhibit the same sequence as is seen in Newton's colour rings.

The phenomena exhibited by plates of uniaxial crystals cut perpendicularly to the optical axis are particularly interesting when studied in *convergent* polarized light. This may be done, for example, by bringing the plates into a so-called microscopic polarizer (Fig. 397). The ray which strikes the plate perpendicularly passes through in the direction of the optical axis without suffering double refraction. Every other ray of the conical beam, however, is doubly refracted. For the reason

that oblique rays have a longer distance to traverse within the crystal, any given ray will exhibit the phenomena of double refraction the more strongly the more obliquely it traverses the crystal. Thus it occurs that greater differences of distance are met, the further one goes from the axis of the cone and at equal distances from the optical axis, where all circumstances influencing the difference of distance are the same, the same difference and accordingly also the same interference colour must occur. The locus of such points will be the circumference of a circle drawn in the field of view about a point corresponding to the axial ray. A succession of lines of equal colour, or of isochromatic curves, are thus seen as coloured rings (Fig. 400)

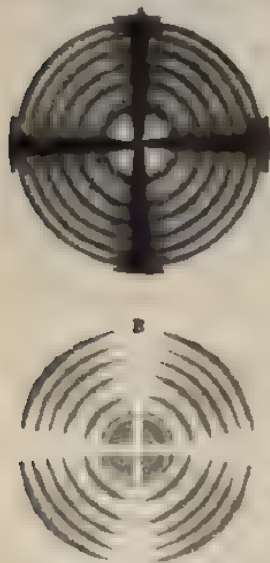


FIG. 400.—Colour Rings in optically Uniaxial Crystals.

The rings appear to be cut through by a black cross (Fig. 400, A) when the planes of vibration of the polarizing apparatus are

crossed. Since now the optical axis is perpendicular to the surface of the crystal, each straight line, PQ, RS, *ab, cd* (Fig. 399), drawn through the centre of the rings corresponds to a principal section. All rays from the polarizer, striking the plate of the crystal, vibrate parallel to RS. They pass, therefore, without decomposition through both principal sections, RS and PQ, with their vibrations parallel to the former and perpendicular to the latter. They are consequently extinguished by the analyzer whose direction of vibration is along PQ. On the other hand, if the direction of vibration of the analyzer is made parallel to that of the polarizer, a white cross (Fig. 400, B) appears in the place of the former black one, and the rings exhibit colours which are complementary to the former. A plate cut from an optically biaxial crystal, with its surfaces perpendicular to the central line of the optical axis, exhibits (Fig. 401) two groups of rings, each of which surrounds an optical axis. The rings of higher order fuse into peculiarly shaped curves (*lemniscates*) enveloping the terminal points of the two axes. When the principal section of the plate passing through the optical axis coincides with one of the two directions of vibration of the polarizer, the double ring system appears to be cut through by a black cross (Fig. 401, A). But when the crystal is turned out of this position the cross dissolves into two dark brushes of hyperbolic form which cut through the rings at right angles (Fig. 401, B).



FIG. 401 — Colour Rings in optically Biaxial Crystals.

By the help of the phenomena of colour in polarized light exhibited exclusively by bodies capable of double refraction, it may be proved that simply refracting bodies, *e.g.* glass, become doubly refracting whenever a strain is produced in them by the application of any sort of force. When a thick square plate of glass is compressed by a small clamp (Fig. 402), in parallel

polarized light (e.g. in Noerremberg's polarizing apparatus, Fig. 396), the glass shows a dark cross with coloured fringes. By strongly heating a piece of glass and then rapidly cooling it



FIG. 402.—  
Compressed  
Glass.

the property of double refraction may be permanently imparted to it. A circular glass plate so treated exhibits coloured rings overlaid by a black cross, precisely as is the case with a plate of Iceland spar cut perpendicularly to the optical axis from its native state. With a square glass plate (Fig. 403) a black cross also appears together with a coloured ring figure in each corner, resembling the eye of a peacock's feather. Since glass cooled too rapidly breaks very easily, a purchaser of glasswares may avail himself of the colours shown in a polarizer in the selection of the most durable articles by discarding all pieces which exhibit colour when tested by

the apparatus. The double refraction of cooled glass as revealed by these colour phenomena, is, however, essentially different from that of crystals. The ring system of a cool glass plate appears even in a *parallel* beam of polarized rays. The



FIG. 403.—  
Colour Phenomena in  
cooled Glass.

differences of distance increasing from the centre outward can, therefore, arise only from the fact that the double refraction with unaltered direction of rays increases toward the edge of the plate. With a crystal, however, the double refraction at each point is the same for the same direction of the rays, and does not change from one point of the crystal to another.

**367. Rotation of the Plane of Polarization**—(*circular polarization, rotatory polarization*).—When a plate cut from a uniaxial crystal perpendicularly to the optical axis is brought into a polarizing apparatus and tested with parallel light (e.g. between two Nicol's prisms), only those alternations of brightness and darkness appear, when the analyzer is rotated, which would have occurred without the plate. This is due to the fact that in the direction of the optical axis no resolution of the vibrations takes place. Rock crystal, or crystallized quartz, however, furnishes an exception to this. A quartz plate of proper thickness cut as

above suggested, appears coloured in the polarizer, and on rotating the analyzer its colours pass successively through red, orange, yellow, green, blue, indigo, and violet (*r, o, y, g, b, i, v*). When the coloured light coming from the analyzer is dispersed by a prism, a dark stripe is seen in the spectrum (or in case of thick plates several stripes) which moves along the spectrum during the rotation, destroying its colours successively. The analyzer, however, can extinguish only vibrations perpendicular to its plane of vibration. In the white light from the polarizer all colours have the same direction of vibration (indicated by an arrow in Fig. 404), and would, therefore, were the quartz plate not present, be wholly extinguished by the crossed analyzer. But in the presence of the quartz plate only one colour can vanish at a time. To extinguish the red rays, if the plate be 3.75 mm. thick, the analyzer must be rotated from the crossed position by  $60^\circ$  and the plate then shows the corresponding complementary green. In the light from the quartz plate the direction of vibration of the red rays must be perpendicular to the present position of the analyzer. The effect of the quartz has, therefore, been such as to rotate the plane of vibration through an angle of  $60^\circ$ , so that it now assumes the position, *rr'* (Fig. 404, upper part). Similarly, it is found that the plane of vibration of the yellow rays has suffered a rotation of  $90^\circ$  (*gg'*), and that of the violet a rotation of  $165^\circ$  (*vv'*). The effect of the quartz is, therefore, shown by a rotation of the plane of vibration of the polarized rays, by amounts differing for the various colours and increasing from red to violet. This separation of the colours according to the directions of their vibrations effects a decomposition of white light into its constituents, i.e. a sort of dispersion, which has received the designation *rotatory dispersion*. For any primary colour the rotation is proportional

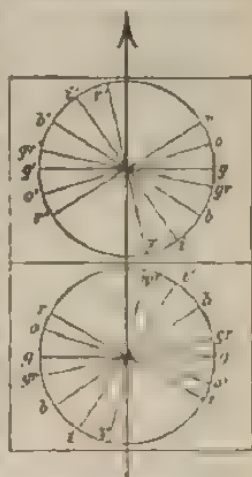


FIG. 404. — Rotation of the Plane of Vibration of Polarized Light.

to the thickness of the plate. Knowing its value, then, for a given thickness, it may be given for any other thickness. For the colours corresponding to the principal Fraunhofer lines, a quartz plate 1 mm. thick produces the following rotations—

B	C	D	E	F	G	H
15°	17°	22°	27	32°	42°	51°

To extinguish the colours in the spectrum successively from red to violet, many rock crystals require a rotation of the analyzer in the direction of the hands of the clock (*dextrorsum*). Others, however, require an opposite rotation to accomplish the same (Fig. 404, lower part). The former are called *dextrogyrate*, or *right-handed*, and the latter, *levogyrate*, or *left-handed* crystals. Both sorts rotate the plane of vibration of the same simple colour by equal amounts with equal thicknesses of plate.

The various colours which appear successively on rotating the analyzer, become simultaneously visible in the following experiment.

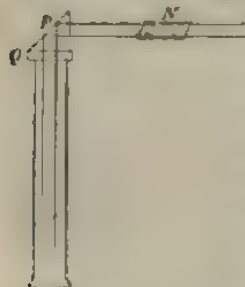


FIG. 405.—Polarized by Diffuse Reflectors.

A sufficient quantity of an alcoholic solution of resin is poured into a cylindrical glass vessel (Fig. 405) containing water to render the water slightly milky. The mouth of the vessel is then covered with a quartz plate, *Q*, and upon this is placed a reflecting prism, *P*, which turns a polarized beam. *NP*, coming from a Nicol's prism, *N*, downward through the quartz plate into the liquid. The diffuse light radiating from this beam is now coloured, and shows simultaneously all the

various colours, which would occur one after the other by rotating the analyzer of the polarizing apparatus. A cloudy liquid exerts a somewhat different polarizing effect. The finely divided particles in the liquid reflect only the light, whose vibrations are perpendicular to the plane of reflection, *i.e.* to the plane determined by the incident and reflected rays. If, therefore, in Fig. 404, the circle represents the cross-section of the cylinder, in any given direction, *e.g.* in the direction indicated by the arrow, only the vibrations perpendicular to this direction are reflected. In the case cited, only the yellow vibrations,

$gg'$ , are completely reflected, and of the remainder only the component directed along  $gg'$ . The same is true of all other directions, so that the clouded liquid shows to the observer a different mixture of colours in every different direction.

To understand the progress of events during the rotation of the plane of vibration in quartz, let us call to mind the circular pendulum (Fig. 406), which traverses the circle,  $BC'B'CB$ , with uniform velocity, provided that, after being drawn from the position of equilibrium,  $A$ , to  $B$ , it receives a suitable impulse in the direction  $Bb$ . Considering a to and fro motion of the pendulum as an entire vibration, the pendulum has just completed a *quarter of a vibration* when it receives the impulse  $Bb$ . It follows, therefore, that two rectilinear vibratory motions at right angles to each other, one of which precedes the other by  $\frac{1}{4}$  of a vibration, compound into a circular vibration. In the case illustrated in the drawing the circular motion is in the direction of the hands of the clock (or toward the right). If the impulse had been imparted in the opposite direction, or if the pendulum had been set in vibration at first toward  $AC'$ , and, then, having reached  $C'$ , an impulse had been given in the direction  $C'e$ , parallel to  $AB$ , the circular motion would have been toward the left. If the impulse had been either more or less powerful than what was assumed above, or if it had been imparted while the pendulum was between  $A$  and  $B$ , the pendulum would have described an *elliptic* path. On the other hand, a rectilinear motion arises when the side-thrust occurs at the instant the pendulum passes through its position of equilibrium,  $A$ , or when the one motion either does not exist at all or precedes the other by an integral number of half-vibrations.



FIG. 406.—Circular Pendulum.

These circumstances of motion may be realized with luminous vibrations by the aid of thin plates of crystals. Mica is

specially well suited to this purpose, since it is easily prepared in very thin sheets. When a thin mica plate is placed in the polarizing apparatus so that the directions of vibration,  $ab$  and  $cd$



FIG. 407.—Resolution of Vibrations.

(Fig. 407), of the rays which are propagated in it with unequal velocities by virtue of its double refraction, form angles of  $45^\circ$  with the direction,  $RS$ , of the polarizer, two equally bright rays will pass outward through the plate, the one of which vibrates in the direction  $ab$ , and the other in the direction  $cd$ . The particle of ether lying at  $(O)$ , on the surface of exit from the plate, like the pendulum, is

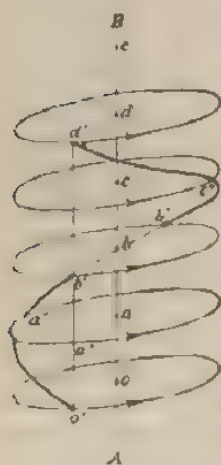


FIG. 408.—Circularly Polarized Ray

possessed of two simultaneous perpendicular motions, and it therefore moves in a circle, ellipse, or straight line, according to the start the one component has of the other. If this advantage amounts to one-fourth of a vibration, as occurs when

one ray precedes the other by virtue of its greater velocity by one-fourth of a wavelength, the particle of ether takes up a circular motion, right or left handed according as the ray vibrating toward  $ab$ , or toward  $cd$ , precedes. This motion is then imparted to the successive particles of ether along the direction of the ray. Each begins its circuit a little later than the other and moves in a circle whose plane is perpendicular to the ray, so that, if at any instant all simultaneous positions of the particles of ether were connected by a curve, a wave line,  $o'a''b''c''d''$  (Fig. 408), would result. This line winds spirally upward about the wave-ray, each wavelength ( $o'd'' = od$ ) corresponding to one complete turn of the spiral. A ray of

such character is said to be *circularly polarized* and in contradistinction to such rays, the polarized rays hitherto considered, whose vibrations are in straight lines perpendicular to

the direction of the ray and in one definite plane through the ray, are said to be rectilinearly polarized. A circularly polarized ray, having the same constitution in all directions, shows no peculiar behaviour in any special direction, as is the case with a rectilinearly polarized ray. It comports itself under the analyzer apparently like an unpolarized ray. But when it is transmitted through a mica plate one quarter of a wave-length in thickness, since the difference of distance of the two waves,  $ab$  and  $cd$  (Fig. 407), which equals  $\frac{1}{2}$  wave-length, is thereby destroyed or made equal to  $\frac{1}{2}$  wave-length, it is transformed into rectilinearly polarized light. Under these same circumstances unpolarized light maintains its character as such.

If a pendulum weight, while at a distance,  $AB$  (Fig. 409), from its position of equilibrium,  $A$ , receives two equal and opposite impulses towards  $Bb$  and  $Bb'$ , each of which combined with the impulse of the pendulum in the direction  $BA$ , would produce a circular motion, the one right-handed and the other left-handed, since the two impulses destroy each other, the pendulum will vibrate back and forth in the straight line  $BB'$ .



Fig. 409 Combined effect of two opposite Circular Vibrations.

If the second impulse had come later, after the pendulum, by reason of the first impulse, had described the circular arc,  $Bb$ , a rectilinear motion along  $rr'$  would have ensued. Applying this conception to luminous vibrations, it is seen that from the combined effect of two opposite circularly polarized rays in other respects similarly constituted, a rectilinearly polarized ray must result, and conversely, that any rectilinearly polarized ray is resolvable into two equally bright opposite circularly polarized rays, or such a ray may be replaced by them. This idea, based on the general laws of motion, would remain without practical significance, were there no bodies which act differently upon right-handed and left-handed circular light. Quartz is such a body. Fresnel explained the rotation of the plane of vibration produced by quartz as due to the different velocities with which opposite circularly polarized rays are

propagated along the axis of the crystal (circular double refraction). A rectilinearly polarized ray must then, on entering a plate of rock crystal, be resolved into two opposite circular rays, which, after traversing the plate with unequal rapidity, reunite on exit into a rectilinearly polarized ray, whose plane of vibration deviates toward the right, or left, of that of the incident ray, according as the right-handed, or the left-handed circular impulse, advanced the more rapidly in the plate, and was the first to set the ether particle at the point of exit into vibration.

The power of rotating the plane of vibration of polarized light resides in but few solids besides quartz, *e.g.* sodium chlorate, cinnabar, and sulphate of strychnine possess this property. On the contrary, it is possessed by many liquids. The following substances rotate the plane of vibration right-handed: German oil of turpentine, lemon oil, alcoholic solution of camphor, water



FIG. 410.—Tube for Liquids.

solutions of sugar cane, of grape sugar, of dextrine, tartaric acid, etc. While French oil of turpentine, water of the bay cherry, water solutions of gum arabic, of quinine, of inuline, of morphine, of strychnine, etc., rotate the plane of vibration to the left. Most oils of ether also possess this capability. Since the rotatory power of liquids is much smaller than that of quartz, to observe it satisfactorily, much thicker layers of them must be used. Tubes (Fig. 410) closed at the ends with plane glass plates are, therefore, filled with the liquid. The amount of rotation increases with the thickness of the layer, *i.e.* with the length of the tube and also with the degree of concentration of the solution. It has been found that, with a tube of 20 cm. length, the rotation for each gram of sugar per 100 cu. cm. of solution equals one and one-third degrees (for sodium light). From the observed angle of rotation the quantity of sugar contained in a given solution, is then readily found. As a means of accurately measuring very small rotations, Soleil's double quartz

plate is used (*double plate*, Fig. 411). It consists of two quartz plates cut perpendicularly to the optical axis and cemented beside each other. One of these plates is right-handed and the other left-handed, and both are 3.75 mm. thick.

With this thickness the yellow rays suffer a rotation of  $90^\circ$  (Fig. 404), and are, therefore, extinguished when the plate is between two parallel Nicol's prisms, so that both parts show the same violet colour. Since in this colour the yellow, to which the human eye is most sensitive, is wanting, the least rotation of one of the Nicol's plates colours the one portion of the plate reddish and the other bluish. On this account the violet colour is called the *transition colour*. If, in addition to the double plate, a tube filled with a solution of sugar is placed between the parallel Nicols, since this solution rotates the plane of vibration to the right, for the *dextrogyrate* part of the plate the rotation is increased, while for the *levogyrate*, it is diminished. On the former

plate the orange colours are destroyed and on the latter the green. The former, therefore, appears more bluish and the latter more reddish. To determine the amount of rotation, one Nicol is turned until in both parts of the plate the same violet colouration is restored. Apparatus designed to determine in this way the constituents of a

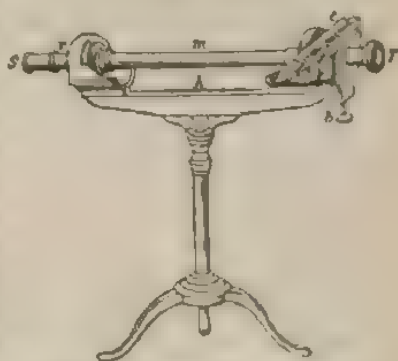


FIG. 412. - Soliel's Saccharimeter

sugar solution are called *saccharimeters*. That of Mitscherlich is the instrument just described. Soliel's saccharimeter (Fig. 412) contains the double plate at *r*, between the two Nicol's prisms, *S* and *T*, whose planes of vibration are fixed parallel to each other. The alteration of colour produced by the tube, *m*, filled with the sugar solution, instead of being equalized by the rotation of the polariscope, *T*, it is adjusted by means of the *compensator*



FIG. 411.  
Double Plate.

at  $e$ . The ray, passing through  $n$ , continues through a right-handed quartz plate,  $Q$  (Fig. 413), and then through two wedges,  $N$  and  $N'$ , cut from a left-handed rock crystal. The wedges may be displaced with reference to each other by means of a milled head,  $b$  (Fig. 412). When the prisms are



FIG. 413.—  
Compensator.

pushed completely together they represent a quartz plate of the thickness of the plate,  $Q$ , and, therefore, in this condition, they neutralize its right-handed rotation. If they are drawn out of this position toward the one side, or the other, the distance through which a ray of light has to pass in both wedges is increased, or decreased. The wedges together form accordingly a left-handed quartz plate, whose thickness may be varied at pleasure within certain limits and which may be made equal to, greater, or less than, the right-handed plate,  $Q$ . The alteration of thickness may be read by the aid of the pointer,  $v$ , from the small scale,  $e$ , to  $\frac{1}{10}$  mm. After having equalized the difference of colour between the two parts of the double plate which was produced by the right-handed rotation of the sugar solution, the reading of the scale indicates the thickness of a quartz plate, which has the same rotatory power as the sugar solution. Since it is known that a sugar solution containing 16.35 grams of sugar per 100 cu. cm. produces in the tube 20 cm. long a rotation just as strong as a quartz plate 1 mm. thick, it is only necessary to multiply the number read off by 16.35 to obtain the weight of sugar contained in 100 cu. cm. When the liquid to be investigated is coloured, the two portions of the plate will exhibit a less satisfactory colour. The apparatus is, therefore, provided with a device consisting of a quartz plate and a prism of Iceland spar by which the most highly sensitive colour is produced. In Soleil's instrument this device is placed upon the ocular,  $T$ , and in the apparatus by Voutzke it is placed in front of the polarizer,  $S$ .

In more recent times the *semi-obscuring apparatus* has come into use as a saccharimeter. The instrument is so called because, instead of producing equal colourations of the two parts of the

field of view, equal obscurations of the parts are effected, and the difficulties attending the discernment of colours are hereby avoided. The *semi-obscuring* saccharimeter of Laurent (Fig. 414) contains as a polarizer a prism, A, of Iceland spar, rotating by means of the lever, B, about the axis of the instrument. The

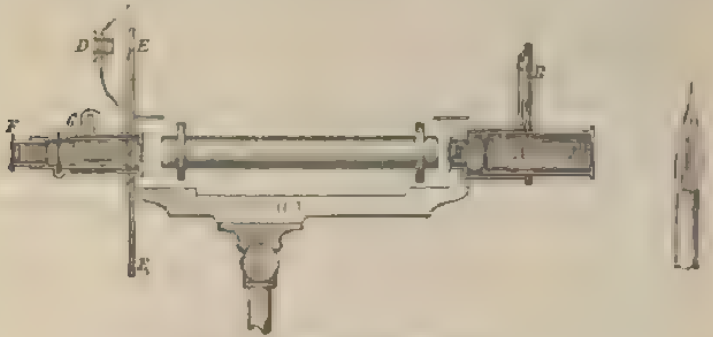


FIG. 414.—Laurent's Saccharimeter.

analyzer is likewise a movable Nicol's prism, C, whose position may be read from a graduated circle, EE, by means of a vernier and microscope, D. The lenses, F and G, form a small telescope, which is directed toward the round opening at H. The left half of this opening is covered by a thin quartz plate, Q (Fig.

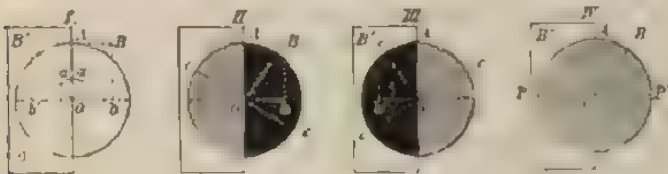


FIG. 415.—For Laurent's Saccharimeter.

415, I.), ground parallel to the optical axis. The thickness of the plate is such that the difference of distance of the two rays produced by double refraction equals half a wave-length of yellow light. The apparatus is illuminated by the yellow light of a sodium flame, which, before impinging upon the

polarizer, must pass through a plate, *J* (Fig. 414), of doubly refracting calcium bichromate, whereby it is deprived of its relatively feeble green, blue, and violet rays, and it arrives at *A* as nearly in the condition of simple yellow light as possible. If now the plane of vibration of the polarizer is in the position, *OB* (Fig. 415, I.), forming with the direction of the axis, *OA*, of the quartz plate an angle,  $\alpha$ , the vibration, *OB*, of the uncovered part of the field on the right may be resolved into the component vibrations, *OA* and *Ob*; for the (left) portion which is covered by the quartz plate, we may substitute the components, *OA* and *Ob'*, the latter of which, on account of the difference of distance of half a wave-length produced by the quartz, is in exactly the opposite phase to that of the vibration, *Ob*. The components, *OA* and *Ob'*, combine in the left portion of the field into the direction, *OB'*, while in the right portion the original direction, *OB*, remains unaltered. If now the plane of vibration of the analyzer is placed in the direction, *Oe*, perpendicular to *OB* (Fig. 415, II.), the right portion of the field is completely obscured, while the left still transmits light. But when the plane of vibration of the analyzer is brought in to the position, *Oe'* (Fig. 415, III.), perpendicular to *OB'*, the left portion becomes dark and the right appears bright. If, finally, the plane of vibration, *OP* (Fig. 415, IV.), is made perpendicular to *OA*, both portions appear equally bright. The latter position corresponds to the zero of graduation, and it is evident that an abrupt change of brightness of the two portions of the field must occur whenever the analyzer is turned out of this position in either direction. If now a tube, filled with a sugar solution and closed at both ends by glass plates, be placed between the opening, *H* (Fig. 414), and the analyzer, *C*, while the analyzer stands at zero, the two portions of the field appear unequally bright, because the sugar solution rotates the two directions of vibration, *OB* and *OB'*, in the same sense (toward the right), and to restore equal brightness the analyzer must be rotated by the same amount. From this angle of rotation the percentage of sugar contained in solution is easily obtained. In practice the graduation of the circle, *EE*, is such as to indicate the quantity of sugar directly.

To determine the angle of rotation for sugar solutions as

well as for other liquids, the *polaristrobometer* of Wild (Fig. 416, model) is used. The tube, *rr*, contains a Savart's polariscope, consisting of two quartz plates, 20 mm. thick, cut at an angle of  $45^\circ$  to the optical axis. The principal sections of the crystals cross at right angles, and form, with the plane of vibration of the ocular Nicol, *a*, an angle of  $45^\circ$ . There are also in the tube two lenses, *l* and *m*, which act as a feebly magnifying astronomical telescope. The arrow indicates the position occupied by the reticle. This part of the apparatus alone reveals the least traces of polarized light, and is therefore called a "polariscope;" for, looking through it at a spot whence polarized

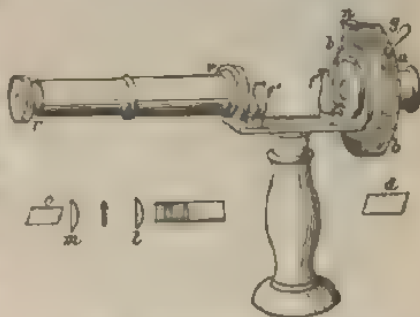


FIG. 416.—Polaristrobometer of Wild.

light proceeds, rectilinear, coloured interference bands are seen, which become more pronounced the more perfectly the individual rays are polarized. Wild's form of the instrument has, at *a*, another Nicol's prism *d*, whose case is fixed within the graduated circle, *bb*, while both case and circle may be turned by means of the handle, *g*, past the fixed pointer, *n*. If the Nicol occupy such position that its plane of vibration coincides with the principal section of the quartz plate, and therefore forms with the ocular Nicol an angle of  $45^\circ$ , the bands are invisible. They reappear, however, immediately when the tube, filled with the liquid, is inserted between the springs, *f* and *f'*. The disk, *bb*, together with the Nicol, *a*, is turned until the bands again disappear, and the rotation may be read from the pointer, *n*. This rotation is equal and opposite to that of the liquid. The setting upon the vanishing position of the bands may be quite accurately made, especially by using the homogeneous light of a sodium flame in a dark room. The saccharimeters, as also the polaristrobometer described above, are used in the manufacture of sugar to determine the percentage of sugar in the sap to be reduced, and also in medicinal practice as a diabetometer

to determine the amount of sugar in the urine of sufferers from diabetes.

**368. Magnetic Rotation of the Plane of Polarization (Faraday, 1845).**—Let a piece (*g*) of Faraday's "heavy glass" (silicious lead borate), or a tube filled with carbon disulphide and closed at the ends with plane glass plates, be brought between two masses of iron which form the poles of a powerful electromagnet, and which are bored through along the connecting line of the poles in the direction, *ad* (Fig. 417)



FIG. 417.—Magnetic Rotation of the Polarizing Plane.

for convenience in observing. When a ray of light, which has been previously polarized at *d* by a Nicol's prism, is sent through the glass, or the tube, no light can be seen through a

second Nicol's prism at *a*, whose principal section is perpendicular to that of the first, so long as the electromagnet is not energized. But when a galvanic current is sent through its coil, and the electromagnet is energized, the field of view becomes bright, and the Nicol at *a* must be rotated through a certain angle to extinguish the light. The plane of vibration of the ray polarized by the glass, under the influence of the magnet, has consequently suffered a rotation equal to that by which the Nicol must be turned to produce darkness. The rotation is shown still more plainly by the unlike colouration of the two portions of a Soleil's double plate (367). The same effect may be produced without a magnet by merely surrounding the piece of glass with windings of wire and passing through them a strong galvanic current. The rotation will be found to occur in the direction in which the current traverses the wire. All transparent bodies, including gases, exhibit this property, though in a lower degree than Faraday's glass. The magnetic rotation is proportional to the component in the direction of the lines of force of the length of the path of the ray in the field, and also to the strength of the field. It increases also with the refrangibility, as is the case with natural rotation.

The plane of polarization is also rotated when the polarized ray is reflected from the surface of a magnetic pole (Kerr).

**369. Other Resemblances of Electrical and Luminous Phenomena.**—Transparent, nonconducting (dielectric) bodies, both solid and liquid, when placed between two oppositely electrified poles, become double refracting (Kerr, 1875), as will be seen from the following experiment. Two metal spheres connected with the electrodes of an inductor, are placed in a horizontal line in a glass trough having parallel walls, and filled with carbon disulphide. A ray of light, polarized by a Nicol, with its plane of vibration inclined at 45° to the horizontal, is passed between the spheres. So long as the two spheres remain unelectrified this ray will be extinguished by a second Nicol, whose plane of vibration is crossed with that of the first. But when the apparatus is put in operation, the spaces between the spheres again appear bright upon a screen against which the image of the trough is projected by a lens. The reason for this is that the vibrations from the polarizing Nicol are resolved parallel and perpendicularly to the connecting line of the poles, as occurs with glass when it is compressed or stretched in this direction.

If the spherical electrodes of a sparking inductor, or of an influence machine, are separated so far that no sparks pass, discharges will occur immediately on illuminating the *negative* electrode by light, such as the electric arc light, or the magnesium light, which is rich in waves of short wave-length, and especially in ultra-violet rays (Hertz, 1887). This *light electrical* discharge, in which a negatively charged conductor rapidly loses its charge when illuminated with the more refrangible rays and the positive conductor does so only gradually, is shown by the following experiment (given by Elster and Geitel). A wide glass tube, terminating in bulbs and almost completely exhausted of air, contains a little mercury, in which sodium is dissolved (sodium amalgam). The amalgam which, before the experiment was in the blackened bulb, is transferred into the other bulb by inclining the tube. The latter is transparent. The tube is then clamped horizontally, so that the platinum electrodes which are tused into it

lie, the one above and the other below. The lower electrode, which is coated with the amalgam, is connected by wires with a gold-leaf electroscope and with the negative electrode of a Zamboni's pile; while the upper electrode is connected either with the positive pole of the pile, or directly with the ground. The amalgam, and the electroscope connected with it, are then charged with negative electricity, and the gold leaves diverge so long as the transparent bulb is shaded. But they collapse immediately when the amalgam surface is illuminated with electric, or solar light, or even with bright daylight, to diverge again, however, so soon as the bulb is shaded.

Since the illumination causes, or, rather, accelerates the passage of the negative electricity from a conductor, an insulated unelectrified conductor must become positively charged when illuminated with rays of high refrangibility (light electrical excitation).

### 370. Electrical Vibrations—Electro-magnetic Theory of Light.

—As has been mentioned above (188), the discharge of a condenser, *e.g.* of a Leyden jar, is *oscillatory* in its character, *i.e.* it takes place through the agency of *electrical vibrations*. The electricity flowing at the beginning of the discharge from the one coating to the other, awakens, by self-induction (264) in the circuit itself, an opposite extra-current, which charges the condenser oppositely, and thereby causes a renewed discharge in the opposite direction, which action is repeated indefinitely. This action takes place very rapidly, so that in the short duration of a spark discharge, a large number of such oscillations to and fro, or of electrical vibrations, follow each other with diminishing intensity. The duration of a vibration, *i.e.* the time ( $T$ ) of an oscillation to and fro, depends upon the capacity ( $C$ ), and upon the coefficient of self-induction ( $L$ ) of the circuit ( $T = 2\pi \sqrt{CL}$ ). A sparking inductor may be used to produce such vibrations, by connecting its electrodes through slender wires with two brass bars facing each other in a straight line (*i.e.* with the "primary conductor"), between the bulbed ends of which the sparks pass. The capacity of the bars may be augmented by connecting them with metallic surfaces. In this way vibrations may be obtained whose

duration amounts to only a few hundred-millionths of a second. These induce likewise, in a second closed conductor, electrical vibrations, whose existence is revealed by the passage of a current of small sparks, when this "secondary conductor" is broken by a thin layer of air. When the primary conductor is placed against a metal wall, the secondary, in the immediate neighbourhood of the wall, shows no spark. But, when removed from the wall toward the primary conductor, sparks occur, and attain their greatest vigour at a definite distance. Beyond this distance, the sparks gradually diminish in intensity, and disappear completely at double the distance, then rise to a second maximum at three times the distance, and so on. The experiment shows that the inducing action is propagated through space in waves, and, consequently, that time is consumed in the propagation; otherwise, it would be incomprehensible that, between the wall and the primary conductor, both the incident and the reflected waves could produce by interference a condition with nodes and regularly alternating antinodes, in a word, a stationary wave motion. Since the half wave-length is the distance between two adjacent nodes (a few centimeters to several meters), and the duration of vibration (consequently also the frequency), may be computed from the dimensions of the primary conductor, the velocity of propagation readily results as the product of wave-length into frequency, and equals 300,000 km. The velocity of propagation of these "rays of electrical force" is, therefore, equal to that of light rays (Hertz, 1888).

Let the primary conductor be now brought into a position, such that the direction of its length lies in the caustic curve of a large cylindrical concave mirror of sheet zinc. Moreover, let the inductor be placed behind the mirror, and the conducting wires pass insulated through its walls. The rectilinear secondary conductor is now brought into the caustic curve of a second equal concave mirror, so that the two wires leading to the place of the spark pass insulated through the mirror. The spark is produced behind the mirror for the sake of convenient observation. If the two mirrors exactly face each other (at a distance of 6-10 m.), small sparks are seen to pass a sphere

and a point at the breaking point of the secondary conductor between. The sparks immediately cease when the second mirror is slightly displaced to one side. Hence it follows that electrical rays emitted by a primary conductor are reflected parallel to one another, exactly as are rays of light, by the first mirror and are collected in the caustic of the second, when the optical axes of the mirrors coincide. The sparking also ceases when a screen of conducting material is brought between the facing mirrors. Nonconducting (dielectric) bodies, on the contrary, do not stop the passage of the rays. They pass directly through a wooden wall, or door.

The occurrence of the small sparks may be exhibited to a wider circle of spectators as follows (Boltzmann). The ball of the secondary sparking-point is connected with a gold-leaf electroscope, and the point with one pole of a Zamboni's pile, the other pole of which is connected with the earth. So long as no sparks pass, the electroscope remains uncharged. Instantly



FIG. 418.—  
Zehnder's Tube.

the sparks begin to appear, however, they form a conducting bridge between ball and point, and the gold leaves diverge. Zehnder's method was to place the sparking point of the secondary conductor between two electrodes,  $H_1$  and  $H_2$  (Fig. 418), lying near each other in a Geissler tube and nearer the negative,  $K$ , of the two principal electrodes,  $A$  and  $K$ , which also project into the tube. These are connected with the poles of an accumulator of 600 elements, whose potential is so regulated that the current of the accumulator passes, and the tube lights up, only when sparks appear between  $H_1$  and  $H_2$ , and thereby diminish the resistance of the gas.

That the electric rays are formed of transverse vibrations parallel to the length of the primary conductor, and, accordingly, in an optical sense, are rectilinearly polarized, is at once evident from the manner of their origin. Special proof is also furnished by the following experiments: Turning the receiving mirror about a beam of rays as axis until its caustic is perpendicular to that of the former, the secondary sparks grow

gradually weaker, and vanish completely in the crossed position. The mirrors are, therefore, related to each other as polarizer and analyzer. Furthermore, let a wooden frame, with parallel copper wires stretched across it, be placed perpendicularly to the beam between the mirrors, whose caustics are parallel to each other. When the wires lie perpendicularly to the caustics, the frame works practically no detriment to the passage of the secondary sparks, but, when its wires are parallel to the caustic, the rays are completely quenched. The wire screen has then an influence upon these rays similar to that of a tourmaline plate upon a rectilinearly polarized ray of light.

To prove that the electrical rays are refracted on passing from air into another dielectric, Hertz placed a large prism of hard pitch (1·5 m. in height, 600 kg. in weight) between the mirrors. The second mirror then had to be turned laterally into a definite position, to produce secondary sparks, whence it follows that the electrical beam of rays from the first mirror was deflected by the prism.

The "rays of electrical force" follow, then, the same laws of propagation, reflection, and refraction as do light rays. The hypothesis is, therefore, unavoidable, that both are to be regarded as motions of one and the same medium, whether that medium be termed "ether" or not. Electrical rays may be said to be light rays of very great wave-length, or that light rays are electrical rays of very short lengths, which is the view taken by *Maxwell's electromagnetic theory of light*. The remarkable relationship resulting from Maxwell's theory, that for transparent nonconductors the *dielectric constant equals the square of the index of refraction*, has been experimentally verified in numerous cases.

**371. The Eye.**—The human eye is enveloped by a firm, tough membrane (*sclerotica*, commonly called the "white of the eye." In front of the ball of the eye is a round, transparent window, resembling a small watch-glass, and called the *cornea*. Behind the cornea is the *iris*, the coloured part of the eye, stretched like a curtain, and having at its centre a dark, circular aperture, called the *pupil*. Through the pupil, the light is admitted into the eye. Close behind the pupil, is a

lens composed of several transparent layers, called the *crystalline lens*, *dd* (Fig. 419). The *anterior chamber*, bounded by the *retina* and the crystalline lens, is filled with a transparent *aqueous humour*, and the larger, *posterior chamber*, contains a transparent mass, resembling the white of an egg, the so-called



FIG. 419.—Eye.

*vitreous humour*, or *vitreous body*. The inner wall of the posterior chamber has a double coating, the *choroid*, lying next within the *sclerotica*, and above this the *retina*, the latter being merely a continuation of the *optic nerve*. Where the optic nerve enters the retina, the latter is insensitive to light, and this place is, therefore, termed the *blind spot* (*punctum cecum*). The action of the eye is similar to that of a small camera obscura. The crystalline lens forms upon the retina, small inverted images, *ab*, of external objects, *AB*. These images are for a brief time fixed photographically, by the decomposing action of this light, upon the *visual purple*, or *rhodopsin* (Boll. Kuehne, 1876), a purple colouring matter, covering the retina, and readily bleached by light. These images, however, soon disappear because this colouring matter is renewed rapidly from the circulation of the blood. The stimulus produced by this image is transmitted by the optic nerve to the brain, the seat of conscious-

ness. The *yellow spot* lies almost at the centre of the retina, and in the middle of this is a depression called the *fovea centralis*. Only that portion of the retinal image which falls upon the yellow spot is sharply defined. All other portions are seen more indistinctly the farther they lie from the fovea centralis. The line connecting the fovea with the centre of the cornea is called the *axis of the eye* (*visual axis*). In order that the image of a point shall fall upon the yellow

spot, and, accordingly, be distinctly seen, the axis of the eye, or the line of sight, must be directed toward the point. Every straight line,  $aA$ ,  $bB$ , connecting a point of the image with the corresponding point of the object, is called a *line of direction*. All lines of direction intersect at a point upon the axis of the eye lying close in front of the back surface of the crystalline lens, *crossing-point*, in which the principal points and nodes of the lens-system, formed by the refracting media of the eye, almost coincide. The angle formed by the lines of direction drawn from the external points,  $A$  and  $B$ , of the object is called the *visual angle*. By means of it the size of the retinal image, and, accordingly, also the *apparent size* of the object, are judged.

**372. The Reduced Eye.**—According to Listing (1845), the real eye so far as regards its effect, may be reduced to a single refracting surface, having a coefficient of refraction of 1.34 and a radius of curvature of 5.2 mm. The vertex of the surface must be 1.3 mm. behind the vertex of the cornea. Its second focal distance (the distance of the centre of the retina from the vertex of the surface) must be 20.1 mm., and the first focal distance 15.2 mm. The centre of curvature of this reduced eye very nearly coincides with the crossing-point of the rays in the real eye.

**373. Accommodation.**—A normal (*emmetropic*) eye is so constructed that, in a state of repose, the images of very remote objects (*e.g.* of the stars) are produced upon the retina. But through the action of a certain muscle of the eye the crystalline lens may be more sharply curved and pushed slightly forward, so as to *accommodate* the eye to smaller distances. The nearest point for which this accommodation is possible is about 10 to 15 cm. from the eye, and it is called the *nearest point of distinct vision*. In a *short-sighted (myopic)* eye the axis of the ball is too long from the front to the rear. Rays coming from a remote point unite in front of the retina, and produce upon the latter a blurred dispersion circle, instead of a distinct punctual image. The *longest distance* for which such an eye can accommodate itself is called its *greatest distance of distinct vision*. Eyes whose axes are too short from front to rear are *long-sighted (hypermetropic)*. They focus the rays from distant points behind the retina, and produce, therefore, upon it, a circle of dispersion. They are, however, capable of focussing a convergent beam upon the

retina. All these difficulties are removed, or compensated, by means of *spectacles*. By the use of a concave lens, which spreads out the rays, the short-sighted eye is rendered capable of seeing at a distance. The long-sighted eye requires for distinct vision, at a short distance, a convex lens which converges the rays. In old age all eyes may lose the power of adaptation for near objects by relaxation of the muscles and become *far-sighted* (*presbyopic*), so that even a normal eye then requires a convex lens for vision at a short distance. The distance



FIG. 420.  
Measurement  
of Distance of  
Distinct Vision.

between the nearest and farthest points of distinct vision (the latter with a normal eye lies at an infinite distance) is called the *range of accommodation*. Within this range lies the point whose distance from the eye is called the *distance of distinct vision*. By the latter is meant the distance at which ordinary type can be read most conveniently. It is about twenty-five cm. for the normal eye. To ascertain the limits of distinct vision, let one end of a white thread be fastened above to a black background, and the other end be placed against the lower eyelid, while the eye looks along the stretched thread. The eye will see the thread in the form, A or B (Fig. 420), according as it is short, or long-sighted. At *o* the thread appears broadest and quite indistinct, tapers then to the point, *a*, which may be easily indicated with the finger, where it is smallest and most distinct. With short-sightedness, it runs from *a* (nearest point of distinct vision) to *a'* (farthest point of distinct vision) slender and distinct (A). For a long-sighted eye, however, it continues slender and distinct from *a*, entirely

to the end (*Ohm*).

When a slender object, as, for example, a pin, is observed through two small apertures, at a distance less than the diameter of the pupil, it appears single, only in case the eye is accommodated to it. It appears double at all other distances, because the beams passing through the apertures intersect

in front of, or behind, the retina (Scheiner, 1619). This experiment may be performed objectively by means of a lens, in front of which two slits are placed.

**374. Binocular Vision.**—To see a point *singly* with both eyes, it is necessary that the lines passing from it through the nodes of the eyes shall meet either the yellow spot itself, or *corresponding points* of the two retinas, *i.e.* points situated equally distant from the yellow spots and on the same side of it. Binocular vision very materially assists the judgments of vision of near objects. With the right eye we see a near object against a different part of the background from that against which the left eye projects the object, and these points draw more closely together the farther the object is from the eye. By long experience we learn to judge of the distance of an object unconsciously from the distance between these points of projection. For example, it is quite difficult to thread a needle, or to touch the upwardly-directed tip of the index finger of the one hand held at arm's length with the downwardly directed tip of the other index finger from above, when one of the eyes is closed. Both of the difficulties disappear, however, when both eyes are open.

**375. Stereoscope.**—Binocular vision has the further advantage of enabling us to perceive objects as extended bodies and not as surfaces, as is the case with monocular vision. With the right eye, an extended object (*e.g.* a truncated pyramid, Fig. 421) is seen somewhat more from the right (R), and with the left, a little more from the left side (L). By making these images fuse in consciousness into a single impression, we obtain the impression of the extendedness of the object. If now, a plane drawing of a body is presented to each eye as the object would have been imaged upon the retina, and the images of the drawings are made to fall upon corresponding places of the retinas, the two impressions must combine into the single composite impression, which would be produced by the object itself, when viewed immediately, and with both eyes. Wheatstone combined these



FIG. 421. — Stereoscopic Images of a Truncated Pyramid

images by means of a reflecting stereoscope (Fig. 422). This consists of two mirrors,  $ab$  and  $ac$ , placed at right angles to each other. The observer looks with the left eye,  $l$ , into the former, and with the right eye,  $r$ , into the latter mirror. The drawings of the object for the left and the right eye respectively are placed at the sides,  $d$  and  $e$ . The rays proceeding from corresponding points of the drawings are now reflected by the mirrors so as to make them appear to emanate from a single point,  $m$ , behind the mirrors. The rays fall upon corresponding places of the retinas, and the observer receives the impression of an extended body situated at  $m$ . A more convenient, and therefore more extensively used, instrument is the *Brewster refracting stereoscope* (Fig. 423).



FIG. 422.—Wheatstone's Stereoscope

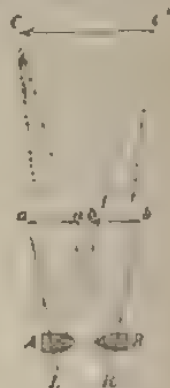


FIG. 423.—Brewster's Stereoscope

The two halves of lenses,  $A$  and  $B$ , are turned with their thick ends outward and fastened in the upper surface of a small casing, at the bottom of which are placed the two views,  $aa'$  and  $bb'$ , of the same object, mounted beside each other upon a card. These views are obtained by photographing the object from two different positions, the camera obscura playing in one position the *role* of the left eye, and in the other, that of the right. It is evident from the figure that the refraction of the light in the half lenses brings the two views to overlap and melt into a single impression. By the aid of photography, the stereoscope is capable of reproducing to the eye, statues,

monuments, and views of various kinds with remarkable truth to nature.

**376. Persistence of Retinal Impressions.**—The production of a luminous sensation in the eye requires time both to form and to disappear (about  $\frac{1}{4}$  of a second). A flying musket-ball is not seen. A glowing splinter swung in a circle has the appearance of a closed circle of fire. The spokes of a rapidly revolving wheel cannot be distinguished, and the surface of a disk painted with alternately black and white sectors, appears uniformly grey, when rotated rapidly upon a centrifugal machine. When the sectors are painted with colours resembling as closely as possible the spectral colours, the disk appears of a greyish-white colour when rotated rapidly, since all colour-impressions mix at each point of the image on the retina (*colour-disk*). If a disk be twirled rapidly by means of two cords attached at two diametrically opposite points of its circumference about this diameter so that the sides are presented alternately to view, drawings upon both sides are seen simultaneously. For example, if upon one side a dark band is drawn in the direction of the cord, and upon the other a similar band perpendicularly to the cord, a black cross will be seen; or if upon one side a bird is painted, and a cage upon the other when the disk is rapidly rotated, the bird appears to be in the cage (*Thaumatrope*).

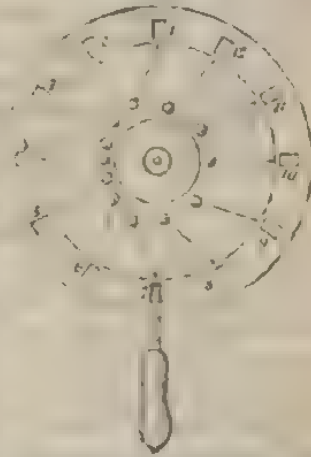


FIG. 424.—Stroboscopic Disk

**377. The Stroboscope** (*phœnakistoscope*, *phantoscope*) consists of a disk of pasteboard (Fig. 424) through the circumference of which a number of holes, e.g. twelve, are cut. Upon this a second smaller disk is fastened, upon which an object, e.g. a pendulum, is painted in as many successive positions as there are holes. When this disk is turned with its painted surface toward a mirror, on looking through one of the openings

(e.g. the uppermost) into the mirror, while the disk is rapidly rotated, one image after another is seen as the apertures pass successively before the eye, and each succeeding image follows so quickly after the preceding, that the impression produced by the former persists until the following takes its place. These images of the successive positions, passing the one into the other, produce beneath the upper opening the appearance of a vibrating pendulum. Since each image of the disk is effaced by the one next succeeding it, not only the uppermost image, but all the rest as well seem to be in vibration. Inasmuch as each pendulum apparently passes its position of equilibrium a little later than the preceding, the impression of a wavelike motion propagated through the row of pendulums is produced. This phenomenon may be made visible simultane-

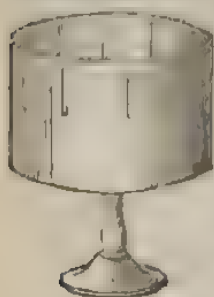


FIG. 425.—Stroboscopic Cylinder.

ously to a number of spectators, by condensing a bright beam of light (sunlight, electric, or Drummond's light) turned against the back surface of the disk by means of a lens upon one of the peripheral apertures and by reflecting, then, the cone of rays emerging from the aperture upon the front surface of the disk. The *stroboscopic cylinder* is another form of the phenakistoscope (*wheel of life*, *zoetrope*, FIG. 425). This consists of an open cylinder, which can be rotated about its vertical

axis. At its top are twelve slits. Twelve images of the moving objects are drawn upon a strip of paper in twelve consecutive positions. The slip of paper is then coiled within the cylinder beneath the slits close against the wall. This apparatus dispenses with the mirror, and has the advantage of allowing several persons to view the figures at the same time through the slits from different directions. Not only may wave motions be represented in this way, but also, by the aid of instantaneous photography, the successive phases of the movements of men and animals; the latter may be reproduced in realistic form, thus making the stroboscope, especially in its latter form, a favourite toy.

**378. Irradiation.**—The apparent alteration of size of bright objects seen against a dark ground, or of dark objects upon a bright ground, is called *irradiation*. For example, the lunar crescent appears to belong to a larger circle than does the portion of the moon rendered visible by the *earth shine*. Irradiation is explained by the circumstance that intense luminous impressions spread out upon the retina beyond the positions at which they are directly formed.

**379. Pseudoscopic Phenomena** arise from the involuntary deception attaching to eye estimates which bias the judgment concerning the forms and sizes of objects. The majority of them are due to preconceived opinions formed unconsciously. Gradu-

ated magnitudes appear more extended than ungraduated, because a greater extent of space is involuntarily demanded for an aggregation of observed individuals. The graduated half of the straight line (Fig. 426) for this reason appears the longer, and of the two equal squares of Fig. 427, the one ruled vertically appears the broader, and the horizontally ruled one, the higher. For the same reason, the distance from the observer to the horizon looks greater than that to the zenith, and, hence, we ascribe to the apparent celestial vault the form of a

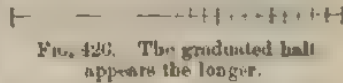


FIG. 426. The graduated half appears the longer.



FIG. 427.—a appears too broad ;  
b, too long.



FIG. 428.—a appears greater  
than b.

flattened hemisphere. When near the horizon, where numerous terrestrial objects intervene, the moon and sun seem more remote, and, therefore (since the visual angle of these bodies remains the same), greater than when near the zenith. Fig. 428 furnishes a simple example of another series of pseudoscopic phenomena. Of the two equal circular segments, the right, a, toward which the upper and lower boundary lines converge,

appears greater than  $b$ , because the observer unconsciously expects a contraction toward this side, and he is thereby led to believe that the figure lying on this side is greater than is actually the case. This deception is again repeated in the series of equal trapezoids (Fig. 429), whose bases,  $ac \dots bd$ , apparently increase from  $a$  toward  $b$ . Imagining the straight lines,  $ab$  and  $cd$ , to be drawn, they appear to diverge in the direction  $ab$ ,



FIG. 429. — Each Trapezoid from  $a$  toward  $b$  appears greater than the preceding.

although in reality they are parallel. In the same way the effect of Fig. 430 is explained (Zoellner's illustration), which is merely a repetition of Fig. 429, leaving out the parallel

sides of the trapezoids, and adding the straight lines,  $ab$  and  $cd$ . Though these lines are actually parallel, they appear to diverge in the direction toward which the cross lines converge. The pseudoscopic phenomena of motion are allied with these. Looking

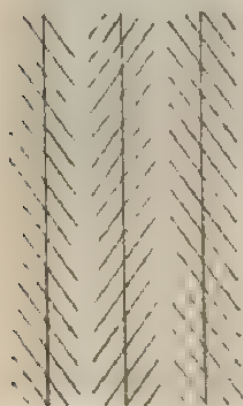


FIG. 430. The Vertical Parallels appear alternately Convergent and Divergent.

from a bridge, for example, into rapidly flowing water, and then glancing at the bridge, or other stationary object, these objects appear to move in the opposite direction. Oppel's *antirrheoscope* may be used to produce this deception artificially. The apparent reversion of relief, especially distinctly perceptible when the illumination of an object is opposite to what the observer supposes, is also a pseudoscopic phenomenon. A coin, for example, looks like a depressed seal when the light of the window is screened away from it, and it is illuminated from the opposite side by a mirror not visible to the observer. Under the same circumstances a cylinder looks like a trough,

a cigar like a hollow tobacco leaf, and the lining of a hat appears to stand outside of the hat (*Wheatstone's pseudoscope*, *Oppel's anaglyphoscope*).

**380. Perception of Colours.**—Although the elementary

colours are individually comparable to simple tones, our *mode* of perceiving colours is totally different from our mode of perceiving tones. While in a mixture of sounds the ear perceives its simple constituents, and recognizes sounds composed of different tones immediately as different, the eye cannot distinguish the white, produced, for example, by mixing simple blue with the complementary simple yellow, from the white produced by three, or more, or all of the spectral colours. It is possible to produce from three properly chosen simple colours, in addition to white and black, all perceptions of colour that exist, and the mixture obtained may be again replaced by a single homogeneous colour producing the same impression. These facts have led to the hypothesis, that the infinitely numerous rays of the spectrum are capable of exciting in the eye only a few different colour perceptions. According to the theory of Young (1807) and Helmholtz (1867), the retina contains three species of nerves, viz. those susceptible to red, green, and violet, all of which, in different degrees, are excited by any homogeneous light ray. According to the theory of Hering, the various kinds of light are capable of exciting three *perception-pairs*: white-black, red-green, and yellow-blue, which stand to each other in such relations of contrariety, that, while the white-perception, for example, consumes a certain material, the black-perception replaces it.



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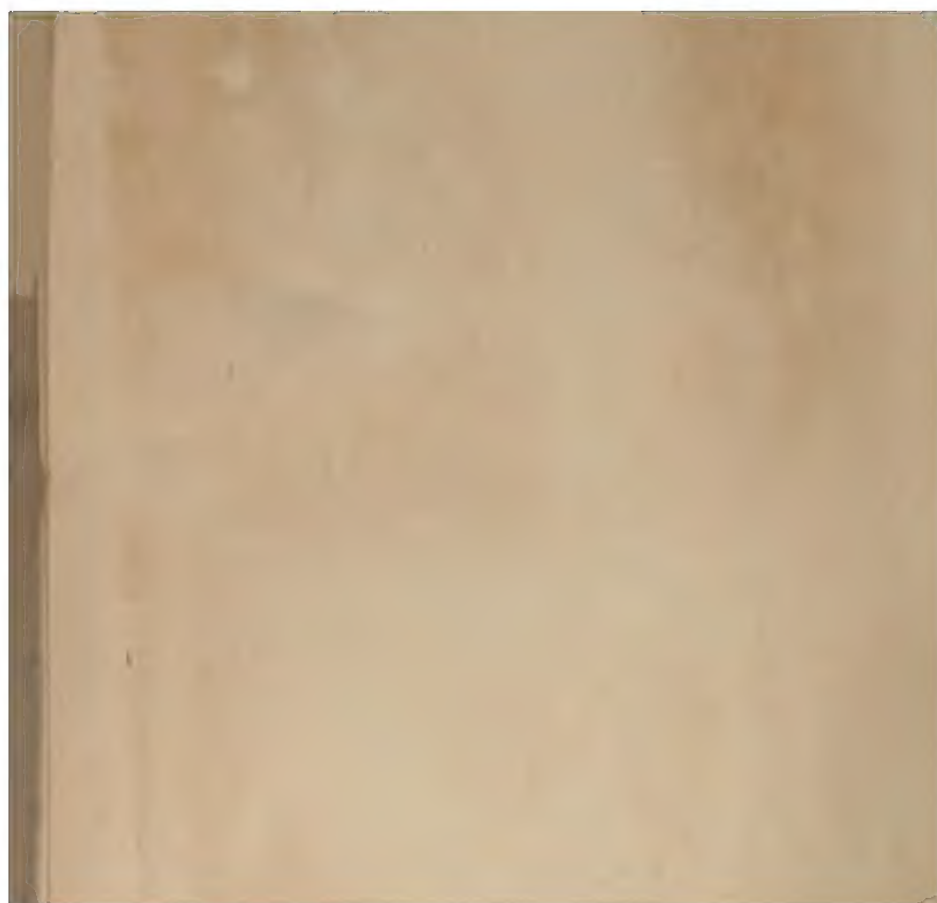
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